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Estimation for ISB p-dim Rayleigh distribution under progressive type-II censored data using different loss functions

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Abstract

In this paper, the Bayesian estimation procedures for the unknown parameter as well as the reliability function of the inverse size biased (ISB) p-dimensional (p-dim) Rayleigh distribution under progressive Type-II censoring scheme are estimated. We consider the maximum likelihood estimator (MLEs) of the unknown parameter. Further, Bayesian estimation of the unknown parameters under Lindley's Approximation (L-Approximation) method and Markov Chain Monte Carlo (MCMC) techniques are used. The Hartigan prior with squared error and general entropy loss functions are considered for Bayesian analysis. Simulation study is performed to compare the proposed estimates.

Keywords:

ISB p-dim Rayleigh distribution;
progressive type-II censoring scheme;
L-Approximation method;
MCMC techniques.

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1. Introduction

In this article, we consider the progressive type-II censoring scheme. The progressive censoring is useful in industrial life testing applications and clinical settings; it allows the removal of surviving experimental units before the termination of the test. Cohen (1963) introduced a general censoring scheme known as progressive type-II censoring in which removal of the experimental units is allowed in between the experiment, hence become very popular in the reliability and life testing experiments. Several authors have investigated different inference problem with progressively censored samples namely, Mann (1971), Thomas and Wilson (1972). The estimation of the parameters of various lifetime distribution based on the progressive Type-II censoring scheme is done by Cohen and Norgaard (1977), Davis and Feldstein (1979). Balakrishnan et al. (2000, 2007) and provides a comprehensive reference on the subject of progressive censoring and its applications. Some recent studies on progressive censoring are initiated by Lee et al. (2011), Krishna and Kumar (2011), Rastogi et al. (2012), Krishna and Malik (2012), Krishna and Kumar (2013), Abou-Elheggag (2013), Singh and Sharma et al. (2013) and Rastogi and Tripathi (2014).

Here, we discuss the progressive type II censoring. Suppose n independent and identically distributed units taken from a continuous distribution are placed on a life test experiment. Let a censoring scheme $\left(R_1 - R_2 - ..... - R_m\right)$ be prefixed in such a manner that immediately after the first failure, R_1 items are randomly removed from n-1 surviving items and, at the time of occurrence of the second failure, the R_2 out of $n - R_1 - 2$ surviving units are withdrawn from the experiment. The test is continued units at the time of mth (1≤ m ≤ n) failure, the remaining surviving units R_m where $R_m = n - m - R_1 - R_2 - ..... - R_{m-1}$ are removed from the experiment. The usual type-II censoring scheme is a particular case of this scheme with $R_1 - R_2 - ..... - R_{m-1} = 0,\ R_m = n - m$. Also, the complete sampling corresponds to the case when $R_1 - R_2 - ..... - R_{m-1} = R_m = 0$.

We discuss the estimation problem, when the progressive censored samples are drawn from the ISB p-dim Rayleigh distribution (see Pandey and Kumari, 2016). The probability density function and cumulative distributionfunction of the ISB p-dim Rayleigh distribution is defined as follows:

$$f(y;\alpha,p) = \frac{2}{\alpha^{(p+1)/2}\Gamma\left(\frac{p+1}{2}\right)}\frac{1}{y^{p+2}}\exp\left(-\frac{1}{\alpha y^2}\right);\ 0 < y < \infty \tag{1.1}$$

$$F(t) = \frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\Gamma\left(\frac{p+1}{2},\frac{1}{\alpha t^2}\right);t > 0 \tag{1.2}$$

The cumulative density function is an upper incomplete gamma function. The reliability function is given by

$$R(t) = \frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\gamma\left(\frac{p+1}{2},\frac{1}{\alpha t^2}\right);t > 0 \tag{1.3}$$

It is assumed that parameter α is unknown and *p* is a positive known quantity. The reliability function is lower incomplete gamma function. Classical and Bayesian estimation of the unknown parameter are studied for ISB p-dim Rayleigh distribution under progressive Type-II censoring. It is observed that the MLE of the unknown parameter cannot be obtained in closed form, hence approximate MLE has been proposed in explicit form. The performance of the MLE and the approximate MLE is very close to each other. Bayes estimates of the unknown parameter based on different symmetric and asymmetric loss functions such as squared error and general entropy, and it is observed that they cannot be obtained in explicit form; hence L-Approximation and MCMC technique has been incorporated.

