International Journal of Engineering, Science and Mathematics
Vol. 5 Issue 1, March 2016, ISSN: 2320-0294, Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Wave Propagation through Loosely Bonded Solid/Solid Interface

Vinod Kaliraman
Department of Mathematics, Chaudhary Devi Lal University, Sirsa-125055, INDIA

Abstract
In this paper, solution of the governing equations of micropolar elastic solid and fluid saturated incompressible porous solid is employed to study the reflection and transmission phenomenon at a loosely bonded interface between micropolar elastic solid half space and fluid saturated porous half space. P-wave or SV-wave is considered to be incident on the plane interface through fluid saturated porous solid half space. The amplitude ratios of various reflected and transmitted waves are derived and computed numerically for a specific model for different values of bonding parameter. The results thus obtained are depicted graphically with angle of incidence of incident wave. It is found that these amplitude ratios depend on angle of incidence of the incident wave and material properties of the medium. Effect of bonding parameter, fluid filled in the pores of fluid saturated porous medium on the amplitude ratios is shown.
Keywords: Porous solid, micropolar elastic solid, reflection, transmission, longitudinal wave, transverse wave, amplitude ratios, empty porous solid, loosely bonded interface.

1. Introduction

Most of natural and man-made materials, including engineering, geological and biological media, possess a microstructure. The ordinary classical theory of elasticity fails to describe the microstructure of the material. To tackle this problem, Suhubi and Eringen (1964), Eringen and Suhubi (1964) developed a theory in which they considered the microstructure of the material and they showed that the motion in a granular structure material is characterized not by a displacement vector but also by a rotation vector. Gautheir (1982) found aluminum-epoxy composite to be a micropolar material. Many problems of waves and vibrations have been discussed in micropolar elastic solid by several researchers. Some of them are Parfitt and Eringen (1969), Tomar and Gogna (1992), Tomar and Kumar (1995), Singh and Kumar (2007), Kumar and Barak (2007) etc.
In the problems of wave propagation at the interface between two elastic half spaces, the contact between them is normally assumed to be welded. However, in certain situations, there are reasons for expecting that bonding is not complete. Murty (1976) discussed a theoretical model for reflection, transmission, and attenuation of elastic waves through a loosely bonded interface between two elastic solid half spaces by assuming that the interface behaves like a dislocation which preserves the continuity of stresses allowing a finite amount of slip. A similar situation occurs at the two different poroelastic solids, as the liquid present in the porous skeleton may cause the two media to be loosely bonded. Vashisth and Gogna (1993), Kumar and Singh (1997) etc. discussed the problems of reflection and transmission at the loosely bonded interface between two half spaces.
The mechanical behaviour of fluid saturated porous material when the material contains liquid filled pores with help of classical theory is inadequate. Due to complicated structures of pores and different motions of solid and liquid phases, it is very complex and difficult to describe the mechanical behaviour of a fluid saturated porous medium. So many researchers tried to overcome this difficulty from time to time. Bowen (1980) and de Boer and Ehlers (1990a, 1990b) developed an interesting theory for porous medium having all constituents to be incompressible. There are sufficient reasons for considering the fluid saturated porous

constituents as incompressible. Therefore, the assumption of incompressible constituents meet the properties appearing the in many branches of engineering and avoids the introduction of many complicated material parameters as considered in the Biot theory because Biot's theory was based on the assumption of compressible constituents.

Based on the theory given by de Boer and Ehlers (1990a, 1990b), many researchers like de Boer and Didwania (2004), de Boer and Liu(1994,1995), Kumar and Hundal (2003), Tajuddin and Hussaini (2006), Kumar et.al.(2011), Kumari (2014) etc. studied some problems of wave propagation in fluid saturated porous media. Using the theory of de Boer and Ehlers (1990) for fluid saturated porous medium and Eringen (1968) theory for micropolar elastic solid, the reflection and transmission of longitudinal wave (P-wave) or transverse wave (SV-wave) at a loosely bonded interface between micropolar elastic solid half space and fluid saturated porous solid half space is discussed. Amplitudes ratios for various reflected and transmitted waves are computed for a particular model and depicted graphically and discussed accordingly. The model considered is assumed to exist in the oceanic crust part of the earth and the propagation of wave through such a model will be of great use in the fields related to earth sciences.

2. Formulation of the problem

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and the plane interface z=0 separating the fluid saturated porous half space M_1 $[z > 0]$ and micropolar elastic solid half spaceM_2 [z<0]. A longitudinal wave or transverse wave propagates through the medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to incident longitudinal or transverse wave, we get two reflected waves in the medium M_1 and three transmitted waves in medium M_2. See fig.1

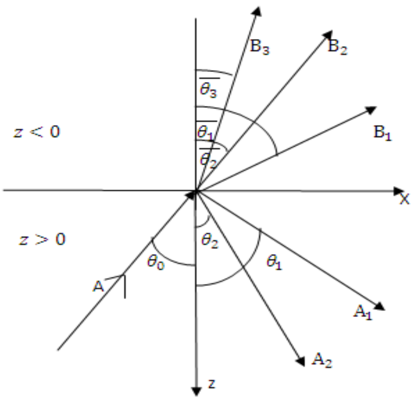


Fig.1 Geometry of the problem.

3. Basic equations and constitutive relatio

3.1. For medium M_2 (Micropolar elastic solid half space)

The equation of motion in micropolar elastic medium are given by Eringen (1968) as

$$({c_1}^2 + {c_3}^2)\nabla^2\phi = \frac{\partial^2\phi}{\partial t^2} \quad (1)$$

$$({c_2}^2 + {c_3}^2)\nabla^2\vec{U} + {c_3}^2\nabla\times\vec{\Phi} = \frac{\partial^2\vec{U}}{\partial t^2} \quad (2)$$

$$({c_4}^2\nabla^2 - 2{\omega_0}^2)\vec{\Phi} + {\omega_0}^2\nabla\times\vec{U} = \frac{\partial^2\vec{\Phi}}{\partial t^2} \quad (3)$$

Where

$$c_1{}^2 = \frac{\lambda+2\mu}{\rho}\;; \quad c_2{}^2 = \frac{\mu}{\rho}\;; \quad c_3{}^2 = \frac{\kappa}{\rho}\;; \quad c_4{}^2 = \frac{\gamma}{\rho j}\;; \quad \omega_0{}^2 = \frac{\kappa}{\rho j} \tag{4}$$

