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Propagation of Elastic Waves at Solid/Solid Interface

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Abstract
In this paper, the reflection and transmission of plane waves from a plane surface separating a micropolar elastic solid half space and a fluid saturated porous solid half space is studied. Longitudinal or transverse waves impinge obliquely at the interface. Amplitude ratios of various reflected and transmitted waves are obtained and computed numerically for a specific model and results obtained are depicted graphically with angle of incidence of incident wave. It is found that these amplitude ratios depend on angle of incidence of the incident wave and material properties of medium. A special case when fluid saturated porous solid half space reduced to empty porous solid half space has also been deduced and discussed from the present investigation.
Keywords: Porous solid, micropolar elastic solid, reflection, transmission, longitudinal wave, transverse wave, amplitude ratios, empty porous solid.
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1. Introduction

Most of natural and man-made materials, including engineering, geological and biological media, possess a microstructure. The ordinary classical theory of elasticity fails to describe the microstructure of the material. To tackle this problem, Eringen and Suhubi (1964) developed a theory in which they considered the microstructure of the material and they showed that the motion in a granular structure material is characterized not by a displacement vector but also by a rotation vector. Gautheir (1982) found aluminum-epoxy composite to be a micropolar material. Many problems of waves and vibrations have been discussed in micropolar elastic solid by several researchers. Some of them are Tomar and Gogna (1992), Tomar and Kumar (1995), Singh and Kumar (2007), Kumar and Barak (2007) etc.
In the case of bodies with definite internal structure i.e. sand, fissured rocks, cemented sandstones, limestone's and other sediments permeated by groundwater or oil (i.e. for porous materials), the existing theories needs to be upgraded. Bowen (1980) and de Boer and Ehlers (1990a, 1990b) developed an interesting theory for porous medium having all constituents to be incompressible. There are sufficient reasons for considering the fluid saturated porous constituents as incompressible. For example, consider the composition of soil in which the solid constituents as well as liquid constituents which are generally water or oils are incompressible. Therefore, the assumption of incompressible constituents meets the properties appearing the in many branches of engineering.
Based on this theory, many researchers like de Boer and Liu (1994), Liu (1999), de Boer and Didwania (2004), Tajuddin and Hussaini (2006), Kumar and Hundal (2007), Kumar et.al. (2011), Kumari (2013, 2014) etc. studied some problems of wave propagation in fluid saturated porous media.
Using the theory of de Boer and Ehlers (1990) for fluid saturated porous medium and Eringen (1968) for micro polar elastic solid, the reflection and transmission phenomenon of longitudinal and transverse waves at an interface between micropolar elastic solid half space

and fluid saturated porous half space is studied. A special case when fluid saturated porous half space medium reduced to empty porous solid half space medium has been deduced and discussed from the present investigation.

2. Formulation of the problem

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and the plane interface z=0 separating the fluid saturated porous half space M_1 $[z > 0]$ and micropolar elastic solid half spaceM_2 [z<0]. A longitudinal wave or transverse wave propagates through the medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to incident longitudinal or transverse wave, we get two reflected waves in the medium M_1 and three transmitted waves in medium M_2. See fig.1

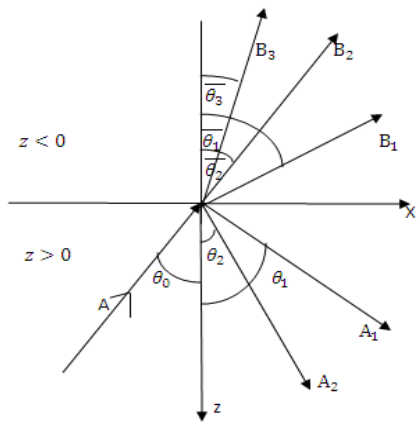


Fig.1 Geometry of the problem.

3. Basic equations and constitutive relations

3.1. For medium M_2 (micropolar elastic solid)

The equation of motion in micropolar elastic medium are given by Eringen (1968) as

$$(c_1^2 + c_3^2)\nabla^2\phi = \frac{\partial^2\phi}{\partial t^2}, \tag{1}$$

$$(c_2^2 + c_3^2)\nabla^2\vec{U} + c_3^2\nabla\times\vec{\Phi} = \frac{\partial^2\vec{U}}{\partial t^2}, \tag{2}$$

$$(c_4^2\nabla^2 - 2\omega_0^2)\vec{\Phi} + \omega_0^2\nabla\times\vec{U} = \frac{\partial^2\vec{\Phi}}{\partial t^2}, \tag{3}$$

Where

$$c_1^2 = \frac{\lambda+2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}, \quad c_3^2 = \frac{\kappa}{\rho}, \quad c_4^2 = \frac{\gamma}{\rho j}, \quad \omega_0^2 = \frac{\kappa}{\rho j} \tag{4}$$