By symmetricity, it is implied that overestimation and underestimation are equally serious. Squared error loss or quadratic loss function (SELF) classified as a symmetric function associates equal importance to the losses due to overestimation and underestimation of equal magnitude and is evaluated as

$$L(\alpha,\hat{\alpha}) = (\alpha - \hat{\alpha})^2 \tag{1.4}$$

The asymmetric loss function, GELF due to Calabria and Pulcini (1994) is defined as

$$L(\delta) = \left(\delta^q - q\log_e(\delta) - 1\right);\ \delta = \frac{\hat{\alpha}}{\alpha}\ and\ q = 1 \tag{1.5}$$

and is useful in the situations where it is worse to underestimate (or overestimate) the potential of an event than to overestimate (or underestimate) the unknown parameter.

The aim of this paper is to discuss the inverse size biased p-dimensional Rayleigh distribution with MLEs and Bayes estimation procedures for one parameter and reliability function under progressive type-II censored data with SELF and GELF. We present the derivation of the maximum likelihood estimation (MLE) of the unknown parameter and reliability function in section 2. In Section 3, we obtain Bayesian estimation under L-Approximation Method and MCMC technique with SELF and GELF. The simulation study is provided in section 4. The conclusion is given in section 5.

2. Maximum Likelihood Estimators

In this section, the maximum likelihood estimation (MLE) of the unknown parameter α and reliability function of ISB p-dim Rayleigh distribution using progressive type-II censored samples are derived. The asymptotic confidence interval of the parameter is constructed.

Suppose that $\underline{Y} = Y_{1:m:n}, Y_{2:m:n}, \ldots\ldots, Y_{m:m:n}$ is a progressive type-II censored sample of size *m* from a sample of size *n* taken from ISB p-dim Rayleigh distribution with pdf in equation (1.1) and $\underline{R} = (R_1, R_2, \ldots\ldots, R_m)$ denote the corresponding numbers of units removed from the test. The likelihood function based on the progressive type-II censored sample (see Balakarishana and Aggarwala, 2000) is given by

$$f(y_{1:m:n}, y_{2:m:n}, \ldots\ldots, y_{m:m:n}) = A\prod_{i=1}^{m} f_{Y_{i:m:n}}(y_{i:m:n})\left[1 - F_{Y_{i:m:n}}(x_{i:m:n})\right]^{R_i} \tag{2.1}$$

where, $A = n(n - R_1 - 1)(n - R_1 - R_2 - 2)\ldots.\ (n - R_1 - \ldots. - R_{m-1} - m + 1)$ is a constant.

Using equations (1.1) and (1.2) the likelihood function (2.1) can be expressed as

$$l(\underline{y}|\alpha) = A\frac{2^m}{\alpha^{m(p+1)/2}\left(\Gamma\left(\frac{p+1}{2}\right)\right)^m}\prod_{i=1}^{m}\left(\frac{1}{y_{i:m:n}^{p+2}}\right)\exp\left(-\sum_{i=1}^{m}\left(\frac{1}{\alpha y_{i:m:n}^2}\right)\right)$$

$$\times\prod_{i=1}^{m}\left\{1 - \frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\Gamma\left(\frac{p+1}{2}, \frac{1}{\alpha\, y_{i:m:n}^2}\right)\right\}^{R_i} \tag{2.2}$$

where, $\underline{y} = (y_{1:m:n}, y_{2:m:n}, \ldots\ldots, y_{m:m:n})$. Natural logarithmic of the likelihood function of equation (2.2) is taken and after simplification, the log likelihood function is given as

$$\ln l = \ln(A) + m\ln(2) - mP\ln(\alpha) - m\ln(\Gamma P) + K_2 - \frac{1}{\alpha}K_1 + \sum_{i=1}^{m} R_i \ln\left\{\Gamma P - \Gamma\left(P, \frac{1}{\alpha}K_3\right)\right\} \tag{2.3}$$

where, $K_1 = \sum_{i=1}^{m}\left(\frac{1}{y_{i:m:n}^2}\right)$, $K_2 = \sum_{i=1}^{m}\ln\left(\frac{1}{y_{i:m:n}^{p+2}}\right)$, $K_3 = \frac{1}{y_{i:m:n}^2}$, $P = \left(\frac{p+1}{2}\right)$