Parfitt and Eringen (1969) have shown that equation (1) corresponds to longitudinal wave propagating with velocity $\bar{v}_1$, given by $\bar{v}_1{}^2 = c_1{}^2 + c_3{}^2$ and equations (2)-(3) are coupled equations in vector potentials $\vec{U}$ and $\vec{\Phi}$ and these correspond to coupled transverse and micro-rotation waves. If $\frac{\omega^2}{\omega_0{}^2} > 2$, there exist two sets of coupled-wave propagating with velocities $1/\lambda_2$ and $1/\lambda_3$; where

$$\lambda_2{}^2 = \frac{1}{2}\left[B - \sqrt{B^2 - 4C}\right], \quad \lambda_3{}^2 = \frac{1}{2}\left[B + \sqrt{B^2 - 4C}\right], \tag{5}$$

where

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2{}^2 + c_3{}^2)} + \frac{1}{c_4{}^2}\;; \qquad C = \left(\frac{1}{c_4{}^2} - \frac{2q}{\omega^2}\right)\frac{1}{(c_2{}^2 + c_3{}^2)}\;;$$

$$p = \frac{\kappa}{\mu+\kappa}\;; \quad q = \frac{\kappa}{\gamma} \tag{6}$$

Considering a two dimensional problem by taking the following components of displacement and micro rotation as

$$\vec{u} = (u, 0, w), \qquad \vec{\Phi} = (0, \Phi_2, 0) \tag{7}$$

where

$$u = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z} \qquad w = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x} \tag{8}$$

and components of stresses are as under

$$t_{zz} = (\lambda + 2\mu + \kappa)\frac{\partial^2\phi}{\partial z^2} + \lambda\frac{\partial^2\phi}{\partial x^2} + (2\mu + \kappa)\frac{\partial^2\psi}{\partial x\,\partial z} \tag{9}$$

$$t_{zx} = (2\mu + \kappa)\frac{\partial^2\phi}{\partial x\,\partial z} - (\mu + \kappa)\frac{\partial^2\psi}{\partial z^2} + \mu\frac{\partial^2\psi}{\partial x^2} - \kappa\Phi_2 \tag{10}$$

$$m_{zy} = \gamma\frac{\partial\Phi_2}{\partial z} \tag{11}$$

3.2. For medium M_1 (Fluid saturated incompressible porous solid half space)

Following de Boer and Ehlers (1990b), the governing equations in a fluid-saturated incompressible porous medium are

$$\mathrm{div}(\eta^S\dot{\mathbf{x}}_S + \eta^F\dot{\mathbf{x}}_F) = 0. \tag{12}$$

$$\mathrm{div}\mathbf{T}_E^S - \eta^S\,\mathrm{grad}\,p + \rho^S(\mathbf{b} - \ddot{\mathbf{x}}_s) - \mathbf{P}_E^F = 0, \tag{13}$$

$$\mathrm{div}\mathbf{T}_E^F - \eta^F\,\mathrm{grad}\,p + \rho^F(\mathbf{b} - \ddot{\mathbf{x}}_F) + \mathbf{P}_E^F = 0, \tag{14}$$

where $\dot{\mathbf{x}}_i$ and $\ddot{\mathbf{x}}_i$ $(i = S, F)$ denote the velocities and accelerations, respectively of solid (S) and fluid (F) phases of the porous aggregate and p is the effective pore pressure of the incompressible pore fluid. ρ^S and $\rho^F$are the densities of the solid and fluid phases respectively and **b** is the body force per unit volume. $\mathbf{T}_E^S$ and $\mathbf{T}_E^F$ are the effective stress in the solid and fluid phases respectively, $\mathbf{P}_E^F$ is the effective quantity of momentum supply and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \tag{15}$$

If $\mathbf{u}_S$ and $\mathbf{u}_F$ are the displacement vectors for solid and fluid phases, then

$$\dot{x}_S = \dot{\mathbf{u}}_S\;; \quad \ddot{\mathbf{x}}_s = \ddot{\mathbf{u}}_s\;; \quad \dot{\mathbf{x}}_F = \dot{\mathbf{u}}_F; \quad \ddot{\mathbf{x}}_F = \ddot{\mathbf{u}}_F \tag{16}$$

The constitutive equations for linear isotropic, elastic incompressible porous medium are given by de Boer, Ehlers and Liu (1993) as

$$\mathbf{T}_E^S = 2\mu^S\mathbf{E}_S + \lambda^S(E_S.\mathbf{I})\mathbf{I}, \tag{17}$$

$$\mathbf{T}_E^F = 0, \tag{18}$$

$$\mathbf{P}_E^F = -\mathbf{S}_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S), \tag{19}$$

where λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and $\mathbf{E}_S$ is the linearized Langrangian strain tensor defined as

$$\mathbf{E}_S = \frac{1}{2}(\text{grad}\,\mathbf{u}_S + \text{grad}^T\mathbf{u}_S), \tag{20}$$

In the case of isotropic permeability, the tensor $\mathbf{S}_v$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990b) as

$$\mathbf{S}_v = \frac{(\eta^F)^2\gamma^{FR}}{K^F}\mathbf{I}, \tag{21}$$

where γ^{FR} is the specific weight of the fluid and K^F is the Darcy's permeability coefficient of the porous medium.