Parfitt and Eringen (1969) have shown that equation (1) corresponds to longitudinal wave propagating with velocity $\bar{v}_1$, given by $\bar{v}_1^{\,2} = c_1^2 + c_3^2$, and equations (2)-(3) are coupled equations in vector potentials $\vec{U}$ and $\vec{\Phi}$ and these correspond to coupled transverse and micro-rotation waves. If $\frac{\omega^2}{\omega_0^2} > 2$, there exist two sets of coupled-wave propagating with velocities $1/\lambda_2$ and $1/\lambda_3$, where

$$\lambda_2^2 = \frac{1}{2}\left[B - \sqrt{B^2 - 4C}\right], \quad \lambda_3^2 = \frac{1}{2}\left[B + \sqrt{B^2 - 4C}\right], \tag{5}$$

where

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2{}^2 + c_3{}^2)} + \frac{1}{c_4{}^2} \quad , \qquad C = \left(\frac{1}{c_4{}^2} - \frac{2q}{\omega^2}\right)\frac{1}{(c_2{}^2 + c_3{}^2)} \quad ,$$
$$p = \frac{\kappa}{\mu + \kappa} \quad , \quad q = \frac{\kappa}{\gamma} \tag{6}$$

Considering a two dimensional problem by taking the following components of displacement and micro rotation as

$$\vec{u} = (u.0.w), \qquad \vec{\Phi} = (0, \Phi_2, 0), \tag{7}$$

where

$$u = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z} \quad , \qquad w = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x} \quad , \tag{8}$$

and components of stresses are as under

$$t_{zz} = (\lambda + 2\mu + \kappa)\frac{\partial^2\phi}{\partial z^2} + \lambda\frac{\partial^2\phi}{\partial x^2} + (2\mu + \kappa)\frac{\partial^2\psi}{\partial x\,\partial z} \quad , \tag{9}$$

$$t_{zx} = (2\mu + \kappa)\frac{\partial^2\phi}{\partial x\,\partial z} - (\mu + \kappa)\frac{\partial^2\psi}{\partial z^2} + \mu\frac{\partial^2\psi}{\partial x^2} - \kappa\Phi_2 \quad , \tag{10}$$

$$m_{zy} = \gamma\frac{\partial\Phi_2}{\partial z} \tag{11}$$

3.2. For medium M_1 (fluid saturated incompressible porous solid half space)

Following de Boer and Ehlers (1990b), the governing equations in a fluid-saturated incompressible porous medium are

$$\mathrm{div}(\eta^S\dot{\mathbf{x}}_S + \eta^F\dot{\mathbf{x}}_F) = 0. \tag{12}$$
$$\mathrm{div}\mathbf{T}_E^S - \eta^S\,\mathrm{grad}\,p + \rho^S(\mathbf{b} - \ddot{\mathbf{x}}_s) - \mathbf{P}_E^F = 0, \tag{13}$$
$$\mathrm{div}\mathbf{T}_E^F - \eta^F\,\mathrm{grad}\,p + \rho^F(\mathbf{b} - \ddot{\mathbf{x}}_F) + \mathbf{P}_E^F = 0, \tag{14}$$

where $\dot{\mathbf{x}}_i$ and $\ddot{\mathbf{x}}_i$ $(i = S, F)$ denote the velocities and accelerations, respectively of solid (S) and fluid (F) phases of the porous aggregate and p is the effective pore pressure of the incompressible pore fluid. ρ^S and $\rho^F$are the densities of the solid and fluid phases respectively and **b** is the body force per unit volume. $\mathbf{T}_E^S$ and $\mathbf{T}_E^F$ are the effective stress in the solid and fluid phases respectively, $\mathbf{P}_E^F$ is the effective quantity of momentum supply and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1 \tag{15}$$

If $\mathbf{u}_S$ and $\mathbf{u}_F$ are the displacement vectors for solid and fluid phases, then

$$\dot{x}_S = \dot{\mathbf{u}}_S \ ; \ \ddot{\mathbf{x}}_s = \ddot{\mathbf{u}}_s \ ; \quad \dot{\mathbf{x}}_F = \dot{\mathbf{u}}_F \ ; \quad \ddot{\mathbf{x}}_F = \ddot{\mathbf{u}}_F \tag{16}$$

The constitutive equations for linear isotropic, elastic incompressible porous medium are given by de Boer, Ehlers and Liu (1993) as

$$\mathbf{T}_E^S = 2\mu^S\mathbf{E}_S + \lambda^S(\mathrm{E}_S.\mathbf{I})\mathbf{I} \tag{17}$$
$$\mathbf{T}_E^F = 0 \tag{18}$$
$$\mathbf{P}_E^F = -\mathbf{S}_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) \tag{19}$$

where λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and $\mathbf{E}_S$ is the linearized Langrangian strain tensor defined as

$$\mathbf{E}_S = \frac{1}{2}(\mathrm{grad}\,\mathbf{u}_S + \mathrm{grad}^T\mathbf{u}_S) \tag{20}$$

In the case of isotropic permeability, the tensor $\mathbf{S}_v$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990b) as

$$\mathbf{S}_v = \frac{(\eta^F)^2\gamma^{FR}}{K^F}\mathbf{I} \tag{21}$$

where γ^{FR} is the specific weight of the fluid and K^F is the Darcy's permeability coefficient of the porous medium.