The MLE of *α* is obtained by the setting the first partial derivatives of equation (2.3) equal to zero with respective to *α*, respectively, these simultaneous equations is,

$$\frac{\partial \ln l}{\partial \alpha} = -mP\frac{1}{\alpha} + \frac{1}{\alpha^2}K_1 - \sum_{i=1}^{m}\frac{R_i\Phi\Gamma\left(P, \frac{1}{\alpha}K_3\right)}{\left\{\Gamma P - \Gamma\left(P, \frac{1}{\alpha}K_3\right)\right\}} = 0 \tag{2.4}$$

where, $\Phi\Gamma\left(P, \frac{1}{\alpha}K_3\right) = \frac{\partial\Gamma\left(P, \frac{1}{\alpha}K_3\right)}{\partial\alpha}$

Subsequently, the MLEs of the reliability function, at a given time *t* are given, respectively, by

$$\hat{R}_{ML}(t) = \frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\gamma\left(\frac{p+1}{2}, \frac{1}{\hat{\alpha}_{ML}t^2}\right); t > 0 \tag{2.5}$$

where, $\hat{\alpha}_{ML}$ is the MLE of *α*. Since, equation (2.4) cannot be solved analytically; therefore, we used Newton Raphson iterative algorithm method.

2.1 Asymptotic Confidence Interval

To obtain the asymptotic confidence interval (ACI), we consider Fisher information matrix *I(α)* which is given by:

$$I(\hat{\alpha}) = -E\left(\frac{\partial^2 \ln l}{\partial\alpha^2}\right)_{\alpha=\hat{\alpha}}$$

Now, the ACI of α is given by

$$\text{var}(\hat{\alpha}) = \frac{1}{I(\hat{\alpha})}$$

The exact mathematical expression for the expectation of Fisher Information Matrix does not exist thus, using the concept of large sample theory we have obtained ACI. $100\left(1-\frac{\mu}{2}\right)\%$ confidence interval of the unknown parameter *α* is given by

$$[\hat{\alpha}_L, \hat{\alpha}_U] = \hat{\alpha}_{ML} \pm Z_{\mu/2}\sqrt{\text{var}(\hat{\alpha}_{ML})}$$

where, $Z_{\mu/2}$ is the upper $(\mu/2)^{th}$ percentile of the standard normal distribution.

3. Bayesian Estimation

This section deals with Bayes estimate for unknown parameter α and reliability function of ISB p-dim Rayleigh distribution under SELF and GELF on progressive type-II censored data. In Bayesian analysis, the parameter of interest is to be considered as a random variable and follows the prior distribution. We assume the asymptotically invariant prior, proposed by Hartigan (1964) whose form is as follows:

$$g(\alpha) = \frac{1}{\alpha^3}, \alpha > 0 \tag{3.1}$$

Now the joint posterior density function $\prod\left(\underline{y}|\alpha\right)$ as from equation (2.2) and (3.1) we get,

$$\prod\left(\underline{y}|\alpha\right) = \frac{L(\underline{y}|\alpha, p)g(\alpha)}{\int_\alpha L(\underline{y}|\alpha, p)g(\alpha)\,d\alpha}$$

$$\prod(\underline{y}|\alpha)=\frac{2^m K^{-1}}{\alpha^{(m(p+1)+6)/2}\left(\Gamma\left(\frac{p+1}{2}\right)\right)^m}\prod_{i=1}^{m}\left(\frac{1}{y_{i:m:n}^{p+2}}\right)\exp\left(-\sum_{i=1}^{m}\left(\frac{1}{\alpha y_{i:m:n}^2}\right)\right)\prod_{i=1}^{m}\left\{1-\frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\Gamma\left(\frac{p+1}{2},\frac{1}{\alpha\, y_{i:m:n}^2}\right)\right\}^{R_i}$$