Making the use of (16) in equations (12)-(1 4), and with the help of (17)-(20), we obtain

$$\text{div}(\eta^S\dot{\mathbf{u}}_S + \eta^F\dot{\mathbf{u}}_F) = 0, \tag{22}$$

$$(\lambda^S + \mu^S)\text{grad div}\,\mathbf{u}_S + \mu^S\text{div grad}\,\mathbf{u}_S - \eta^S\text{grad}\,p + \rho^S(\mathbf{b} - \ddot{\mathbf{u}}_s) + S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0, \tag{23}$$

$$-\eta^F\text{grad}\,p + \rho^F(\mathbf{b} - \ddot{\mathbf{u}}_F) - S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0. \tag{24}$$

For the two dimensional problem, we assume the displacement vector $\mathbf{u}_i$ $(i = F, S)$ as

$$\mathbf{u}_i = (u^i, 0, w^i) \quad \text{where} \quad i = F, S. \tag{25}$$

Equations (22) - (24) with the help of eq. (25) in absence of body forces take the form

$$\eta^S\left[\frac{\partial^2 u^S}{\partial x\,\partial t} + \frac{\partial^2 w^S}{\partial z\,\partial t}\right] + \eta^F\left[\frac{\partial^2 u^F}{\partial x\,\partial t} + \frac{\partial^2 w^F}{\partial z\,\partial t}\right] = 0, \tag{26}$$

$$\eta^F\frac{\partial p}{\partial x} + \rho^F\frac{\partial^2 u^F}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{27}$$

$$\eta^F\frac{\partial p}{\partial z} + \rho^F\frac{\partial^2 w^F}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{28}$$

$$(\lambda^S + \mu^S)\frac{\partial\theta^S}{\partial x} + \mu^S\nabla^2 u^S - \eta^S\frac{\partial p}{\partial x} - \rho^S\frac{\partial^2 u^S}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{29}$$

$$(\lambda^S + \mu^S)\frac{\partial\theta^S}{\partial z} + \mu^S\nabla^2 w^S - \eta^S\frac{\partial p}{\partial z} - \rho^S\frac{\partial^2 w^S}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{30}$$

where

$$\theta^S = \frac{\partial(u^S)}{\partial x} + \frac{\partial(w^S)}{\partial z} \tag{31}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \tag{32}$$

Also, ${t_{zz}}^S$ and ${t_{zx}}^S$ the normal and tangential stresses in the solid phase are as under

$${t_{zz}}^S = \lambda^S\left(\frac{\partial u^S}{\partial x} + \frac{\partial w^S}{\partial z}\right) + 2\mu^S\frac{\partial w^S}{\partial z} \tag{33}$$

$${t_{zx}}^S = \mu^S\left(\frac{\partial u^S}{\partial z} + \frac{\partial w^S}{\partial x}\right) \tag{34}$$

The displacement components u^j and w^j are related to the dimensional potential ϕ^j and ψ^j as

$$u^j = \frac{\partial\phi^j}{\partial x} + \frac{\partial\psi^j}{\partial z}; \quad w^j = \frac{\partial\phi^j}{\partial z} - \frac{\partial\psi^j}{\partial x}; \quad j = S, F. \tag{35}$$

Using equation (35) in equations (26)-(30), we obtain the following equations determining ϕ^S, ϕ^F, ψ^S, ψ^F and p as:

$$\nabla^2\phi^S - \frac{1}{{C_1}^2}\frac{\partial^2\phi^S}{\partial t^2} - \frac{S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\frac{\partial\phi^S}{\partial t} = 0, \tag{36}$$

$$\phi^F = -\frac{\eta^S}{\eta^F}\phi^S \tag{37}$$

$$\mu^S\nabla^2\psi^S - \rho^S\frac{\partial^2\psi^S}{\partial t^2} + S_v\left[\frac{\partial\psi^F}{\partial t} - \frac{\partial\psi^S}{\partial t}\right] = 0 \tag{38}$$

$$\rho^F\frac{\partial^2\psi^F}{\partial t^2} + S_v\left[\frac{\partial\psi^F}{\partial t} - \frac{\partial\psi^S}{\partial t}\right] = 0 \tag{39}$$

$$(\eta^F)^2 p - \eta^S\rho^F\frac{\partial^2\phi^S}{\partial t^2} - S_v\frac{\partial\phi^S}{\partial t} = 0 \tag{40}$$

where

$$C_1 = \sqrt{\frac{(\eta^F)^2(\lambda^S + 2\mu^S)}{(\eta^F)^2\rho^S + (\eta^S)^2\rho^F}} \tag{41}$$

Assuming the solution of the system of equations (36) - (40) in the form

$$(\phi^S, \phi^F, \psi^S, \psi^F, p) = ({\phi_1}^S, {\phi_1}^F, {\psi_1}^S, {\psi_1}^F, p_1)\exp(i\omega t) \tag{42}$$

where ω is the complex circular frequency.

Making the use of (42) in equations (36)-(40), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{{C_1}^2} - \frac{i\omega S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\right]{\phi_1}^S = 0 \tag{43}$$

$$[\mu^S\nabla^2 + \rho^S\omega^2 - i\omega S_v]{\psi_1}^S = -i\omega S_v{\psi_1}^F \tag{44}$$

$$[-\omega^2\rho^F + i\omega S_v]{\psi_1}^F - i\omega S_v{\psi_1}^S = 0 \tag{45}$$

$$(\eta^F)^2 p_1 + \eta^S\rho^F\omega^2{\phi_1}^S - i\omega S_v{\phi_1}^S = 0 \tag{46}$$

$${\phi_1}^F = -\frac{\eta^S}{\eta^F}{\phi_1}^S \tag{47}$$

Equation (43) corresponds to longitudinal wave propagating with velocity v_1, given by

$${v_1}^2 = \frac{1}{G_1} \tag{48}$$

where

$$G_1 = \left[\frac{1}{{C_1}^2} - \frac{iS_v}{\omega(\lambda^S + 2\mu^S)(\eta^F)^2}\right] \tag{49}$$

From equation (44) and (45), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{{v_2}^2}\right]{\psi_1}^S = 0 \tag{50}$$

Equation (50) corresponds to transverse wave propagating with velocity v_2, given by ${v_2}^2 = 1/G_2$

Where $$G_2 = \left\{\frac{\rho^S}{\mu^S} - \frac{iS_v}{\mu^S\omega} - \frac{{S_v}^2}{\mu^S(-\rho^S\omega^2 + i\omega S_v)}\right\} \tag{51}$$

