
EFFECTS OF SLIP CONDITIONS, WALL PROPERTIES WITH THE HEAT TRANSFORMS ON PERISTALTIC FLOW OF A JEFFREY FLUID IN NON UNIFORM CHANNEL

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Abstract

In this paper attempt is made to show the effects of both wall slip properties and heat transfer on the peristaltic flow of a Jeffrey fluid in a porous channel with flexible wall properties have been investigated under the assumptions of long wavelength and low Reynolds number. A closed form solution has been seen at stream function, heat transfer coefficient, velocity, and temperature. The effects of various physical parameters on velocity and temperature are analyzed through graphs and the results are discussed in detail.

Keywords:

Peristaltic flow, Slip conditions, Wall properties, Heat transfer, Jeffrey fluid and Non- uniform channel.

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1. Introduction

The word peristaltic stem derived from the Greek word peristaltic. It means clasping and compressing. In peristaltic pumping process fluid transport in distensible tube is simultaneously occur as the progressive waves travel among wall of the tube. When stimulated at any point contractile rings emerge and spread in the circular muscle tubes. In this way, peristaltic occurs in the gastrointestinal tract, the bile ducts, other glandular ducts throughout the body, the ureters, and many other smooth muscle tubes of the body. It has many applications in medicine and industry.

Peristalsis mechanism has recently become a very important subject of scientific research in both mechanical and physiological situations the first research work carried out by Lathan [1] in this

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field. Also the literature on peristalsis is extensive. Some investigations [2-5] are presented in the references.

Most of the biofluids like blood are observed to behave like non-Newtonian fluids. In fact there is no unique model to describe all non-Newtonian fluids in physiological systems, several models are proposed [6-12] to explain the behavior of these biofluids. Several researchers studied the peristaltic flow with heat transfer due to its wide applications in fluid mechanics. Vajravelu et al. [13] analyzed the peristaltic flow and heat transfer in a vertical porous annulus with long wavelength approximation. Srinivas and Kothandapani [14] carried their research on the influence of slip conditions, wall properties and heat transfer on MHD peristaltic flow through porous space with compliant walls.

Hayat et al. [15] carried out his investigations in the MHD peristaltic channel flow of a Jeffrey fluid with compliant walls and porous medium.

The present study deals with the influence of both wall slip conditions and heat transfer on peristaltic flow of a Jeffrey fluid in a non-uniform channel with elastic wall properties have been analyzed under the assumptions of long wavelength and low-Reynolds number.

2. Mathematical formulation of the problem

Let us consider the peristaltic flow of a Jeffrey fluid through a two-dimensional channel of uniform thickness. The walls of the channel are assumed to be flexible and are taken as stretched membranes, on which travelling sinusoidal waves of moderate amplitude are imposed.

The wall geometry of the surface is given by

$$\bar{\eta}(\bar{x}, \bar{t}) = d(x) - a \sin \frac{2\pi}{\lambda} (\bar{x} - c\bar{t}) \quad (1)$$

where $d(x) = d + m\bar{x}$, $\bar{m} < 1$

The governing equations governing the flow are given by

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (2)$$

$$\rho \left(\frac{\partial \bar{u}}{\partial \bar{t}} + u \frac{\partial \bar{u}}{\partial \bar{x}} + v \frac{\partial \bar{u}}{\partial \bar{y}} \right) = - \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} - \frac{\partial}{\partial \bar{y}} \left(\tau_0 - \frac{\bar{\mu}}{1 + \lambda_1} \frac{\partial \bar{u}}{\partial \bar{y}} \right) \quad (3)$$

$$\rho \left(\frac{\partial \bar{v}}{\partial \bar{t}} + u \frac{\partial \bar{v}}{\partial \bar{x}} + v \frac{\partial \bar{v}}{\partial \bar{y}} \right) = - \frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\bar{\mu}}{1 + \lambda_1} \left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \right) \quad (4)$$