Making the use of (16) in equations (12)-(1 4), and with the help of (17)-(20), we obtain

$$\mathrm{div}(\eta^S \dot{\mathbf{u}}_S + \eta^F \dot{\mathbf{u}}_F) = 0, \quad (22)$$

$$(\lambda^S + \mu^S)\mathrm{grad}\,\mathrm{div}\,\mathbf{u}_S + \mu^S \mathrm{div}\,\mathrm{grad}\,\mathbf{u}_S - \eta^S \mathrm{grad}\,p + \rho^S(\mathbf{b} - \ddot{\mathbf{u}}_s) + S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0, \quad (23)$$

$$-\eta^F \mathrm{grad}\,p + \rho^F(\mathbf{b} - \ddot{\mathbf{u}}_F) - S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0. \quad (24)$$

For the two dimensional problem, we assume the displacement vector $\mathbf{u}_i$ $(i = F, S)$ as

$$\mathbf{u}_i = (u^i, 0, w^i) \quad \text{where} \quad i = F, S. \quad (25)$$

Equations (22) - (24) with the help of eq. (25) in absence of body forces take the form

$$\eta^S \left[\frac{\partial^2 u^S}{\partial x\,\partial t} + \frac{\partial^2 w^S}{\partial z\,\partial t}\right] + \eta^F \left[\frac{\partial^2 u^F}{\partial x\,\partial t} + \frac{\partial^2 w^F}{\partial z\,\partial t}\right] = 0, \quad (26)$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \quad (27)$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \quad (28)$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \quad (29)$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \quad (30)$$

where

$$\theta^S = \frac{\partial(u^S)}{\partial x} + \frac{\partial(w^S)}{\partial z} \quad (31)$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \quad (32)$$

Also, ${t_{zz}}^S$ and ${t_{zx}}^S$ the normal and tangential stresses in the solid phase are as under

$${t_{zz}}^S = \lambda^S \left(\frac{\partial u^S}{\partial x} + \frac{\partial w^S}{\partial z}\right) + 2\mu^S \frac{\partial w^S}{\partial z} \quad (33)$$

$${t_{zx}}^S = \mu^S \left(\frac{\partial u^S}{\partial z} + \frac{\partial w^S}{\partial x}\right). \quad (34)$$

The displacement components u^j and w^j are related to the dimensional potential ϕ^j and ψ^j as

$$u^j = \frac{\partial \phi^j}{\partial x} + \frac{\partial \psi^j}{\partial z}\;; \quad w^j = \frac{\partial \phi^j}{\partial z} - \frac{\partial \psi^j}{\partial x}\;; \quad j = S, F. \quad (35)$$

Using equation (35) in equations (26)-(30), we obtain the following equations determining ϕ^S, ϕ^F, ψ^S, ψ^F and p as:

$$\nabla^2 \phi^S - \frac{1}{{C_1}^2}\frac{\partial^2 \phi^S}{\partial t^2} - \frac{S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\frac{\partial \phi^S}{\partial t} = 0, \quad (36)$$

$$\phi^F = -\frac{\eta^S}{\eta^F}\phi^S \quad (37)$$

$$\mu^S \nabla^2 \psi^S - \rho^S \frac{\partial^2 \psi^S}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \quad (38)$$

$$\rho^F \frac{\partial^2 \psi^F}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \quad (39)$$

$$(\eta^F)^2 p - \eta^S \rho^F \frac{\partial^2 \phi^S}{\partial t^2} - S_v \frac{\partial \phi^S}{\partial t} = 0, \tag{40}$$

where

$$C_1 = \sqrt{\frac{(\eta^F)^2(\lambda^S + 2\mu^S)}{(\eta^F)^2\rho^S + (\eta^S)^2\rho^F}} \tag{41}$$

Assuming the solution of the system of equations (36) - (40) in the form

$$(\phi^S, \phi^F, \psi^S, \psi^F, p) = \left(\phi_1{}^S, \phi_1{}^F, \psi_1{}^S, \psi_1{}^F, p_1\right)\exp(i\omega t), \tag{42}$$

where ω is the complex circular frequency.

Making the use of (42) in equations (36)-(40), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{C_1{}^2} - \frac{i\omega S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\right]\phi_1{}^S = 0, \tag{43}$$

$$[\mu^S\nabla^2 + \rho^S\omega^2 - i\omega S_v]\psi_1{}^S = -i\omega S_v \psi_1{}^F, \tag{44}$$

$$[-\omega^2\rho^F + i\omega S_v]\psi_1{}^F - i\omega S_v\psi_1{}^S = 0, \tag{45}$$

$$(\eta^F)^2 p_1 + \eta^S\rho^F\omega^2\phi_1{}^S - i\omega S_v\phi_1{}^S = 0, \tag{46}$$

$$\phi_1{}^F = -\frac{\eta^S}{\eta^F}\phi_1{}^S. \tag{47}$$

Equation (43) corresponds to longitudinal wave propagating with velocity v_1, given by

$$v_1{}^2 = \frac{1}{G_1} \tag{48}$$

where

$$G_1 = \left[\frac{1}{C_1{}^2} - \frac{iS_v}{\omega(\lambda^S + 2\mu^S)(\eta^F)^2}\right]. \tag{49}$$

From equation (44) and (45), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{v_2{}^2}\right]\psi_1{}^S = 0, \tag{50}$$