In medium M_2

$$\phi = B_1\exp\{i\delta_1(x\sin\bar\theta_1 - z\cos\bar\theta_1) + i\bar\omega_1 t\}, \tag{52}$$

$$\psi = B_2\exp\{i\delta_2(x\sin\bar\theta_2 - z\cos\bar\theta_2) + i\bar\omega_2 t\}$$
$$+ B_3\exp\{i\delta_3(x\sin\bar\theta_3 - z\cos\bar\theta_3) + i\bar\omega_3 t\}, \tag{53}$$

$$\Phi_2 = EB_2 \exp\{i\delta_2(x \sin\bar{\theta}_2 - z\cos\bar{\theta}_2) + i\bar{\omega}_2 t\}$$
$$+ FB_3 \exp\{i\delta_3(x \sin\bar{\theta}_3 - z\cos\bar{\theta}_3) + i\bar{\omega}_3 t\}, \qquad (54)$$

where

$$E = \frac{{\delta_2}^2\left({\delta_2}^2 - \frac{\omega^2}{({c_2}^2+{c_3}^2)} + pq\right)}{\text{deno.}} \qquad (55)$$

$$F = \frac{{\delta_3}^2\left({\delta_3}^2 - \frac{\omega^2}{({c_2}^2+{c_3}^2)} + pq\right)}{\text{deno.}} \qquad (56)$$

and

$$\text{deno.} = p\left(2q - \frac{\omega^2}{{c_4}^2}\right); \quad {\delta_2}^2 = {\lambda_2}^2\omega^2; \quad {\delta_3}^2 = {\lambda_3}^2\omega^2 \qquad (57)$$

In medium M_1

$$\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\}[A_{01} \exp\{ik_1(x \sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} + A_1 \exp\{ik_1(x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}], \qquad (58)$$

$$\{\psi^S, \psi^F\} = \{1, m_3\}[B_{01} \exp\{ik_2(x\sin\theta_0 - z\cos\theta_0) + i\omega_2 t\} + A_2 \exp\{ik_2(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}], \qquad (59)$$

where

$$m_1 = -\frac{\eta^S}{\eta^F}; \quad m_2 = -\left[\frac{\eta^S{\omega_1}^2\rho^F - i\omega_1 S_v}{(\eta^F)^2}\right]; \quad m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - {\omega_2}^2\rho^F} \qquad (60)$$

and B_1, B_2, B_3 are amplitudes of transmitted P-wave, transmitted coupled transverse and micro-rotation waves respectively. Also A_{01} or B_{01}, A_1 and A_2 are amplitudes of incident P-wave or SV-wave, reflected P-wave and reflected SV-wave respectively and to be determined from boundary conditions.

4. Boundary conditions

The appropriate boundary conditions are the continuity of displacement, micro rotation and stresses at the interface separating media M_1 and M_2. Mathematically, these boundary conditions at z=0 can be expressed as:

$$t_{zz} = {t_{zz}}^S - p; \quad t_{zx} = {t_{zx}}^S; \quad m_{zy} = 0; \quad t_{zx} = k_t(u^S - u); \quad w = w^S \qquad (61)$$

where $k_t = ik_0\mu\tau$ and $\tau = \gamma/(1-\gamma)\sin\theta_0$

where γ is bonding constant $0 \le \gamma \le 1$, $\gamma = 0$ corresponds to smooth surface and $\gamma = 1$ corresponds to welded interface.

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{v_0} = \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\bar{\theta}_1}{\bar{v}_1} = \frac{\sin\bar{\theta}_2}{\bar{v}_2} = \frac{\sin\bar{\theta}_3}{\bar{v}_3}, \qquad (62)$$

where $\bar{v}_2 = \frac{1}{\lambda_2}$; $\bar{v}_3 = \frac{1}{\lambda_3}$

For longitudinal wave,

$$v_0 = v_1; \quad \theta_0 = \theta_1 \qquad (63)$$

For transverse wave,

$$v_0 = v_2; \quad \theta_0 = \theta_2 \qquad (63A)$$

Also

$$\delta_1\bar{v}_1 = \delta_2\lambda_2^{-1} = \delta_3\lambda_3^{-1} = k_1v_1 = k_2v_2 = \omega, \quad \text{at } z = 0 \tag{64}$$

Making the use of potentials given by equations (52)-(54) and (58)-(59) in the boundary conditions given by (61) and using (62)-(64), we get a system of five non homogeneous equations which can be written as

$$\sum_{j=1}^{5} a_{ij} Z_j = Y_i, \qquad (i = 1,2,3,4,5\,) \tag{65}$$

where

$$Z_1 = \frac{B_1}{B_0}; \quad Z_2 = \frac{B_2}{B_0}; \quad Z_3 = \frac{B_3}{B_0}; \quad Z_4 = \frac{A_1}{B_0}; \quad Z_5 = \frac{A_2}{B_0} \tag{66}$$

where $B_0 = A_{01}$ or B_{01} is amplitude of incident P-wave or SV-wave respectively.
Also Z_1 to Z_5 are the amplitude ratios of transmitted longitudinal wave, transmitted coupled transverse wave at an angle $\bar{\theta}_2$, transmitted coupled microrotational wave at an angle $\bar{\theta}_3$, reflected P wave and reflected SV wave, respectively. Also a_{ij} and Y_i are as

$$a_{11} = -\lambda{\delta_1}^2 - (2\mu+\kappa)\left({\delta_1}^2\cos^2\bar{\theta}_1\right), \quad a_{12} = (2\mu+\kappa){\delta_2}^2\sin\bar{\theta}_2\cos\bar{\theta}_2,$$
$$a_{13} = (2\mu+\kappa){\delta_3}^2\sin\bar{\theta}_3\cos\bar{\theta}_3, \qquad a_{14} = k_1^2(\lambda^S + 2\mu^S\cos^2\theta_1) + m_2,$$
$$a_{15} = -2\mu^S k_2^2\sin\theta_2\cos\theta_2, \qquad a_{21} = (2\mu+\kappa){\delta_1}^2\sin\bar{\theta}_1\cos\bar{\theta}_1,$$
$$a_{22} = \mu{\delta_2}^2\cos 2\,\bar{\theta}_2 + \kappa{\delta_2}^2\cos^2\bar{\theta}_2 - \kappa E, \; a_{23} = \mu{\delta_3}^2\cos 2\,\bar{\theta}_3 + \kappa{\delta_3}^2\cos^2\bar{\theta}_3 - \kappa F,$$
$$a_{24} = \mu^S k_1^2\sin 2\theta_1, \qquad a_{25} = \mu^S k_2^2\cos 2\theta_2,$$
$$a_{31} = 0, \quad a_{32} = \delta_2 E\cos\bar{\theta}_2, \quad a_{33} = \delta_3 F\cos\bar{\theta}_3, \quad a_{34} = 0, \quad a_{35} = 0,$$
$$a_{41} = (2\mu+\kappa){\delta_1}^2\sin\bar{\theta}_1\cos\bar{\theta}_1 + k_t i\,\delta_1\sin\bar{\theta}_1,$$
$$a_{42} = \mu{\delta_2}^2\cos 2\,\bar{\theta}_2 + \kappa{\delta_2}^2\cos^2\bar{\theta}_2 - \kappa E + k_t i\,\delta_2\cos\bar{\theta}_2,$$
$$a_{43} = \mu{\delta_3}^2\cos 2\,\bar{\theta}_3 + \kappa{\delta_3}^2\cos^2\bar{\theta}_3 - \kappa F + k_t i\,\delta_3\cos\bar{\theta}_3,$$
$$a_{44} = -k_t i\,k_1\sin\theta_1, \qquad a_{45} = -k_t i\,k_2\cos\theta_2,$$
$$a_{51} = -i\,\delta_1\cos\bar{\theta}_1, \quad a_{52} = i\,\delta_2\sin\bar{\theta}_2, \quad a_{53} = i\,\delta_3\sin\bar{\theta}_3, \quad a_{54} = -i\,k_1\cos\theta_1,$$
$$a_{55} = i\,k_2\sin\theta_2 = 0,$$