$$\zeta \left(\frac{\partial \bar{T}}{\partial t} + u \frac{\partial \bar{T}}{\partial x} + v \frac{\partial \bar{T}}{\partial y} \right) = \frac{k}{\rho} \left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} \right) + 2v \left(\left(\frac{\partial \bar{u}}{\partial x} \right)^2 + \left(\frac{\partial \bar{u}}{\partial y} \right)^2 \right) + \left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right)^2 \quad (5)$$

where \bar{u} , \bar{v} , ρ , μ , p , d , a , λ , c , ζ , k , T , \bar{m} and τ_0 are the axial velocity, transverse velocity, fluid density, viscosity of the fluid, pressure, mean width of the channel, amplitude, wavelength, wave speed, dimensional non-uniformity of the channel, specific heat at constant volume, kinematic viscosity, thermal conductivity of the fluid, temperature and yield stress.

The equation of motion of the flexible wall is expressed as

$$H^*(\bar{\eta}) = \bar{p} - \bar{p}_0 \quad (6)$$

here the operator H^* is used to represent the motion of stretched membrane with viscosity damping forces such that

$$H^* = -\tau \frac{\partial^2}{\partial x^2} + m_1 \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t} \quad (7)$$

where m_1 is the mass per unit area, C is the coefficient of viscous damping forces, τ is the elastic tension in the membrane, p_0 is the pressure on the outside surface of the wall due to the tension in the muscles and h is the dimensional slip parameter. We considered $p_0 = 0$

Continuity of stress at $\bar{y} = \pm \bar{\eta}$ and using x - momentum equation, yield

$$\frac{\partial}{\partial x} H^*(\bar{\eta}) = \frac{\partial \bar{p}}{\partial x} = \mu \frac{\partial^2 \bar{u}}{\partial x^2} - \frac{\partial}{\partial y} \left(\tau_0 - \frac{\mu}{1 + \lambda_1} \frac{\partial \bar{u}}{\partial y} \right) - \rho \left(\frac{\partial \bar{u}}{\partial t} + u \frac{\partial \bar{u}}{\partial x} + v \frac{\partial \bar{u}}{\partial y} \right) \quad (8)$$

$$\bar{u} = -h \frac{\partial \bar{u}}{\partial y} \quad \text{at} \quad \bar{y} = \eta \quad (9)$$

$$\begin{aligned} \frac{\partial T}{\partial y} &= 0 \quad \text{on} \quad y = y_0 \\ T &= T_1 \quad \text{on} \quad y = \eta \end{aligned} \quad (10)$$

The function ψ is defined as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

Consider the following non-dimensional parameters are

$$\left. \begin{aligned} x &= \frac{\bar{x}}{\lambda}, y = \frac{\bar{y}}{d}, \psi = \frac{\bar{\psi}}{cd}, p = \frac{d^2 \bar{p}}{\mu c \lambda}, t = \frac{c \bar{t}}{\lambda}, K = \frac{k}{d^2}, \\ m &= \frac{\lambda \bar{m}}{d}, \delta = \frac{d}{\lambda}, \varepsilon = \frac{a}{d}, \eta = \frac{\bar{\eta}}{d} = 1 + mx + \varepsilon \sin 2\pi(x-t), \\ R &= \frac{\rho cd}{\mu}, \theta = \frac{(T-T_0)}{(T_1-T_0)}, Pr = \frac{\rho v \xi}{k}, Ec = \frac{c^2}{\xi(T_1-T_0)}, \\ E_1 &= \frac{-\tau d^3}{\lambda^3 \mu c}, E_2 = \frac{m_1 c d^3}{\lambda^3 \mu}, E_3 = \frac{c d^3}{\lambda^2 \mu}, \beta = \frac{h}{d} \end{aligned} \right\} \quad (11)$$

where R is the Reynolds number, δ and ε are the dimensionless geometric parameters, Ec is the Eckert number, Pr is the Prandtl number, E_1, E_2 and E_3 are the dimensionless elasticity parameters, β is the slip parameter (Knudsen number) and m is the non-uniform parameter.