Equation (50) corresponds to transverse wave propagating with velocity v_2 given by

$$v_2{}^2 = 1/G_2$$

where

$$G_2 = \left\{\frac{\rho^S}{\mu^S} - \frac{iS_v}{\mu^S\omega} - \frac{S_v{}^2}{\mu^S(-\rho^S\omega^2 + i\omega S_v)}\right\}, \tag{51}$$

In medium M_2

$$\phi = B_1 \exp\{i\delta_1(x\sin\bar{\theta}_1 - z\cos\bar{\theta}_1) + i\bar{\omega}_1 t\}, \tag{52}$$

$$\psi = B_2 \exp\{i\delta_2(x\sin\bar{\theta}_2 - z\cos\bar{\theta}_2) + i\bar{\omega}_2 t\}$$
$$+ B_3 \exp\{i\delta_3(x\sin\bar{\theta}_3 - z\cos\bar{\theta}_3) + i\bar{\omega}_3 t\}, \tag{53}$$

$$\Phi_2 = EB_2 \exp\{i\delta_2(x\sin\bar{\theta}_2 - z\cos\bar{\theta}_2) + i\bar{\omega}_2 t\}$$
$$+ FB_3 \exp\{i\delta_3(x\sin\bar{\theta}_3 - z\cos\bar{\theta}_3) + i\bar{\omega}_3 t\}, \tag{54}$$

where

$$E = \frac{\delta_2{}^2\left(\delta_2{}^2 - \frac{\omega^2}{(c_2{}^2+c_3{}^2)} + pq\right)}{\text{deno.}}, \tag{55}$$

$$F = \frac{\delta_3{}^2\left(\delta_3{}^2 - \frac{\omega^2}{(c_2{}^2+c_3{}^2)} + pq\right)}{\text{deno.}}, \tag{56}$$

and $\quad \text{deno.} = p\left(2q - \frac{\omega^2}{c_4{}^2}\right);\quad \delta_2{}^2 = \lambda_2{}^2\omega^2;\quad \delta_3{}^2 = \lambda_3{}^2\omega^2 \qquad (57)$

In medium M_1

$$\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\}[A_{01}\exp\{ik_1(x\sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} + A_1\exp\{ik_1(x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}], \tag{58}$$

$$\{\psi^S, \psi^F\} = \{1, m_3\}[B_{01}\exp\{ik_2(x\sin\theta_0 - z\cos\theta_0) + i\omega_2 t\} + A_2\exp\{ik_2(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}], \tag{59}$$

where

$$m_1 = -\frac{\eta^S}{\eta^F};\quad m_2 = -\left[\frac{\eta^S\omega_1{}^2\rho^F - i\omega_1 S_v}{(\eta^F)^2}\right];\quad m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - \omega_2{}^2\rho^F}; \tag{60}$$

and B_1, B_2, B_3 are amplitudes of transmitted P-wave, transmitted coupled transverse and micro-rotation waves respectively. Also A_{01} or B_{01}, A_1 and A_2 are amplitudes of incident P-wave or SV-wave, reflected P-wave and reflected SV-wave respectively and to be determined from boundary conditions.

4. Boundary conditions

The appropriate boundary conditions are the continuity of displacement, micro rotation and stresses at the interface separating media M_1 and M_2. Mathematically, these boundary conditions at z=0 can be expressed as:

$$u = u^S,\quad w = w^S,\quad m_{zy} = 0,\quad t_{zz} = t_{zz}{}^S - p,\quad t_{zx} = t_{zx}{}^S \tag{61}$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{v_0} = \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\bar{\theta}_1}{\bar{v}_1} = \frac{\sin\bar{\theta}_2}{\bar{v}_2} = \frac{\sin\bar{\theta}_3}{\bar{v}_3} \tag{62}$$

where $\bar{v}_2 = \frac{1}{\lambda_2},\quad \bar{v}_3 = \frac{1}{\lambda_3}$

For longitudinal wave,

$$v_0 = v_1,\quad \theta_0 = \theta_1, \tag{63}$$

For transverse wave,

$$v_0 = v_2,\quad \theta_0 = \theta_2, \tag{63A}$$

Also

$$\delta_1\bar{v}_1 = \delta_2\lambda_2^{-1} = \delta_3\lambda_3^{-1} = k_1 v_1 = k_2 v_2 = \omega,\quad \text{at } z = 0 \tag{64}$$

Making the use of potentials given by equations (52)-(54) and (58)-(59) in the boundary conditions given by (61) and using (62)-(64), we get a system of five non homogeneous equations which can be written as

$$\sum_{j=1}^{5} a_{ij} Z_j = Y_i,\quad (i = 1,2,3,4,5) \tag{65}$$

where

$$Z_1 = \frac{B_1}{B_0}\ , \quad Z_2 = \frac{B_2}{B_0}\ , \quad Z_3 = \frac{B_3}{B_0}\ , \quad Z_4 = \frac{A_1}{B_0}\ , \quad Z_5 = \frac{A_2}{B_0}\ , \tag{66}$$

where $B_0 = A_{01}$ or B_{01} is amplitude of incident P-wave or SV-wave respectively.
Also Z_1 to Z_5 are the amplitude ratios of transmitted longitudinal wave, transmitted coupled transverse wave at an angle $\bar{\theta}_2$, transmitted coupled microrotational wave at an angle $\bar{\theta}_3$, reflected P wave and reflected SV wave respectively. Also a_{ij} and Y_i are as

