International Journal of Engineering, Science and Mathematics
Vol6. xIssue 8, December 2017,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

DISTINCT MECHANISMS OF KINEMATIC CHAINS OF GROUP B

Dr. Ali Hasan

Abstract

Author's objective is to prepare a catalogue of fixed link and its corresponding equivalent links in the distinct mechanisms in kinematic chains of Group B. It will help the new researchers / designers to select the best mechanism kinematic chain and mechanism to perform the desired task at the conceptual stage of design.

Keywords:

Kinematic Chain;
Equivalent Link;
Distinct Mechanism

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1. Introduction

A number of researchers have discussed structural synthesis in the earlier days. Crossley [1] proposed a collection of 10-link plane chains. During the compiling of this collection, his greatest problem was to distinguish whether two arrangements, which might appear unlike, were actually the same or different. This led to a definition of isomorphism between linkages.. Mruthyunjaya [2] made an effort to develop a fully computerized approach for structural synthesis of kinematic chains. Agrawal and Rao [3] investigated a systematic method of analysis of the mobility properties of the kinematic chains by its loop freedom matrix and its permanent function which are used to identify it. Sethi and Agrawal [4] proposed a classification scheme on the basis of structural properties. Madan and Jain [5] considered the kinematic chains-isomorphism, inversions and degree of similarity using the concept of connectivity. Rao [6] threw the light on the enumeration of distinct planar kinematic chains. They developed a very simple method based on independent loop(s) assorted and their adjacency is reported. Misti [7] presented the position analysis in polynomial form of planar mechanisms with Assur groups of class 3 including revolute and prismatic joints. Uicker and Raicu [8] presented a method for the identification and recognition of equivalence of kinematic chains. Later on, this method failed. Mruthyunjaya and Balasubramanian [9] proved that the method proposed by Uicker and Raicu [8] is not reliable. They proved that the test based on comparison of the characteristic coefficients of the adjacency matrices of the corresponding graphs for detection of isomorphism in kinematic chains failed. Shende and Rao [10] work, which deals with the problem of detection of isomorphism which is frequently encountered in structural synthesis of kinematic chains.Chu Jin-Kui and Cao Wei-Qing [11] proposed a method for identification of isomorphism among kinematic chains and inversions using Link's adjacent-chain-table. Yadav, et.al. [12] Proposed a computer aided detection method of isomorphism among kinematic chains and mechanisms using the concept of modified distance. Yadav, et.al.[13] presented a paper mechanism of a kinematic chain and the degree of structural similarity based on the concept of link path code'. Yadav, et.al.[14] presented a paper 'computer aided detection of isomorphism among binary chains using the link-link multiplicity distance concept. Rao [15] suggested the application of fuzzy logic for the study of isomorphism, inversions, symmetry, parallelism and mobility in kinematic chains with some necessary and sufficient conditions. Kong, et.al. [16] Proposed a new method based on artificial neural network (ANN) to identify the isomorphism of the mechanism kinematic chain. Rao and Deshmukh [17] proposed method does not require any separate test for isomorphism in the generation of kinematic chains. Chang, et.al. [18] proposed method is based on the eigen vectors and eigen values to identify isomorphism of mechanism kinematic chain.. He and Jhang [19] proposed a new method for detection of graph isomorphism based on the quadratic form. Tang and Liu [20] established a method 'the degree code' as a new mechanism identifier. Later on this method also failed. Zhao, et.al [21] put forward and more complete theory of degrees of freedom (DOF) for mechanisms, especially for the complex spatial mechanisms, which may not be solved correctly with traditional theories. Hasan et al. [22] but the concept that these methods are based on seems to be unjustified as either link-link adjacency or joint-joint adjacency hardly differ in nature and are likely to fail at some stage or the other. Hasan [23-24] proposed a new method in which kinematic chains are represented in the form of the Joint-Joint [JJ] matrix. Two structural invariants, sum of absolute characteristic polynomial coefficients and maximum absolute value of the characteristic polynomial coefficient are derived from the characteristic polynomials of the [JJ] matrix of the kinematic chains. Dargar et al. [25-26] proposed Link adjacency value method to identify the isomorphism by calculating the first and second link adjacency values. Rizvi et al. [27-28] presented a new method for distinct inversions and isomorphism based on a link identity matrix and link signature. In [28], the authors gave an algorithm for distinct inversions and isomorphism detection in kinematic chains using link identification number. Alam et al.[29] presented weighted squared path technique to determine the structural similarity and dissimilarity in the kinematic chains.