For incident P wave

$$Y_1 = -a_{14}; \quad Y_2 = a_{24}; \quad Y_3 = a_{34}; \quad Y_4 = -a_{44}; \quad Y_5 = a_{54}$$

For incident SV wave

$$Y_1 = a_{15}; \quad Y_2 = -a_{25}; \quad Y_3 = a_{35}; \quad Y_4 = a_{45}; \quad Y_5 = -a_{55} \tag{67}$$

5. Numerical results and discussion

In order to study in more detail the behaviour of various amplitude ratios, we have computed them numerically for a particular model for which the values of relevant elastic parameters are as follow

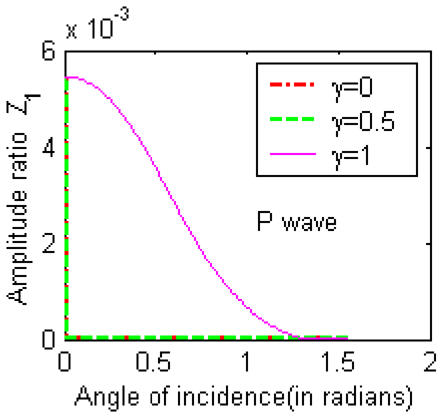
In medium M_2, the physical constants for micropolar elastic solid are taken from Gauthier (1982) as

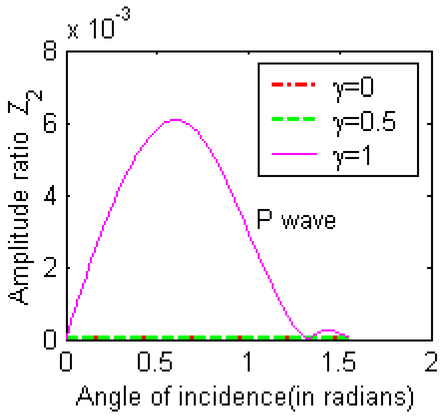
$$\lambda = 7.59\times10^{10}\,\text{N/m}^2, \quad \mu = 1.89\times10^{10}\,\text{N/m}^2, \quad \kappa = 1.49\times10^{8}\,\text{N/m}^2,$$
$$\rho = 2.19\times10^{3}\text{kg/m}^3, \quad \gamma = 2.68\times10^{4}\,\text{N}, \quad j = 1.96\times10^{-6}\text{m}^2,$$
$$\frac{\omega^2}{{\omega_0}^2} = 200. \tag{68}$$

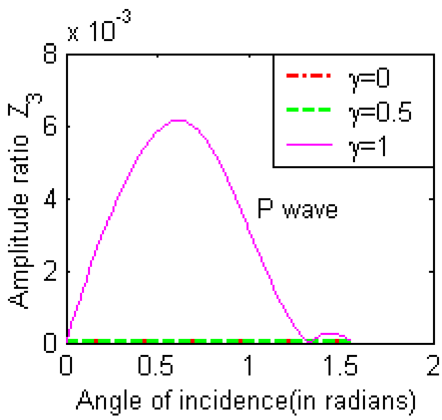
In medium M_1, the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu (1993) as

$$\eta^S = 0.67, \quad \eta^F = 0.33, \quad \rho^S = 1.34\,\text{Mg/m}^3, \quad \rho^F = 0.33\,\text{Mg/m}^3,$$
$$\lambda^S = 5.5833\,\text{MN/m}^2, K^F = 0.01\text{m/s}, \; \gamma^{FR} = 10.00\text{KN/m}^3, \; \mu^S = 8.3750\text{N/m}^2, \tag{69}$$