$$a_{11} = -\lambda{\delta_1}^2 - (2\mu + \kappa)\left({\delta_1}^2 \cos^2\bar{\theta}_1\right)\ , \quad a_{12} = (2\mu + \kappa){\delta_2}^2 \sin\bar{\theta}_2 \cos\bar{\theta}_2\ ,$$
$$a_{13} = (2\mu + \kappa){\delta_3}^2 \sin\bar{\theta}_3 \cos\bar{\theta}_3\ , \quad a_{14} = k_1^2(\lambda^S + 2\mu^S \cos^2\theta_1) + m_2\ ,$$
$$a_{15} = -2\mu^S k_2^2 \sin\theta_2 \cos\theta_2\ , \quad a_{21} = (2\mu + \kappa){\delta_1}^2 \sin\bar{\theta}_1 \cos\bar{\theta}_1\ ,$$
$$a_{22} = \mu{\delta_2}^2 \cos 2\,\bar{\theta}_2 + \kappa{\delta_2}^2 \cos^2\bar{\theta}_2 - \kappa E\ , \quad a_{23} = \mu{\delta_3}^2 \cos 2\,\bar{\theta}_3 + \kappa{\delta_3}^2 \cos^2\bar{\theta}_3 - \kappa F\ ,$$
$$a_{24} = \mu^S k_1^2 \sin 2\theta_1\ , \quad a_{25} = \mu^S k_2^2 \cos 2\theta_2\ ,$$
$$a_{31} = 0\ , \quad a_{32} = \delta_2 E \cos\bar{\theta}_2\ , \quad a_{33} = \delta_3 F \cos\bar{\theta}_3\ , \quad a_{34} = 0\ , \quad a_{35} = 0,$$
$$a_{41} = i\,\delta_1 \sin\bar{\theta}_1\ , \quad a_{42} = i\,\delta_2 \cos\bar{\theta}_2\ , \quad a_{43} = i\,\delta_3 \cos\bar{\theta}_3\ , \quad a_{44} = -i\,k_1 \sin\theta_1\ ,$$
$$a_{45} = -i\,k_2 \cos\theta_2\ , \quad a_{51} = -i\,\delta_1 \cos\bar{\theta}_1\ , \quad a_{52} = i\,\delta_2 \sin\bar{\theta}_2\ , \quad a_{53} = i\,\delta_3 \sin\bar{\theta}_3\ ,$$
$$a_{54} = -i\,k_1 \cos\theta_1\ , \quad a_{55} = i\,k_2 \sin\theta_2 = 0\ ,$$

For incident P wave

$$Y_1 = -a_{14}\ , \quad Y_2 = a_{24}\ , \quad Y_3 = a_{34}\ , \quad Y_4 = -a_{44}\ , \quad Y_5 = a_{54}\ ,$$

For incident SV wave

$$Y_1 = a_{15}\ , \quad Y_2 = -a_{25}\ , \quad Y_3 = a_{35}\ , \quad Y_4 = a_{45}\ , \quad Y_5 = -a_{55} \tag{67}$$

5. Numerical results and discussion

System of equations of five non homogeneous equations obtained above is solved by Cramer rule to obtain the various amplitude ratios of reflected and transmitted waves for incidence of P as well as SV wave. In order to study in more detail the behaviour of various amplitude ratios, we have computed them numerically for a particular model for which the values of relevant elastic parameters are as follow.

In medium M_2 , the physical constants for micropolar elastic solid are taken from Gauthier (1982) as

$$\lambda = 7.59 \times 10^{10}\ \mathrm{N/m^2}\ , \quad \mu = 1.89 \times 10^{10}\ \mathrm{N/m^2}, \quad \kappa = 1.49 \times 10^{8}\ \mathrm{N/m^2},$$
$$\rho = 2.19 \times 10^{3}\mathrm{kg/m^3}, \quad \gamma = 2.68 \times 10^{4}\ \mathrm{N}, \quad j = 1.96 \times 10^{-6}\mathrm{m^2},$$
$$\frac{\omega^2}{{\omega_0}^2} = 200. \tag{68}$$

In medium M_1, the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu (1993) as

$$\eta^S = 0.67, \quad \eta^F = 0.33, \quad \rho^S = 1.34\ \mathrm{Mg/m^3}, \quad \rho^F = 0.33\ \mathrm{Mg/m^3},$$
$$\lambda^S = 5.5833\ \mathrm{MN/m^2}, \quad K^F = 0.01\mathrm{m/s}, \quad \gamma^{FR} = 10.00\mathrm{KN/m^3}, \quad \mu^S = 8.3750\mathrm{N/m^2}, \tag{69}$$

A computer programme in MATLAB has been developed to calculate the modulus of amplitude ratios of various reflected and transmitted waves for the particular model and to depict graphically. The amplitude ratios are computed for the angle of incidence varying from 0^{0} to 90^{0}. The variation of modulus of amplitude ratios Z_i with angle of emergence θ_0 of longitudinal P wave or transverse SV wave are shown in figures (2)-(16). In figures (2) -(11) solid lines show the variations of amplitude ratios $|Z_i|$ when medium-I is incompressible fluid saturated porous medium (FS) and medium-II is micropolar elastic solid whereas dashed lines show the variations of amplitude ratios when medium-I becomes incompressible empty porous solid (EPS). In Figures (2)-(6), there is P wave incident whereas in figures (7)-(11), SV wave is incident. The nature of dependence of modulus of amplitude ratios Z_i is different for different reflected and transmitted waves.