Using non-dimensional quantities the basic equations (1) - (10) reduce to

$$R\delta \left(\frac{\partial^2 \psi}{\partial t \partial y} + \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} \right) = -\frac{\partial p}{\partial x} + \delta^2 \frac{\partial^3 \psi}{\partial x^2 \partial y} - \frac{\partial}{\partial y} \left(\tau_0 - \frac{\mu}{1 + \lambda_1} \frac{\partial^2 \psi}{\partial y^2} \right) \quad (12)$$

$$R\delta^3 \left[\frac{\partial^2 \psi}{\partial t \partial y} + \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial x \partial y} \right] = -\frac{\partial p}{\partial y} + \delta^2 \left(\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^3 \psi}{\partial y \partial y^2} \right) \quad (13)$$

$$R\delta \left[\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right] = \frac{1}{Pr} + \left(\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + E \left(4\delta^2 \left(\frac{\partial^2 \psi}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right)^2 \right) \quad (14)$$

$$\frac{\partial \psi}{\partial y} = \beta \frac{\partial^2 \psi}{\partial y^2} \text{ at } y = \eta \quad (15)$$

$$\begin{aligned} -R\delta \left(\frac{\partial^2 \psi}{\partial t \partial y} + \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} \right) &= \delta^2 \frac{\partial^3 \psi}{\partial x^2 \partial y} - \frac{\partial}{\partial y} \left(\tau_0 - \frac{\mu}{1 + \lambda_1} \frac{\partial^2 \psi}{\partial y^2} \right) \\ &= \left(E_1 \frac{\partial^3 \eta}{\partial x^3} + E_2 \frac{\partial^3 \eta}{\partial x \partial t^2} + E_3 \frac{\partial^2 \eta}{\partial x \partial t} \right) \end{aligned} \quad (16)$$

Further, it is assumed that the zero value of the streamline at the line $y = 0$, i.e.

$$\begin{aligned} \psi_p(0) &= 0, \\ \psi_{yy}(0) &= \tau_0 \text{ at } y = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\partial \theta}{\partial y} &= 0 \text{ on } y = y_0 \\ \theta &= 1 \text{ on } y = \eta \end{aligned} \tag{18}$$

3. Solution of the problem

Applying the long wavelength and low Reynolds number approximation, the basic equations (12) - (18) reduce to

$$\frac{\partial p}{\partial x} = -\frac{\partial}{\partial y} \left(\tau_0 - \frac{\mu}{1 + \lambda_1} \frac{\partial^2 \psi}{\partial y^2} \right) \tag{19}$$

$$\frac{\partial p}{\partial y} = 0 \tag{20}$$

Equation (20) shows that p is not a function of y

$$\frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + E \left(\frac{\partial \psi}{\partial y} \right)^2 = 0 \tag{21}$$

By differentiating equation (19) with respect y we obtain

$$\frac{\partial^2}{\partial y^2} \left(-\tau_0 + \frac{\mu}{1 + \lambda_1} \frac{\partial^2 \psi}{\partial y^2} \right) = 0 \tag{22}$$

From equation (16) we get

$$\frac{\partial}{\partial y} \left(-\tau_0 + \frac{\mu}{1 + \lambda_1} \frac{\partial^2 \psi}{\partial y^2} \right) = \left(E_1 \frac{\partial^3 \eta}{\partial x^3} + E_2 \frac{\partial^3 \eta}{\partial x \partial t^2} + E_3 \frac{\partial^2 \eta}{\partial x \partial t} \right) \tag{23}$$

By solving equation (22) with boundary conditions (15), (17) and (23) we obtain the stream function in the plug flow region as

$$\psi_p = A \left[(y_0^2 - \beta y_0 - \eta y_0) - \frac{1}{2} (y_0^2 - \eta^2 - 2\beta \eta) \right] y \tag{24}$$

and corresponding plug flow velocity is given by

$$u_p = A \left[(y_0^2 - \beta y_0 - \eta y_0) + \frac{1}{2} (y_0^2 - \eta^2 - 2\beta \eta) \right] \tag{25}$$

where

$$y_0 = \frac{\tau_0}{A}$$

and

$$A = -8\xi\pi^3 \left[(E_1 + E_2) \cos 2\pi(x-t) - \frac{E_3}{2\pi} \sin 2\pi(x-t) \right]$$

and the stream function in the non-plug flow region as

$$\begin{aligned} \psi = \tau_0 \left[\frac{(y^2 - y_0^2)}{2} - \beta(y - y_0) - \eta(y - y_0) \right] \\ + A \left[\frac{y^3 - 8y_0^3}{6} - \frac{\eta^2 y}{2} - \beta\eta y - y_0^2(\beta + \eta) \right] \end{aligned} \quad (26)$$