2. Definitions

E-Chain: A chain, in which two polygonal links are directly connected with one joint.
Z-Chain: A chain, in which two polygonal links are connected with the help of one intermediate binary link.
D-Chain: A chain, in which two polygonal links are connected with the help of two intermediate binary links.
V-Chain: A chain, in which two polygonal links are connected with the help of three intermediate binary links.
The Joint-Joint [JJ] Matrix :This matrix (shown in Equation (1)) is based upon the connectivity of the joints through the links and defined, as a square symmetric matrix of size n x n, where n is the number of joints in a kinematic chain.

$$[JJ] = \left\{ L_{ij} \right\}_{n \times n} \qquad \text{---} \quad (1)$$

Where

$$L_{ij} \left\{ \begin{array}{l} = \text{Degree of link between } i^{th} \text{ and } j^{th} \text{ joints those are directly connected} \\ =0, \text{ if joint } i \text{ is not directly connected to joint } j \end{array} \right\}$$

Off course all the diagonal elements $L_{ii} = 0$

Thus the form of [JJ] matrix will be:

$$[JJ] = \begin{pmatrix} 0 & L_{12} & L_{13} & - & - & - & L_{1n} \\ L_{21} & 0 & L_{23} & - & - & - & L_{2n} \\ & - & - & - & - & - & - \\ & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ L_{n1} & L_{n2} & L_{n3} & - & - & - & 0 \end{pmatrix}$$

3. Characteristic polynomial of [JJ] matrix

D (λ) gives the characteristic polynomial of [JJ] matrix. The monic polynomial of degree n is given by equation (2).

$$| (JJ - \lambda I) | = \lambda^{n} + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \text{---------}\ a_{n-1}\lambda + a_n \quad \textbf{(2)}$$

Where; n = number of simple joints in kinematic chain and 1, a_1, a_2, a_{n-1}, a_n are the characteristic polynomial coefficients.The two important properties of the characteristic polynomials are (**a**) The sum of the absolute values of the characteristic polynomial coefficients (SCPC) is an invariant for a [JJ] matrix. i.e. $| 1 | + | a_1 | + | a_2 | + \text{-----} + | a_{n-1} | + | a_n |$ = invariant.(**b**) The maximum absolute value of the characteristic polynomial coefficient (MCPC) is another invariant for a [JJ] matrix.

4. Structural invariants [SCPC] and [MCPC]

The values of characteristic polynomial coefficients are invariants for a [JJ] matrix. To make these [JJ] matrix characteristic polynomial coefficients as a powerful single number characteristic index, new composite invariants have been proposed. These invariants are **'SCPC'** (absolute sum of characteristics polynomials constants) and **'MCPC'** (absolute maximum of characteristics polynomials constants). These invariants are unique for a [JJ] matrix and may be used as identification numbers to detect the isomorphism among simple jointed kinematic chains. The characteristic polynomial coefficients values are the characteristic invariants for the kinematic chains. Many investigators have reported co-spectral graph (non-isomorphic graph having same Eigen spectrum). But these Eigen spectra (Eigen values or characteristic polynomial) have been determined from (0, 1) adjacency matrices. The proposed [JJ] matrix provides distinct set of characteristic polynomial coefficients of the kinematic chains having co-spectral graphs. Therefore, it is verified that the structural invariants **'SCPC' and 'MCPC'** are capable of characterizing all kinematic chains and mechanisms uniquely. Hence, it is possible to detect isomorphism among all the given kinematic chains.

5.ISOMORPHISM OF KINEMATIC CHAINS.