In figure 2, the amplitude ratio corresponding to transmitted longitudinal wave, decreases with increase in angle of incidence. Also, effect of fluid filled in pores is significant. In figure (3), it can be noticed that modulus of amplitude ratio Z_2 , which corresponds to transmitted coefficient for coupled transverse wave is zero at 0^0 angle of emergence. It then increases smoothly and takes it maximum value and then decreases smoothly and takes its minimum value zero at 1.35(radians). After 1.35(radians), it increases very slowly and thereafter it decreases and vanishes at 90^0 (1.6 radians) in both cases. In figure (4), modulus of amplitude ratios Z_3 , which corresponds to transmitted coefficient for coupled microrotational wave, the behaviour is oscillatory for solid line (fluid saturated case). It takes zero value for the angle of incidence varying from 0^0 to 90^0 in empty porous solid case.

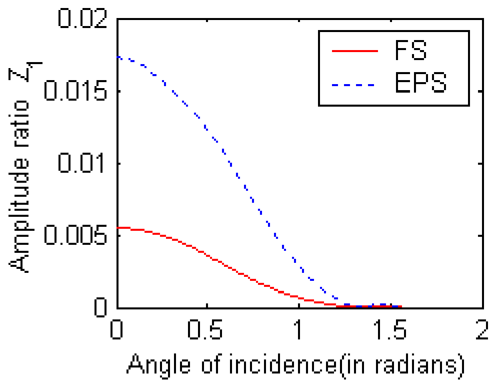
In figure (5), the amplitude ratio corresponding to reflected P wave first decreases from its maxima to minima and then increases to its maxima in both the cases of fluid saturated porous solid and empty porous solid. In figure (6), the amplitude ratio corresponding to reflected SV wave first increases from its minima to maxima and then decreases to its minima in both the cases of fluid saturated porous solid and empty porous solid.

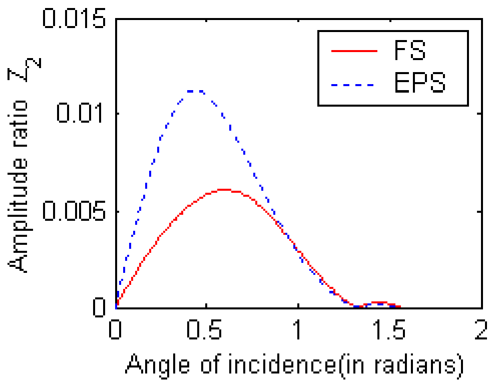
In figure (7) and (8), the amplitude ratios corresponding to transmitted longitudinal wave, transmitted coupled transverse wave, increase sharply at 0.4(radians). Also their behaviour is oscillatory for empty porous solid case. In figure (9), the amplitude ratio $|Z_3|$ which corresponds to transmitted coefficient for coupled microrotational wave, the behaviour is oscillatory for fluid saturated porous solid case and for empty porous solid case; the amplitude ratio is zero for the angle of incidence varying from 0^0 to 90^0.

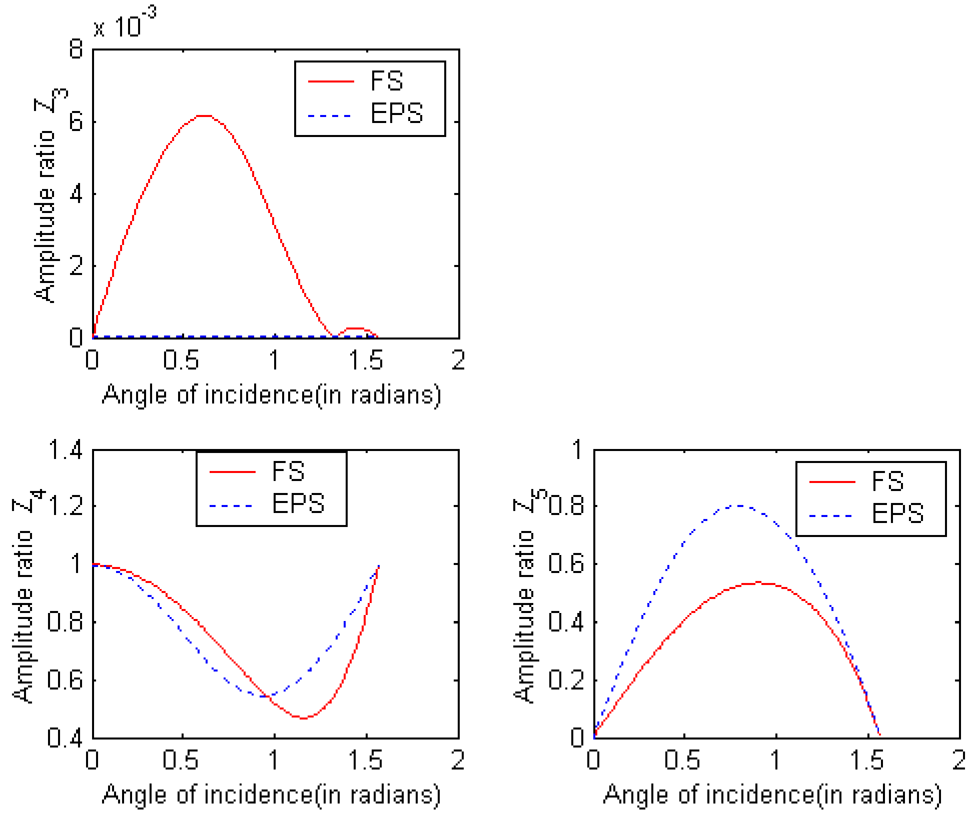
In figure (10), the amplitude ratio corresponding to reflected P wave first increases slowly from its minima to maxima and then decreases slowly to its minima in both the cases of fluid saturated porous solid and empty porous solid. In figure (11), the amplitude ratio corresponding to reflected SV wave first decreases from its maxima to minima and then increases to its maxima in both the cases of fluid saturated porous solid and empty porous solid. Also the behaviour is monotonic in the range 0^0 to 90^0 in figures (10)-(11).