The corresponding velocity in the non-plug flow region is given by

$$u = \tau_0(y - \beta - \eta) + \frac{A}{2}(y^2 - \eta^2 - 2\beta\eta) \quad (27)$$

Using equation (26) in equation (21) subject to the condition (18) we obtain the temperature as

$$\theta = -Br \left[\tau_0^2 \frac{y^3}{6} + \frac{A^2 y^4}{12} + \frac{\tau_0 A y^3}{3} \right] + C_1 y + C_2 \quad (28)$$

$$\text{where } C_1 = Br \left[\tau_0^2 \frac{y_0^2}{2} + \frac{A^2 y_0^3}{3} + \tau_0 A y_0^2 \right]$$

$$C_2 = 1 + Br \left[\tau_0^2 \frac{\eta^3}{6} + \frac{A^2 \eta^4}{12} + \tau_0 A \frac{\eta^3}{3} \right] - C_1 \eta$$

and $Br = Ec Pr$ is the Brinkman number

The coefficient of heat transfer at the wall is given by

$$Nu = -(\theta_y)_{at y=\eta} \quad (29)$$

4. Results and Discussions

Equation (27) gives the expression for velocity as a function of y . Velocity profiles are plotted from Fig.1 to Fig.4 to study the effects of different parameters such as slip parameter β , amplitude ratio ε , non-uniform parameter m and yield stress parameter τ_0 on the velocity distribution.

Fig.1 is plotted for different values of slip parameter β . It is observed that the velocity profiles are parabolic and the velocity increases with increasing β .

From Fig.2 and Fig.3 we noticed that the velocity increases with increasing amplitude ratio ε and decreasing yield stress parameter τ_0 . Fig.4 depicts that the velocity for a divergent channel ($m > 0$) is higher compared with uniform channel ($m = 0$) where as it is lower for a convergent channel ($m < 0$).

Equation (28) gives the expression for temperature as a function of y . Temperature profiles are plotted from Fig.5 to Fig.8 to study the effects of different parameters such as Brinkman number Br , non-uniform parameter m , amplitude ratio ε and yield stress parameter τ_0 on the temperature distribution. It is observed that the temperature profiles are almost parabolic. Fig.5 and Fig.6 are

plotted to study the effect of Brinkman number Br and amplitude ratio ε . We notice that the temperature increases with increasing Brinkman number Br and amplitude ratio ε . Fig.7 shows that the temperature decreases with increasing yield stress parameter τ_0 . Fig.8 reveals that the temperature is higher diverging channel ($m > 0$) compared with uniform ($m = 0$) and convergent ($m < 0$) channels.

The rate of heat transfer (Nu) is calculated in equation (29). The variation in Nusselt number for different values of the interesting non-uniform parameter m , amplitude ratio ε and yield stress parameter τ_0 can be examined through the figures Fig.9-Fig.11. It is noticed that due to peristalsis, the rate of heat transfer (Nu) shows oscillatory behaviour. Figures Fig.10 to Fig.12 depict that the Nusselt number increases with increasing amplitude ratio ε , non-uniform parameter m , and yield stress parameter τ_0 .

Trapping is an interesting phenomenon which refers to closed circulating streamlines that exist at every high flow rates and when occlusions are very large. Streamlines are plotted to study the effect of slip parameter β and non-uniform parameter m on trapping through Fig.12 and Fig.13. From Fig.12 we observe that the number of trapped boluses increases with increasing slip parameter. Fig.13 reveals that the number of trapped boluses increases with increasing non-uniform parameter.