(3.2)

where,

$$K=\int_\alpha \frac{2^m}{\alpha^{(m(p+1)+6)/2}\left(\Gamma\left(\frac{p+1}{2}\right)\right)^m}\prod_{i=1}^{m}\left(\frac{1}{y_{i:m:n}^{p+2}}\right)\exp\left(-\sum_{i=1}^{m}\left(\frac{1}{\alpha y_{i:m:n}^2}\right)\right)\times\prod_{i=1}^{m}\left\{1-\frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\Gamma\left(\frac{p+1}{2},\frac{1}{\alpha\, y_{i:m:n}^2}\right)\right\}^{R_i} d\alpha$$

To find the Bayes estimate of any parametric function, we need the posterior distribution in closed form. But for the distribution we have used, it is not possible to obtain the closed form of marginal, hence, we have used L-Approximation and MCMC technique. These techniques are discussed in the next section.

3.1 L-Approximation Method

This approach is developed by Lindley in 1980 and provides a simplified form of bayes estimator which is easy to use in practice. Now using the L-Approximation method $I(\underline{y})$ can be approximated as

$$I(\underline{y})=\frac{\int_0^\infty p(\alpha)e^{l(\alpha/\underline{y})+\eta(\alpha)}d\alpha}{\int_0^\infty e^{l(\alpha/\underline{y})+\eta(\alpha)}d\alpha} \tag{3.3}$$

where,

$p(\alpha)$ = function of α,

$l(\alpha|\underline{y})$ = the log likelihood function,

$\eta(\alpha,\beta)$ = log of prior distribution of α.

$$I(\underline{y})=p(\hat{\alpha})+\frac{1}{2}[(\hat{p}_{\alpha\alpha}+2\hat{p}_\alpha\hat{\eta}_\alpha)\hat{\sigma}_{\alpha\alpha}]+\frac{1}{2}\left[\hat{p}_\alpha\hat{\sigma}_{\alpha\alpha}\left(\hat{l}_{\alpha\alpha\alpha}\hat{\sigma}_{\alpha\alpha}\right)\right] \tag{3.4}$$

where $\hat{\alpha}$ is MLE of α, $p_\alpha=\frac{\partial p(\alpha)}{\partial\alpha}$, $p_{\alpha\alpha}=\frac{\partial^2 p(\alpha)}{\partial\alpha^2}$, $\hat{l}_\alpha=\frac{\partial l}{\partial\alpha}$, $\hat{l}_{\alpha\alpha}=\frac{\partial^2 l}{\partial\alpha^2}$, $\hat{l}_{\alpha\alpha\alpha}=\frac{\partial^3 l}{\partial\alpha^3}$

$$\hat{\eta}_\alpha=\left.\frac{\partial\log g(\alpha)}{\partial\alpha}\right|_{\alpha=\hat{\alpha},}=-\frac{3}{\hat{\alpha}},\ \sigma_{\alpha\alpha}=-\frac{1}{l_{\alpha\alpha}}$$

The following forms are used to estimate Bayes under SELF for *α* and reliability function which is given as:

(i) Bayes estimate of *α* under SELF

If p(α)=α, then $p_\alpha=1, p_{\alpha\alpha}=0$, the Bayes estimate of *α* under SELF is given by

$$\tilde{\alpha}_{BS}=E(\alpha/\underline{y})=\hat{\alpha}_{ML}+0.5[(\hat{p}_{\alpha\alpha}+2\hat{p}_\alpha\hat{\eta}_\alpha)\hat{\sigma}_{\alpha\alpha}]+0.5\left[\hat{p}_\alpha\hat{\sigma}_{\alpha\alpha}\left(\hat{l}_{\alpha\alpha\alpha}\hat{\sigma}_{\alpha\alpha}\right)\right]$$

(ii) Bayes estimate of reliability function under SELF

If $p(\hat{\alpha}) = \hat{R}(t) = \frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\gamma\left(\frac{p+1}{2}, \frac{1}{\hat{\alpha}t^2}\right)$, Now, the Bayes estimate of reliability function under SELF from equation (2.5) is given by

$$\tilde{R}_{BS}(t) = E\left(\hat{R}(t)/\underline{y}\right)$$

and

$$\tilde{R}_{BS}(t) = \frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\gamma\left(\frac{p+1}{2}, \frac{1}{\hat{\alpha}t^2}\right) + 0.5[(\hat{p}_{\alpha\alpha} + 2\hat{p}_{\alpha}\hat{\eta}_{\alpha})\hat{\sigma}_{\alpha\alpha}] + 0.5[\hat{p}_{\alpha}\hat{\sigma}_{\alpha\alpha}(\hat{l}_{\alpha\alpha\alpha}\hat{\sigma}_{\alpha\alpha})]$$

Next, we estimate Bayes under the GELF for parameter *α* and reliability function using equation (3.4).