A comparison between solid and dotted lines reveals that the effect of fluid filled in pores on amplitude ratios is significant in both the cases of P or SV wave incidence.

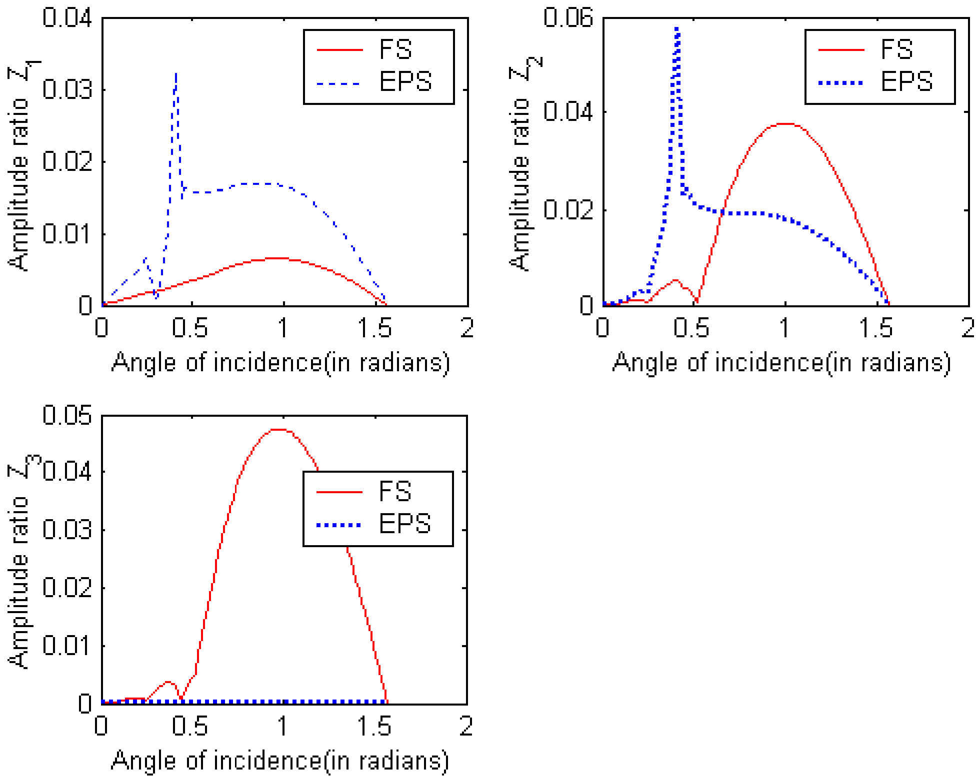
Figures (12)-(16) depicts the effect of incident longitudinal and incident transverse wave on the variation of amplitude ratios. From these figures it is very clear that the amplitude ratios depend on incident wave.







Figures 2-6. Variation of the amplitude ratios with angle of incidence of incident longitudinal wave showing porous effect