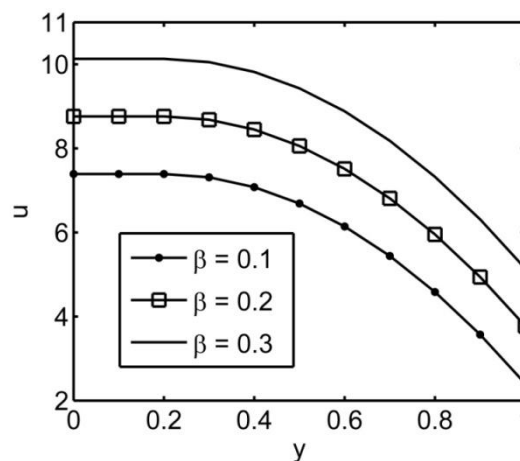


Fig.1 Velocity profiles for different values " β " with fixed

$$y_0 = 0.2, x = 0.2, t = 0.1, m = 0.1, Br = 2, \varepsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$$

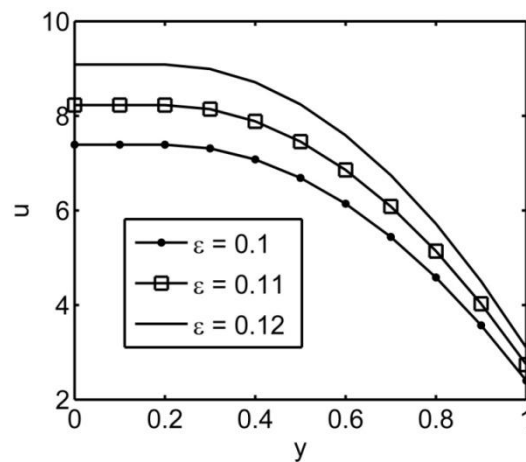


Fig.2 Velocity profiles for different values " ϵ " with fixed

$y_0 = 0.2, x = 0.2, t = 01, m = 0.1, Br = 2, \beta = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

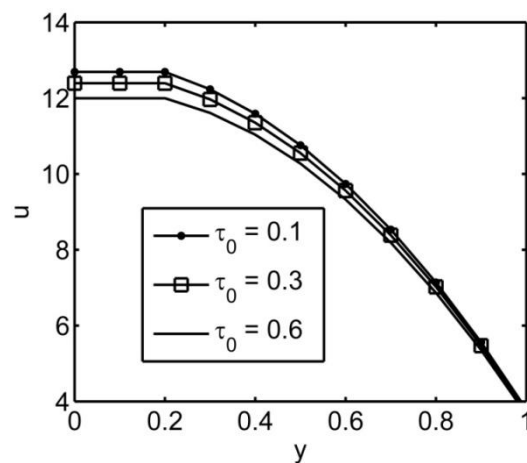


Fig.3 Velocity profiles for different values " τ_0 " with fixed

$x = 0.2, t = 01, m = 0.1, Br = 2, \epsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

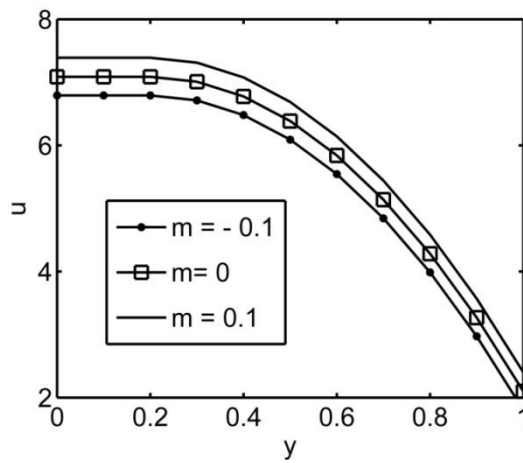


Fig.4 Velocity profiles for different values " m " with fixed

$y_0 = 0.2, x = 0.2, \beta = 0.1, m = 0.1, Br = 2, \varepsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