(iii) Bayes estimate of *α* under GELF

If $p(\alpha) = \alpha^{-q}$, then $p_{\alpha} = -q\alpha^{-(q+1)}$, $p_{\alpha\alpha} = q(q+1)\alpha^{-(q+2)}$, the Bayesian estimate of *α* under GELF loss function is given by

$$\tilde{\alpha}_{BG} = \left[E_{\alpha}\left(\alpha^{-q} / \underline{y}\right)\right]^{-1/q},$$

$$E_{\alpha}\left(\alpha^{-q} / \underline{y}\right) = \hat{\alpha}^{-q} + 0.5[(\hat{p}_{\alpha\alpha} + 2\hat{p}_{\alpha}\hat{\eta}_{\alpha})\hat{\sigma}_{\alpha\alpha}] + 0.5[\hat{p}_{\alpha}\hat{\sigma}_{\alpha\alpha}(\hat{l}_{\alpha\alpha\alpha}\hat{\sigma}_{\alpha\alpha})]$$

(iv) Bayes estimate of Reliability function under GELF

If $p(\hat{\alpha}) = \hat{R}(t)^{-q}$, if $\hat{R}(t) = \frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\gamma\left(\frac{p+1}{2}, \frac{1}{\hat{\alpha}t^2}\right)$, the Bayes estimate of reliability function under GELF loss function is given by

$$\tilde{R}_{BG} = \left[E_{\alpha}\left(R^{-q} / \underline{y}\right)\right]^{-1/q}$$

$$E\left(R_{ML}{}^{-q}/\underline{y}\right) = \hat{R}_{ML}{}^{-q} + 0.5[(\hat{p}_{\alpha\alpha} + 2\hat{p}_{\alpha}\hat{\eta}_{\alpha})\hat{\sigma}_{\alpha\alpha}] + 0.5[\hat{p}_{\alpha}\hat{\sigma}_{\alpha\alpha}(\hat{l}_{\alpha\alpha\alpha}\hat{\sigma}_{\alpha\alpha})]$$

In the next subsection, we use the Markov Chain Monte Carlo technique to compute the Bayes estimates for the parameter α and reliability function.

3.2 MCMC Techniques

The MCMC technique provides the flexibility of extracting the posterior samples from its respective posterior distribution. Metropolis-Hastings algorithm is used to generate samples from the full conditional posterior distributions and then compute the Bayes estimates (Smith and Gelfand (1990), Hastings (1970), Upadhyaya and Gupta (2010)). The marginal posterior distribution is

$$\Pi_1(\underline{y}|\alpha) \propto \prod_{i=1}^{m}\left(\frac{1}{y_{i:m:n}^{p+2}}\right)\exp\left(-\sum_{i=1}^{m}\left(\frac{1}{\alpha y_{i:m:n}^2}\right)\right)\prod_{i=1}^{m}\left\{1 - \frac{1}{\Gamma\left(\frac{p+1}{2}\right)}\Gamma\left(\frac{p+1}{2}, \frac{1}{\alpha\, y_{i:m:n}^2}\right)\right\}^{R_i}$$

Following is the MCMC procedure to generate the sample from above equation:

Step 1: Start with initial value of α say α_0.
Step 2: Set *i=1*.
Step 3: Generate α_i from $\prod_1(\underline{y}|\alpha)$.
Step 4: Repeat Step 2-3, *N* times.