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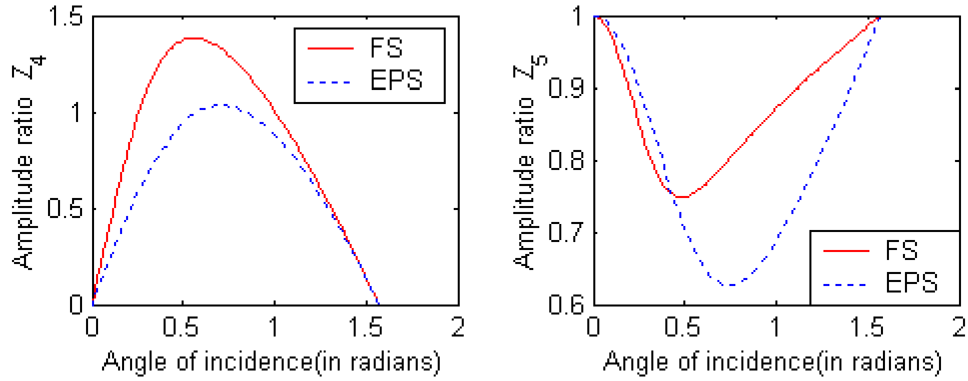
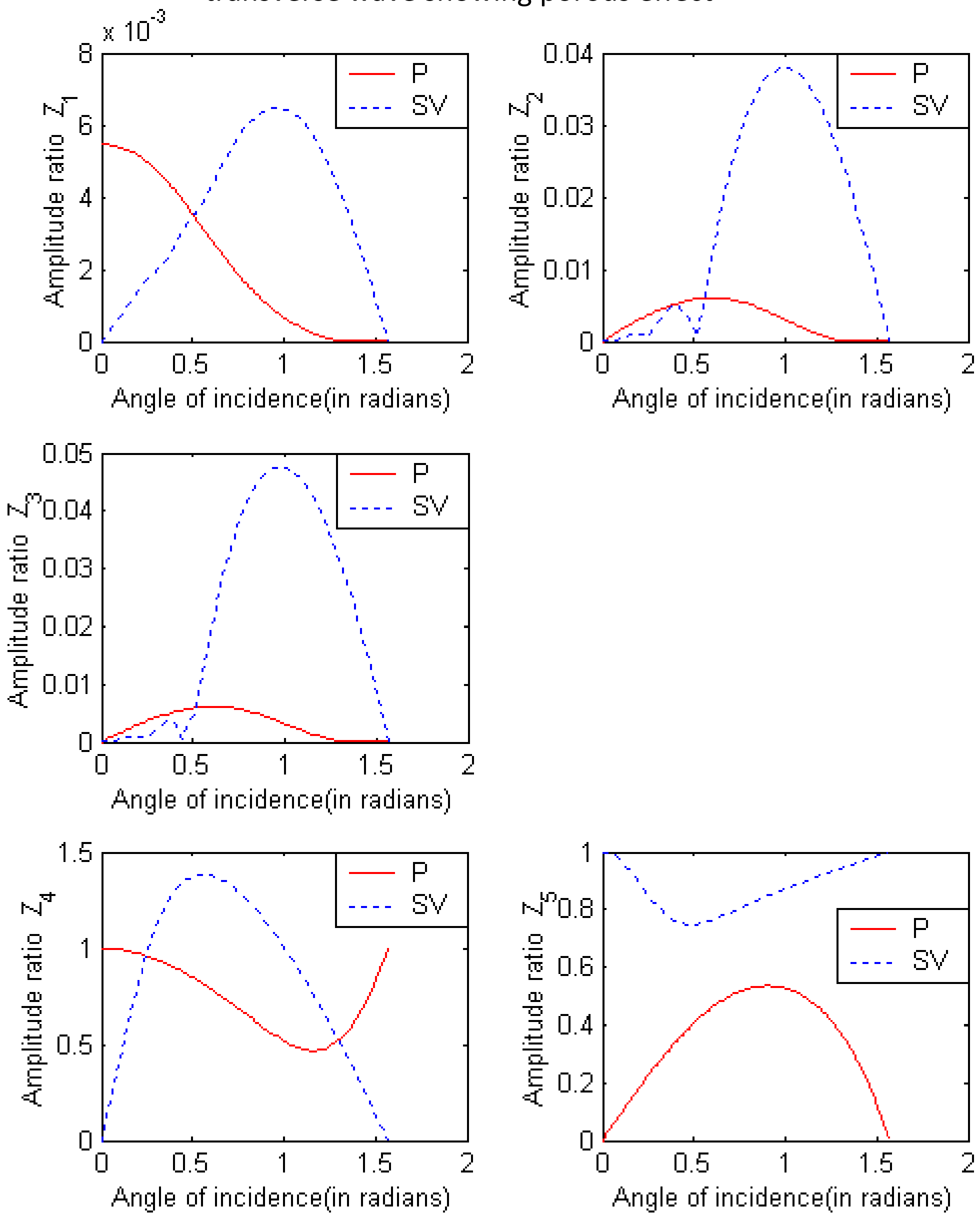


Figures 7-11. Variation of the amplitude ratios with angle of incidence of incident transverse wave showing porous effect



Figures 12-16. Variation of the amplitude ratios with angle of incidence of the incident longitudinal and transverse wave

6. Conclusion

In conclusion, a mathematical study of reflection and transmission coefficients at an interface separating micropolar elastic solid half space and fluid saturated incompressible porous half space is made when longitudinal wave or transverse wave is incident. It is observed that the amplitudes ratios of various reflected and transmitted waves are found to be complex valued. The modulus of amplitudes ratios of various reflected and transmitted waves depend on the angle of incidence of the incident wave and material properties of half spaces. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on the amplitudes ratios. The effect incident wave is significant on amplitude ratios. All the waves are found to depend on incident waves and angle of incidence. The model presented in this paper is one of the more realistic forms of the earth models. The present theoretical results may provide useful information for experimental scientists/researchers/seismologists working in the area of wave propagation in micropolar elastic solid and fluid saturated incompressible porous solid.

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