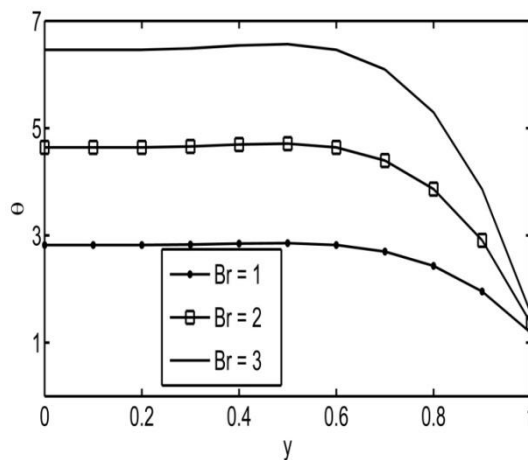


Fig.5 Temperature profiles for different values " Br " with fixed

$y_0 = 0.2, x = 0.2, t = 0.1, m = 0.2, \varepsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

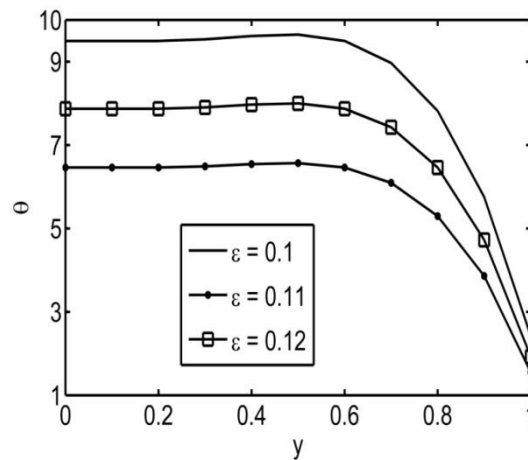


Fig.6 Temperature profiles for different values " ε " with fixed $y_0 = 0.2, x = 0.2, t = 0.1, Br = 2, m = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

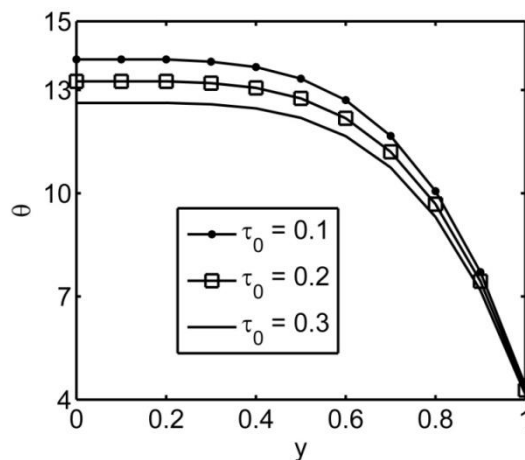


Fig.7 Temperature profiles for different values " τ_0 " with fixed $x = 0.2, t = 0.1, Br = 2, \varepsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

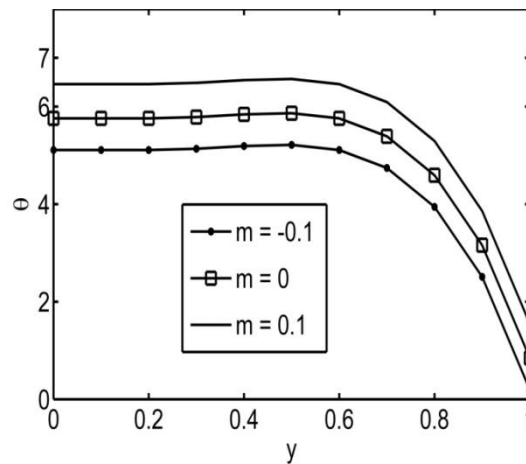


Fig.8 Temperature profiles for different values "m" with fixed $y_0 = 0.2, x = 0.2, t = 0.1, Br = 2, \varepsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