Bayes estimates under SELF for unknown parameter α and reliability function is given by:

$$\tilde{\alpha}_{BSMC} = \frac{1}{N-M}\sum_{i=M+1}^{N}\alpha_i \text{ , and } \tilde{R}(t)_{BSMC} = \frac{1}{N-M}\sum_{i=M+1}^{N}R_i(t)$$

Under GELF, the Bayes estimates for parameter α and reliability function is given by:

$$\tilde{\alpha}_{BGMC} = \left(\frac{1}{N-M}\sum_{i=M+1}^{N}\alpha_i^{-q}\right)^{-\frac{1}{q}} \text{ and } \tilde{R}(t)_{BGMC} = \left(\frac{1}{N-M}\sum_{i=M+1}^{N}R_i^{-q}(t)\right)^{-\frac{1}{q}}$$

4. Simulation Study

In this section, maximum likelihood and Bayes estimators for unknown parameter and reliability function under progressive type-II censoring are considered and hence, compared the performances of these estimators with Monte Carlo simulation, numerically.

We have generated a random sample from ISB p-dim Rayleigh distribution for α=1.5 and *p*=2. The progressive Type-II Censored Samples are generated using algorithm given by Balakrishnan and Sandhu. Three samples of size n=20, 30, 50 are taken and corresponding effective sample sizes are chosen in such a way that the observed samples are 50% and 80% censored. The mean square error (MSE) criterion is considered for comparison of the estimators. MLE of the parameter is obtained by using Newton Raphson method and corresponding reliability function is obtained using invariance property.

Bayes estimate of the parameter α and reliability function are obtained under Hartigan prior using squared error and general entropy loss functions. The Bayes estimates are computed with the help of L-Approximation and Markov Chain Monte Carlo technique. The reliability estimates are evaluated for t = 4, where, actual reliability R(t)=0.2371. Further, Bayes estimators are also derived with respect to asymmetric invariant prior distribution. From the extensive study of simulation, we see that the estimates obtained by MCMC have less variability as compared to the other methods.

The average estimates and corresponding MSE are reported in Tables 1-2. We observed that from the tables the mean square error of all estimators decreases when the sample size *n* and effective sample size *m* increases. Under two approximation techniques, the mean square error of Bayes estimates of the parameter under informative prior following patterns is noticed:

MSEs(*MCMC*) <*MSEs*(*Lindley*) < *MSEs*(*MLE*)

The mean square error of Bayes estimates of the reliability function under two approximation techniques are obtained and suggest that:

MSEs(*MCMC*) <*MSEs*(*Lindley*) < *MSEs*(*MLE*)

Table-1. Average values and MSEs (in Bracket) of all estimates of α for different value of *n* and *m*.