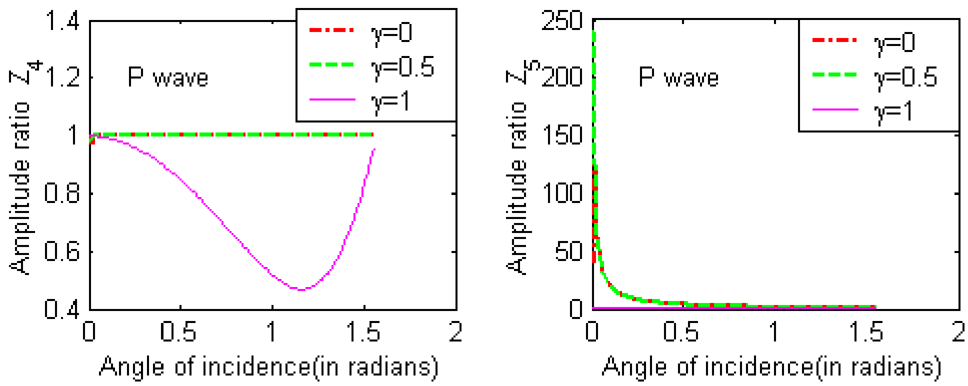
A computer programme in MATLAB has been developed to calculate the modulus of amplitude ratios of various reflected and transmitted waves for the particular model and to depict graphically. In figures (2) - (21) solid lines show the variations of amplitude ratios $|Z_i|$ for welded interface i.e. $\gamma = 1$, dashed lines for bonding parameter $\gamma = 0.5$ and dashed dotted line for smooth interface i.e. $\gamma = 0$. In Figures (2)-(11), there is P wave incident whereas in figures (12)-(21), SV wave is incident. In the figures (2)-(6) and (12)-(16), medium –I is porous solid whereas in figures (7)-(11) and (17)-(21), the medium-I is empty porous solid (EPS), but medium-II is same in all figures i.e. micropolar elastic solid.
In figures (2)-(11), the values of $|Z_i|$, $(i = 1,2,3)$ or modulus of amplitudes ratios corresponding to transmitted waves are greater for bonding parameter $\gamma = 1$ i.e. for welded interface whereas the values of $|Z_i|$, $(i = 4,5)$ are small for modulus of amplitudes ratios corresponding to reflected waves for bonding parameter $\gamma = 1$. Effect of fluid filled in pores of porous solid is significant as it is clear after comparison of figures (2)-(6) and figures (7)-(11). Effect of incident wave is also significant as it is clear by comparing the figures (2)-(6) to corresponding figures (12)-(16). The values of $|Z_i|$, $(i = 1,2,3,4,5)$ are greater for bonding parameter $\gamma = 1$ i.e. for welded interface in case of SV wave incident in comparison to P wave incident. Also, comparing the figures (12)-(16) to figures (17)-(21), the values of $|Z_i|$, $(i = 1,2,3,4,5)$ are large for in case of porous solid than empty porous solid.