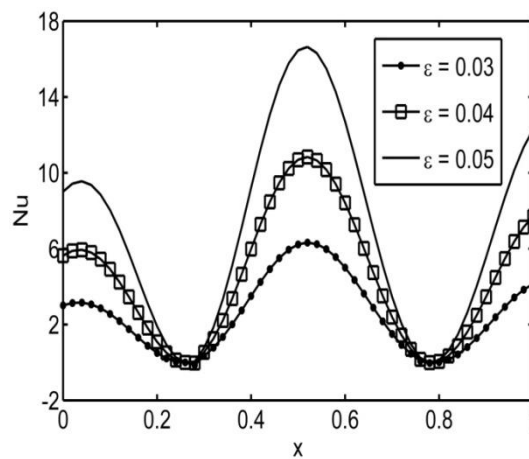


Fig.9 Variation of "Nu" for different values "epsilon" with fixed $\tau_0 = 0.2, x = 0.2, t = 0.1, Br = 2, m = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

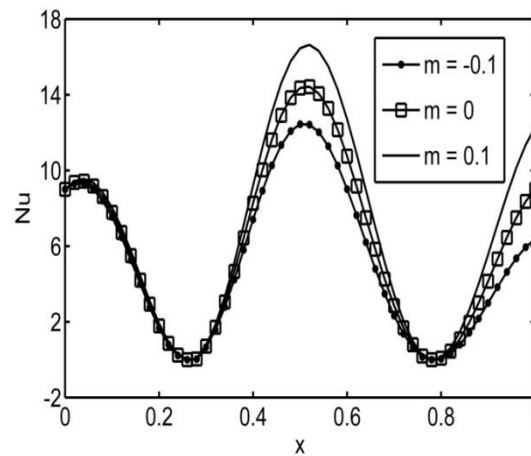


Fig.10 Variation of "Nu" for different values "m" with fixed

$\tau_0 = 0.2, x = 0.2, t = 0.1, Br = 2, \varepsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

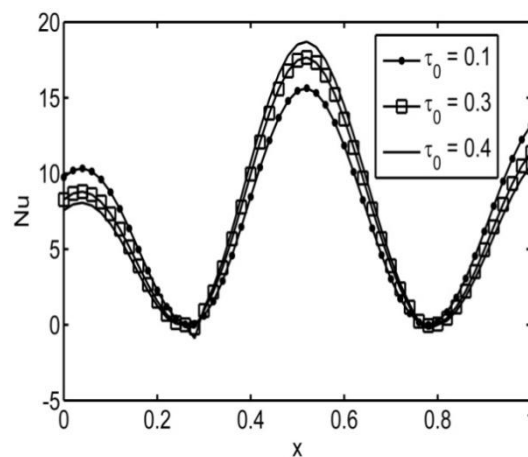


Fig.11 Variation of "Nu" for different values " τ_0 " with fixed

$m = 0.1, x = 0.2, t = 0.1, Br = 2, \varepsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