n	*m*	CS	$\hat{\alpha}_{ML}$	L-Approximation			MCMC technique		
				$\tilde{\alpha}_{BSL}$	$\tilde{\alpha}_{BG_1L}$	$\tilde{\alpha}_{BG_2L}$	$\tilde{\alpha}_{BSMC}$	$\tilde{\alpha}_{BG_1MC}$	$\tilde{\alpha}_{BG_2MC}$
20	10	(10,0*9)	1.5267 (0.0723)	1.5526 (0.0583)	1.4782 (0.0513)	1.5723 (0.0644)	1.5139 (0.0566)	1.4426 (0.0454)	1.5236 (0.0607)
		(2*5,0*5)	1.5636 (0.0663)	1.5895 (0.0513)	1.5246 (0.0476)	1.5948 (0.0571)	1.5368 (0.0502)	1.3248 (0.0413)	1.5676 (0.0532)
		(0*5,2*5)	1.5808 (0.0632)	1.6162 (0.0476)	1.5468 (0.0443)	1.6549 (0.0495)	1.5758 (0.0439)	1.5036 (0.0388)	1.6165 (0.0444)
		(0*9,10)	1.6425 (0.0614)	1.6671 (0.0426)	1.6343 (0.0394)	1.6957 (0.0457)	1.6136 (0.0403)	1.5973 (0.0356)	1.6558 (0.0417)
	20	(0*20)	1.6153 (0.0577)	1.2957 (0.0393)	1.1524 (0.0327)	1.3285 (0.0435)	1.1257 (0.0346)	1.0736 (0.0305)	1.2158 (0.0407)
30	15	(15,0*14)	1.4674 (0.0674)	1.5257 (0.0453)	1.4356 (0.0412)	1.5653 (0.0496)	1.5082 (0.0426)	1.4176 (0.0265)	1.5159 (0.0461)
		(3*5,0*10)	1.5277 (0.0631)	1.5765 (0.0412)	1.4645 (0.0386)	1.6289 (0.0447)	1.5247 (0.0372)	1.4536 (0.0237)	1.5628 (0.0431)
		(0*10,3*5)	1.5823 (0.0597)	1.6298 (0.0366)	1.4756 (0.0339)	1.6458 (0.0387)	1.5765 (0.0317)	1.4836 (0.0201)	1.5747 (0.0335)
		(0*14,15)	1.6597 (0.0476)	1.6648 (0.0308)	1.5227 (0.0284)	1.6925 (0.0357)	1.6238 (0.0283)	1.5172 (0.0193)	1.6435 (0.0312)
	24	(6,0*23)	1.4452 (0.0690)	1.4723 (0.0455)	1.4437 (0.0416)	1.5276 (0.0485)	1.4457 (0.0426)	1.4236 (0.0388)	1.4843 (0.0454)
		(0*22,3*2)	1.4751 (0.0576)	1.5457 (0.0372)	1.4175 (0.0327)	1.5876 (0.0419)	1.5365 (0.0382)	1.5164 (0.0306)	1.5664 (0.0415)
		(2*3,0*21)	1.5077 (0.0521)	1.5762 (0.0346)	1.4935 (0.0276)	1.6373 (0.0456)	1.5658 (0.0358)	1.5463 (0.0257)	1.6125 (0.0346)
		(0*23,6)	1.5232 (0.0415)	1.6324 (0.0297)	1.5548 (0.0247)	1.6723 (0.0321)	1.6083 (0.0257)	1.5387 (0.0215)	1.6427 (0.0307)
	30	(0*30)	1.5433 (0.0657)	1.5857 (0.0478)	1.5436 (0.0427)	1.6258 (0.0484)	1.5486 (0.0426)	1.5235 (0.0372)	1.5765 (0.0441)
50	25	(25,0*24)	1.5153 (0.0572)	1.5465 (0.0373)	1.5034 (0.0324)	1.5749 (0.0435)	1.5134 (0.0343)	1.4686 (0.0286)	1.5356 (0.0367)
		(5*5,0*20)	1.5519 (0.0516)	1.5762 (0.0357)	1.5435 (0.0286)	1.6265 (0.0408)	1.5438 (0.0315)	1.5076 (0.0263)	1.5673 (0.0327)
		(0*20,5*5)	1.5845 (0.0476)	1.6192 (0.0318)	1.5796 (0.0258)	1.6546 (0.0383)	1.5776 (0.0289)	1.5224 (0.0228)	1.6152 (0.0272)
		(0*24,25)	1.6433 (0.0431)	1.6635 (0.0253)	1.6246 (0.0226)	1.6867 (0.0326)	1.6292 (0.0244)	1.5734 (0.0214)	1.6525 (0.0255)
	40	(10,0*39)	1.3862 (0.0496)	1.4484 (0.0322)	1.4242 (0.0259)	1.4786 (0.0334)	1.4137 (0.0272)	1.4063 (0.0227)	1.4636 (0.0282)
		(2*5,0*35)	1.4455 (0.0413)	1.5386 (0.0264)	1.5046 (0.0222)	1.57463 (0.0284)	1.5046 (0.0245)	1.4764 (0.0210)	1.5368 (0.0259)
		(0*35,2*5)	1.4745 (0.0365)	1.5654 (0.0237)	1.5356 (0.0207)	1.6189 (0.0259)	1.5453 (0.0216)	1.5023 (0.0194)	1.5879 (0.0238)
		(0*39,10)	1.5018 (0.0314)	1.6274 (0.0206)	1.5846 (0.0192)	1.6576 (0.0238)	1.5886 (0.0182)	1.5684 (0.0163)	1.6235 (0.0217)
	50	(0*50)	1.5143 (0.0381)	1.5387 (0.0267)	1.5072 (0.0225)	1.5543 (0.0307)	1.5169 (0.0232)	1.4863 (0.0204)	1.5427 (0.0275)