Theorem: Two similar square symmetric matrices have the same characteristic polynomials.
Proof: Let the two kinematic chains are represented by the two similar matrices A and B such that $B = P^{-1}AP$, taking into account that the matrix λI commutes with the matrix P and $| P^{-1} | = | P |^{-1}$. Since the determinant of the product of two square matrices equals the product of their determinants, we have: $| B - \lambda I | = | P^{-1}A P - \lambda I | = | P^{-1}(A - \lambda I) P | = | P^{-1} | \, | (A - \lambda I) | \, | P | = | A - \lambda I |$ .Hence, D (λ) of 'A' matrix = D (λ) of 'B' matrix. D (λ) =characteristic polynomial of the matrix. It means that if D (λ) of two [JJ] matrices representing two kinematic chains is same, their structural invariants 'SCPC' and 'MCPC' will also be same and the two kinematic chains are isomorphic otherwise non-isomorphic chains.

6. ILLUSTRATIVE EXAMPLE 1

The example concerns with 8–bars, 10-joints, 1 F as shown in Fig.1.
The degrees of links of kinematic chain shown in Fig.1 are: d (l_a) =d (l_b) =d (l_c) =d (l_d) =3,d (l_e) = d (l_f) =d (l_g) =d (l_h) =2. The joint-joint matrix representing the kinematic chain [Fig.1] using eq. (1) is given by [JJ]. The set of structural invariants derived from the [JJ] matrix using software MAT LAB for the kinematic chain shown in Fig.1 are: SCPC = 3.0011e+005, MCPC =134784. Fixing link 'a', the first mechanism is developed. The link 'a' is a ternary link having joints 1, 2 and 7. Hence, changing the diagonal element L_{11}, L_{22} and L_{77} from 0 to 1 of [JJ] matrix. The [JJ-a] matrix is obtained .The structural invariants of the first mechanism are derived from the kinematic chain shown in Fig.1 are ; SCPC -a = 3.4310e+005, MCPC -a = 1.6407e+005. Similarly, the links b, c, d, - - - etc are fixed in turn and the diagonal elements of all the corresponding joints in the [JJ] matrix of the fixed link are changed from 0 to one of [JJ] matrix. The structural invariants of other mechanisms are obtained which are; SCPC-b=3.4310e+005,SCPC-c=3.4310e+005,SCPC-d = 3.4310e+005 ,

SCPC-e = 4.5503 e+005, SCPC-f=4.5503e+005,SCPC-g=4.5503e+005,SCPC-h=4.5503e+005,MCPC-b = 164070, MCPC-c=1.6407e+005,MCPC-d=1.6407e+005,MCPC-e=2.3069e+005,MCPC–f=2.3069e+005,MCPC–g = 2.3069e+005 ,MCPC -h = 2.3069e+005.

Observing the structural invariants for the above eight mechanisms, it is found that the structural invariants of link – a, b, c and d are same. Hence, they are treated as equivalent links and form only one distinct mechanism. Similarly, the structural invariants of link- e, f, g and h are same, hence form second distinct mechanism. Therefore, 2 distinct mechanisms are obtained from kinematic chain shown in Fig.1.

$$[JJ]=\begin{pmatrix} 0&3&0&3&3&0&3&0&0&0\\ 3&0&3&0&0&0&3&3&0&0\\ 0&3&0&3&0&0&0&3&0&3\\ 3&0&3&0&3&0&0&0&0&3\\ 3&0&0&3&0&2&0&0&0&0\\ 0&0&0&0&2&0&2&0&0&0\\ 3&3&0&0&0&2&0&0&0&0\\ 0&3&3&0&0&0&0&0&2&0\\ 0&0&0&0&0&0&0&2&0&2\\ 0&0&3&3&0&0&0&0&2&0 \end{pmatrix}, \quad [JJ\text{-}a]=\begin{pmatrix} 1&3&0&3&3&0&3&0&0&0\\ 3&1&3&0&0&0&3&3&0&0\\ 0&3&0&3&0&0&0&3&0&3\\ 3&0&3&0&3&0&0&0&0&3\\ 3&0&0&3&0&2&0&0&0&0\\ 0&0&0&0&2&0&2&0&0&0\\ 3&3&0&0&0&2&1&0&0&0\\ 0&3&3&0&0&0&0&0&2&0\\ 0&0&0&0&0&0&0&2&0&2\\ 0&0&3&3&0&0&0&0&2&0 \end{pmatrix}$$