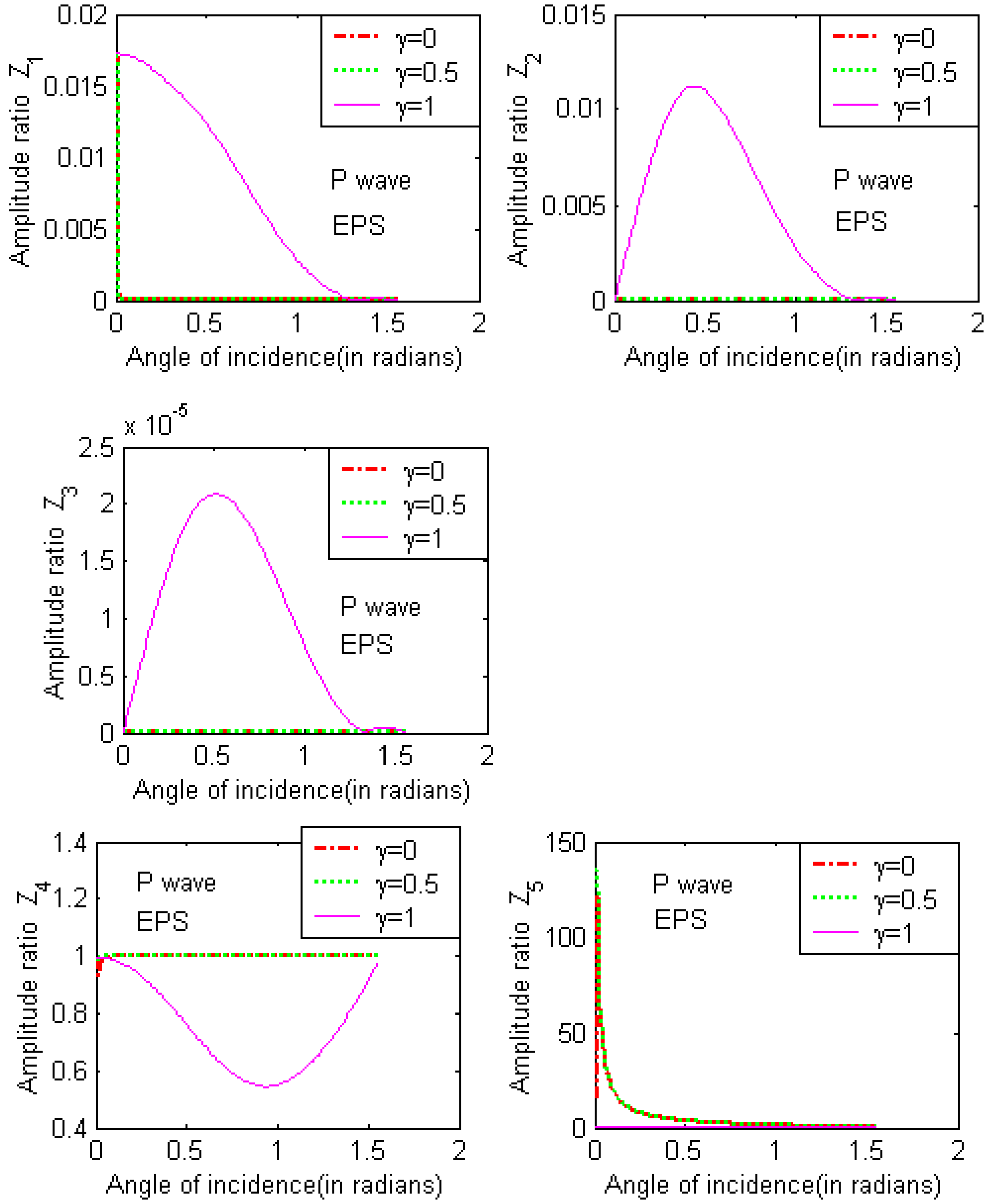




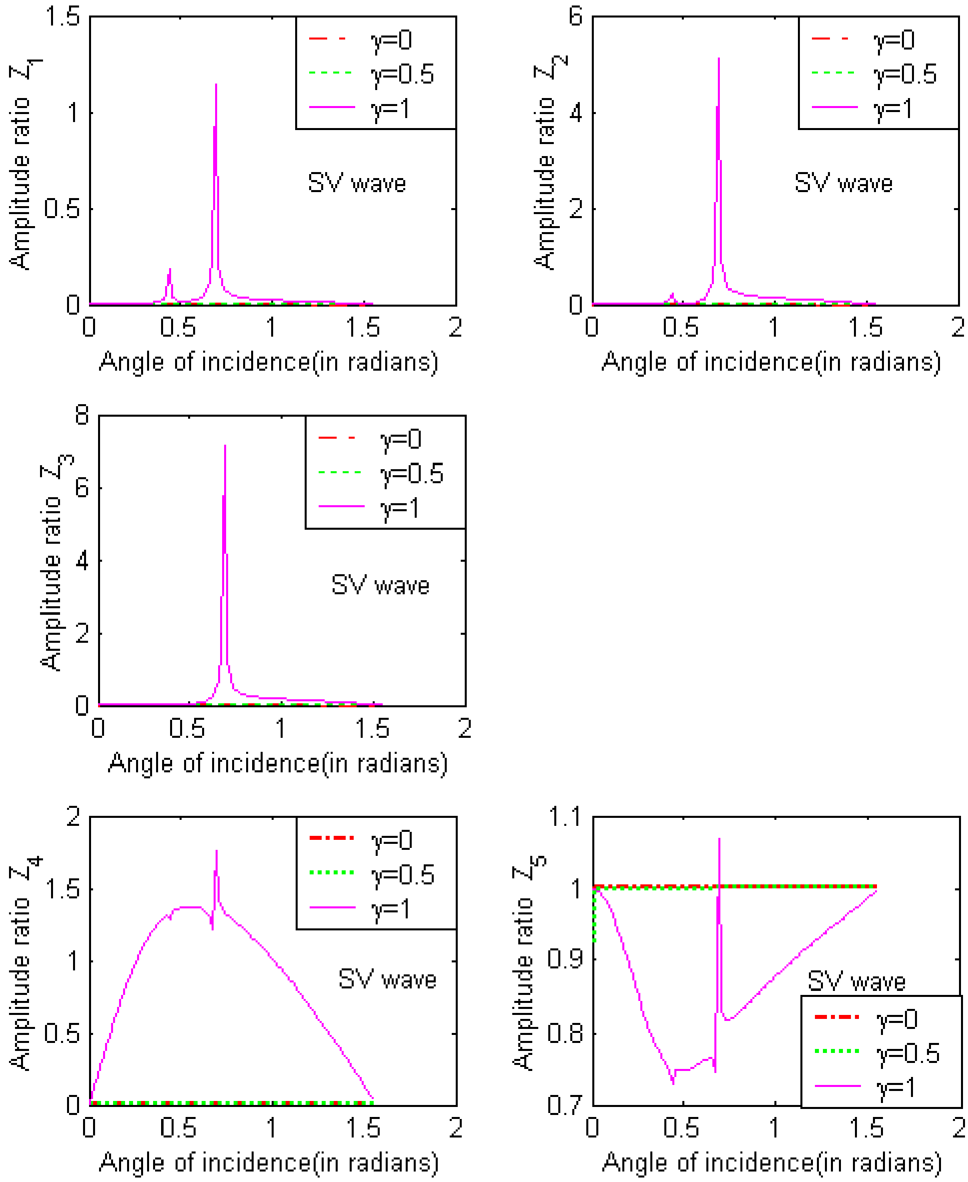
International Journal of Engineering, Science and Mathematics
Vol. 5, Issue 1, March 2016, ISSN: 2320-0294, Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A



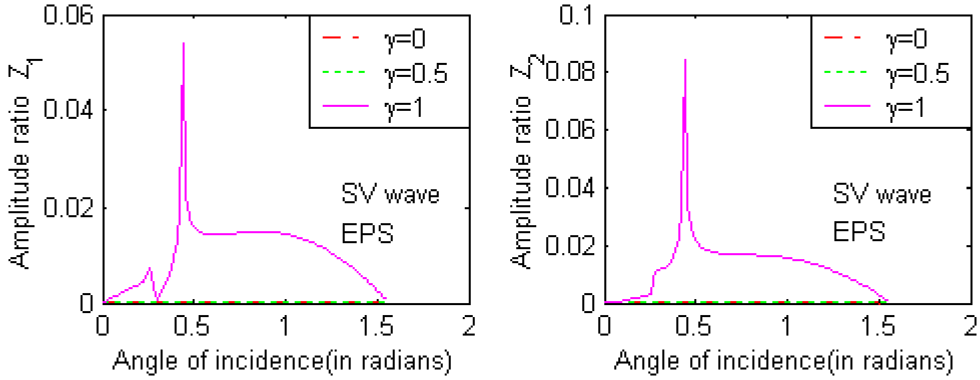
Figures 2-6. Variation of the amplitude ratios with angle of incidence of incident longitudinal wave

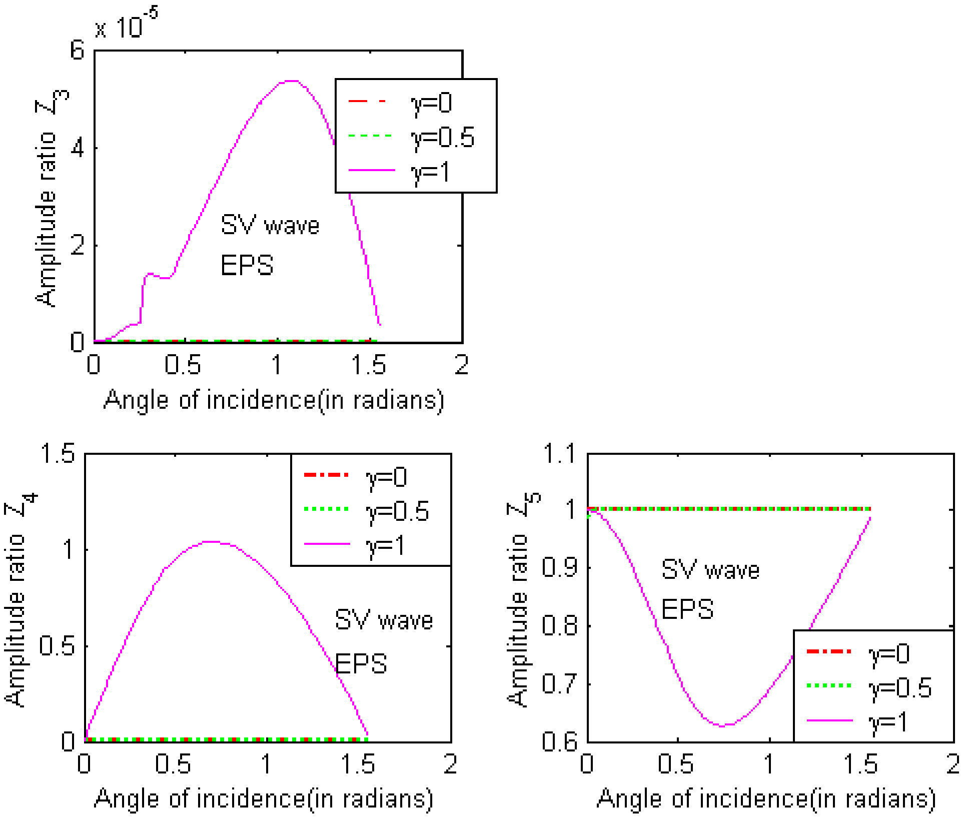


Figures 7-11. Variation of the amplitude ratios with angle of incidence of the incident longitudinal showing porous effect