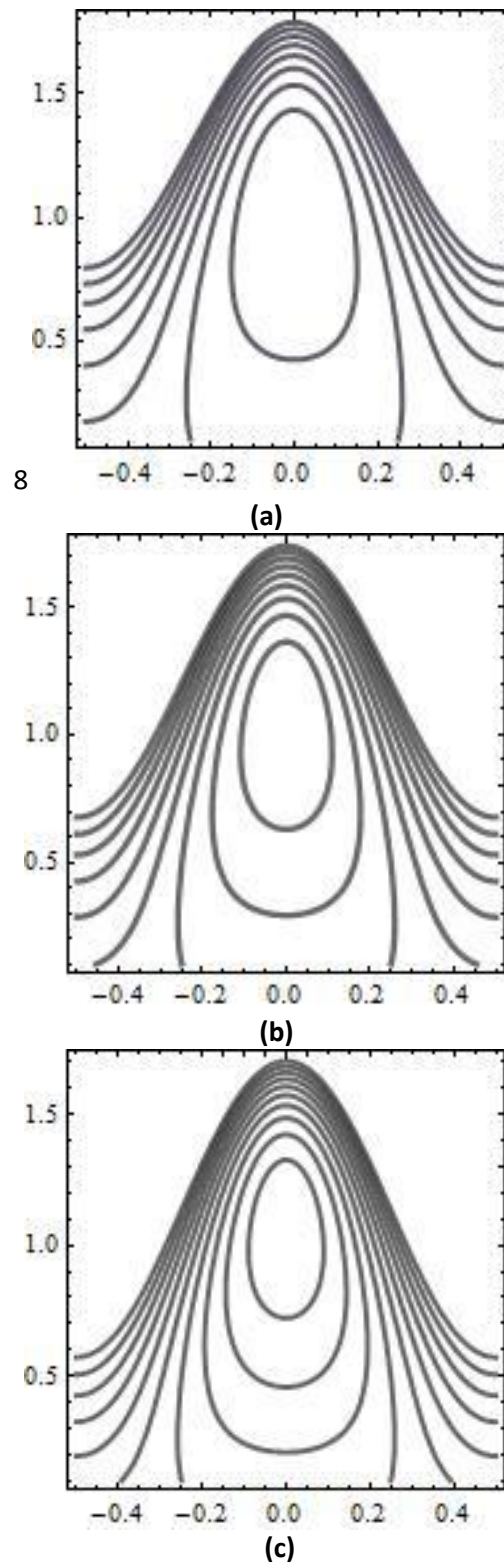
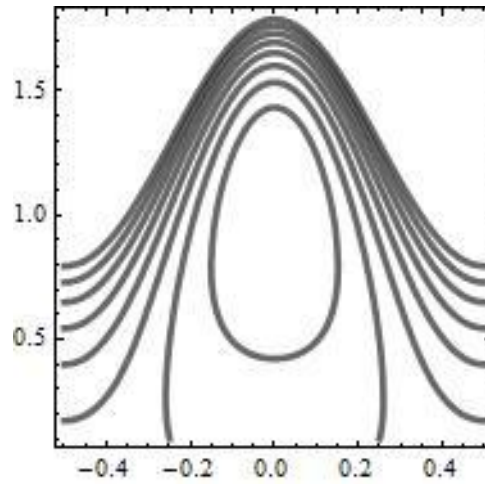
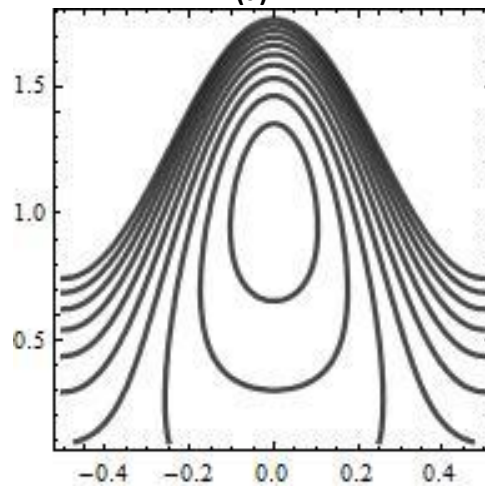


Fig.12 Stream lines for different values of (a) $\beta = 0$, (b) $\beta = 0.4$, (c) $\beta = 0.6$

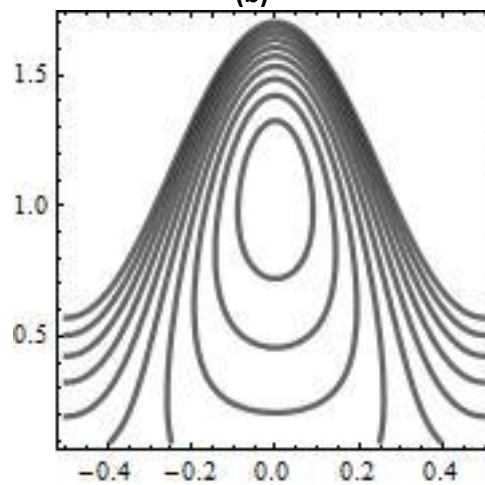
$$y_0 = 0.2, t = 0.5, m = 0.2, \varepsilon = 0.1, E_1 = 0.2, E_2 = 0.4, E_3 = 0.6$$



(a)



(b)



(c)

Fig.13 Stream lines for different values of (a) $m = -0.1$, (b) $m = 0$, (c) $m = 0.1$

$$y_0 = 0.2, t = 0.5, \beta = 0.2, \varepsilon = 0.1, E_1 = 0.2, E_2 = 0.4, E_3 = 0.6$$

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