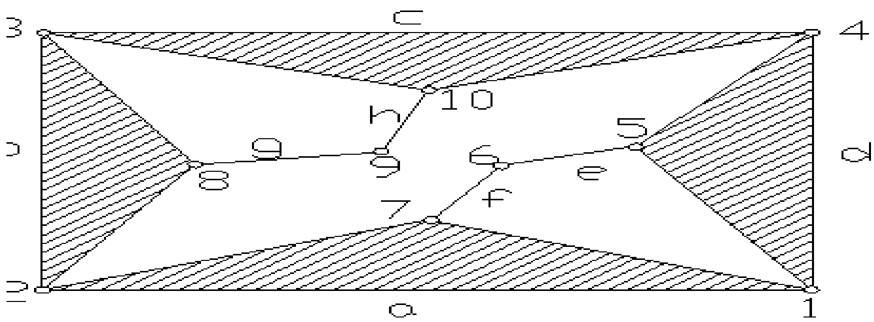

Fig. 1: Eight –Link, Single dof Kinematic Chain

7. Results

The proposed invariants [SCPC] and [MCPC] are used as the identification number of the kinematic chains having simple joints. The identification numbers of all 1-dof kinematic chains up to 10-Links are with the author .These invariants are also able to detect isomorphism among the kinematic chains with multiple joints also. All the KC of Group B from Jensen [30] is redrawn in Table-1 while Equivalent Links and Distinct Mechanisms are given in Table 2.

8. CONCLUSIONS

In this paper, a simple, efficient, and reliable method to identify equivalent links and distinct mechanisms is proposed. It incorporates all features of the kinematic chains and as such, violation of the isomorphism test is rather difficult. The method has been found to be successful in distinguishing all known 16 kinematic chain of 8-links, 230 kinematic chain of 10-links having 1-F. The advantage is that they are very easy to compute using MATLAB software. It is not essential to determine both the structural invariants to compare two chains, only in case the [SCPC] is same then it is needed to determine [MCPC] for both kinematic chains. The [JJ] matrices can be written with very little effort, even by mere inspection of the chain. The proposed test is quite general in nature and can be used to detect isomorphism of not only planar kinematic chains of one degree of freedom, but also kinematic chains of multi degree of freedom.