Figures 12-16. Variation of the amplitude ratios with angle of incidence of incident transverse wave





Figures 17-21. Variation of the amplitude ratios with angle of incidence of incident transverse wave showing porous effect

6. Conclusion

In conclusion, a mathematical study of reflection and transmission coefficients at loosely bonded interface separating micropolar elastic solid half space and fluid saturated incompressible porous half space is made when longitudinal wave or transverse wave is incident. It is observed that

1. The amplitudes ratios of various reflected and transmitted waves are found to be complex valued.
2. The modulus of amplitudes ratios of various reflected and transmitted waves depend on the angle of incidence of the incident wave and material properties of half spaces.
3. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on the amplitudes ratios.
4. The effect of incident wave is significant on amplitude ratios. All the amplitudes ratios are found to depend on incident waves.
5. The effect of bonding parameter for loosely bonded interface is significant either longitudinal wave is incident or transverse wave is incident.

The model presented in this paper is one of the more realistic forms of the earth models. The present theoretical results may provide useful information for experimental scientists/researchers/seismologists working in the area of wave propagation in micropolar elastic solid and fluid saturated incompressible porous solid.

7. References

[1] Bowen, R.M., Incompressible porous media models by use of the theory of mixtures, J. Int. J. Engg. Sci. 18, 1129-1148, 1980.

[2] de Boer, and Didwania, A. K., Two phase flow and capillarity phenomenon in porous solid- A Continuum Thermomechanical Approach, Transport in Porous Media (TIPM), 56, 137-170, 2004.

[3] de Boer, R. and Ehlers, W., The development of the concept of effective stress, Acta Mechanica A 83, 77-92,1990.

[4] de Boer, R. and. Ehlers, W ,Uplift, friction and capillarity-three fundamental effects for liquid- saturated porous solids, Int. J, Solid Structures B, 26,43-57,1990.

[5] de Boer, R., Ehlers, W. and Liu, Z., One dimensional transient wave propagation in fluid saturated incompressible porous media, Arch. App. Mech. 63, 59-72, 1993.

[6] de Boer, R. and Liu, Z., Plane waves in a semi-infinite fluid saturated porous medium, Transport in Porous Media, 16 (2), 147-173, 1994.

[7] Eringen, A.C. and Suhubi, E.S., Nonlinear theory of simple micro-elastic solids I, International Journal of Engineering Science,2,189-203,1964.

[8] Eringen, A.C., Linear theory of micropolar viscoelasticity, International Journal of Engineering Science, 5,191-204, 1968.

[9] Gautheir, R.D., Experimental investigations on micropolar media, Mechanics of micropolar media (eds) O Brulin, R K T Hsieh (World Scientific, Singapore), p.395, 1982.

[10] Kumar, R., Barak, M., Wave propagation in liquid-saturated porous solid with micropolar elastic skelton at boundary surface, Applied Mathematics and Mechanics, 28(3), 337-349, 2007.

[11] Kumar, R. and Hundal, B.S., Wave propagation in a fluid saturated incompressible porous medium, Indian J. Pure and Applied Math.4, 51-65, 2003.

[12] Kumar, R., Miglani, A. and Kumar, S., Reflection and Transmission of plane waves between two different fluid saturated porous half spaces, Bull. Pol. Ac., Tech., 227-234, 59(2), 2011.

[13] Kumar, R. and Singh, B., Reflection and transmission of elastic waves at a loosely bonded interface between an elastic and micropolar elastic solid, Indian Journal of Pure and Applied Mathematics,28,1133-1153,1997.

[14] Kumari, N., Reflection and transmission of longitudinal wave at micropolar viscoelastic solid/fluid saturated incompressible porous solid interface, Journal of Solid Mechanics, 6(3), 240-254, 2014.

[15] Kumari, N., Reflection and transmission phenomenon at an imperfect boundary of viscoelastic solid and fluid saturated incompressible porous solid, Bulletin of Mathematics and Statistics Research, 2(3), 306-319, 2014.

[16] Murty, G.S., Reflection, transmission and attenuation of elastic waves at a loosely bonded interface of two half spaces, Geophys. J.R. Astrom. Soc., 44, 389-404, 1976.

[17] Parfitt, V.R, and Eringen, A.C., Reflection of plane waves from the flat boundary of a micropolar elastic half space, J. Acoust. Soc. Am., 45, 1258-1272, 1969.

[18] Singh, B. and Kumar, R. Wave reflection at viscoelastic-micropolar elastic interface, Applied Mathematics and Computation, 185, 421-431,2007.

[19] Tomar S. K, and Gogna, M. L., Reflection and refraction of longitudinal microrotational wave at an interface between two different micropolar elastic solids in welded contact, Int. J. Eng. Sci. 30:1637-1646, 1992.

[20] Tomar, S. K. and Kumar, R., Reflection and refraction of longitudinal displacement wave at a liquid micropolar solid interface, Int. J. Eng. Sci. 33:1507-1515, 1995.

[21] Tajuddin, M. and Hussaini, S.J., Reflection of plane waves at boundaries of a liquid filled poroelastic half-space, J. Applied Geophysics 58, 59-86, 2006.

[22] Vashisth, A.K., and Gogna, M.L., The effect of loose boundaries on wave propagation in a porous solid: reflection and refraction of seismic waves across a plane interface, international Journal of Solid Structure, 30, 2485-2499, 1993.