Table-1: 10 Links 13 Joints 1F Kinematic Chains of GROUP B

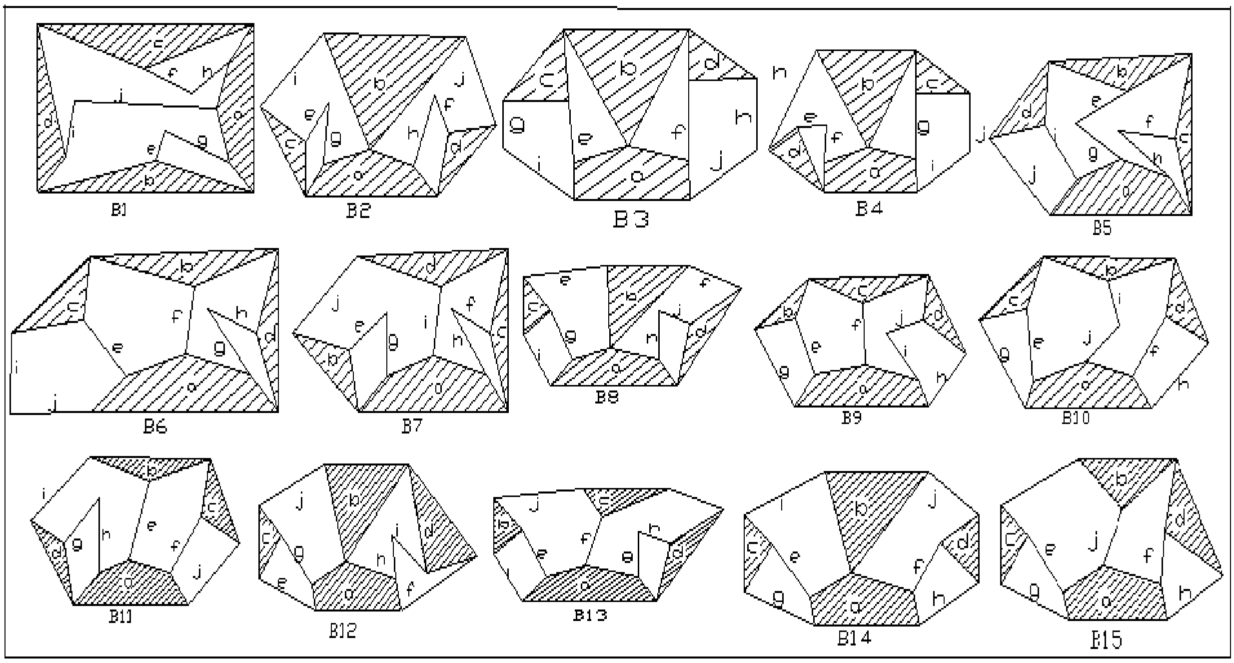

Table-2: Equivalent links and D.M. (Distinct Mechanisms) of KC of GROUP B

KC No.	Structural Invariants	$n_5 n_4 n_3 n_2$	EZDV Chains	Equivalent links	D.M.
1	SCPC(B 1) = 2.9434e+007 MCPC(B 1) = 1.2179e+007	1036	4030	a, b=c, d, e=f, g=h, i, j.	7
2	SCPC(B 2) = 8.3443e+007 MCPC(B 2) = 3.0804e+007	1036	3220	a, b, c=d, e=f, g=h, i=j	6
3	SCPC(B 3) = 1.0459e+008 MCPC(B 3) = 3.6124e+007	1036	3220	a, b, c=d, e=f, g=h, i=j.	6
4	SCPC(B 4) = 7.5611e+007 MCPC(B 4) = 2.2178e+007	1036	3220	a, b, c, d, e, f, g, h, i, j.	10
5	SCPC(B 5) = 9.8280e+007 MCPC(B 5) = 2.9778e+007	1036	3220	a, b, c, d, e, f, g, h, i, j.	10
6	SCPC(B 6) = 1.3308e+008 MCPC(B 6) = 4.4750e+007	1036	3220	a, b, c, d, e, f, g, h, i, j.	10
7	SCPC(B 7) = 9.3750e+007 MCPC(B 7) = 3.0903e+007	1036	3220	a, b, c, d, e, f, g, h, i, j.	10
8	SCPC(B 8) = 1.3743e+008 MCPC(B 8) = 5.2656e+007	1036	2410	a, b, c, d, e, f, g, h, i, j.	10
9	SCPC(B 9) = 3.5777e+008 MCPC(B 9) = 1.4640e+008	1036	2410	a, b, c, d, e=f, g, h, i, j.	9
10	SCPC(B 10) = 3.1304e+008 MCPC(B 10) = 1.0261e+008	1036	2410	a, b, c=d, e=f=g=h, i, j.	6
11	SCPC(B 11) = 1.6131e+008 MCPC(B 11) = 4.9551e+007	1036	2410	a, b, c, d, e, f, g, h, i, j.	10
12	SCPC(B 12) = 3.1755e+008 MCPC(B 12) = 8.5510e+007	1036	2410	a, b, c, d, e, f=g, h, i, j.	9
13	SCPC(B 13) = 2.0396e+008 MCPC(B 13) = 8.6620e+007	1036	1510	a, b, c, d, e=f, g, h, i, j.	9
14	SCPC(B 14) = 3.4929e+008 MCPC(B 14) = 1.3072e+008	1036	1600	a, b, c=d, e=f, g=h, i=j.	6
15	SCPC(B 15) = 6.7910e+008 MCPC(B 15) = 3.0248e+008	1036	1600	a, b, c, d, e=f, g=h, i, j.	8
Total distinct mechanisms =					126

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