Table-2. Average values and MSEs (in Bracket) of all estimates of *R(t)* for different values of *n* and *m* at time t=4.

n	*m*	CS	$\hat{R}_{ML}$	L-Approximation $\tilde{R}_{BSL}$	L-Approximation $\tilde{R}_{BG_1L}$	L-Approximation $\tilde{R}_{BG_2L}$	MCMC technique $\tilde{R}_{BSMC}$	MCMC technique $\tilde{R}_{BG_1MC}$	MCMC technique $\tilde{R}_{BG_2MC}$
20	10	(10,0*9)	0.2757 (0.0296)	0.3245 (0.0246)	0.2873 (0.0212)	0.3563 (0.0272)	0.2936 (0.0226)	0.2595 (0.0185)	0.3259 (0.0232)
		(2*5,0*5)	0.3113 (0.0239)	0.3426 (0.0207)	0.3177 (0.0175)	0.3727 (0.0215)	0.3269 (0.0184)	0.2825 (0.0157)	0.3406 (0.0198)
		(0*5,2*5)	0.3460 (0.0209)	0.3764 (0.0175)	0.3305 (0.0154)	0.3943 (0.0182)	0.3627 (0.0167)	0.3159 (0.0132)	0.3773 (0.0178)
		(0*9,10)	0.3804 (0.0181)	0.4153 (0.0157)	0.3731 (0.0141)	0.4371 (0.0168)	0.4086 (0.0132)	0.3325 (0.0114)	0.4122 (0.0145)
	20	(0*20)	0.2460 (0.0248)	0.2729 (0.0212)	0.2386 (0.0176)	0.3058 (0.0234)	0.2457 (0.0182)	0.2094 (0.0127)	0.2658 (0.0227)
30	15	(15,0*14)	0.2708 (0.0128)	0.3235 (0.0102)	0.3074 (0.0095)	0.3595 (0.0117)	0.3121 (0.0097)	0.2743 (0.0091)	0.3324 (0.0108)
		(3*5,0*10)	0.3424 (0.0110)	0.3682 (0.0093)	0.3574 (0.0084)	0.3870 (0.0096)	0.3415 (0.0091)	0.3214 (0.0219)	0.3624 (0.0092)
		(0*10,3*5)	0.3691 (0.0097)	0.3927 (0.0086)	0.3784 (0.0081)	0.4236 (0.0089)	0.3640 (0.0083)	0.3528 (0.0214)	0.4055 (0.0081)
		(0*14,15)	0.4351 (0.0085)	0.4628 (0.0081)	0.4117 (0.0078)	0.4743 (0.0084)	0.4151 (0.0076)	0.3847 (0.0015)	0.4436 (0.0076)
	24	(6,0*23)	0.2866 (0.0156)	0.3134 (0.0125)	0.2827 (0.0105)	0.3302 (0.0133)	0.2851 (0.0122)	0.2449 (0.0101)	0.3124 (0.0131)
		(0*22,3*2)	0.3245 (0.0127)	0.3463 (0.0097)	0.3327 (0.0093)	0.3734 (0.0101)	0.3276 (0.0089)	0.2836 (0.0086)	0.3424 (0.0093)
		(2*3,0*21)	0.3324 (0.0112)	0.3632 (0.0092)	0.3528 (0.0088)	0.4025 (0.0096)	0.3354 (0.0085)	0.3307 (0.0082)	0.3785 (0.0091)
		(0*23,6)	0.3518 (0.0106)	0.4158 (0.0090)	0.3841 (0.0082)	0.4474 (0.0093)	0.3876 (0.0083)	0.3467 (0.0076)	0.4272 (0.0087)
	30	(0*30)	0.3236 (0.0108)	0.3574 (0.0096)	0.3152 (0.0085)	0.3857 (0.0098)	0.3476 (0.0092)	0.2964 (0.0073)	0.3625 (0.0095)
50	25	(25,0*24)	0.2640 (0.0134)	0.3243 (0.0121)	0.3024 (0.0107)	0.3845 (0.0132)	0.3152 (0.0118)	0.2636 (0.0106)	0.3662 (0.0143)
		(5*5,0*20)	0.3284 (0.0112)	0.3536 (0.0103)	0.3376 (0.0096)	0.4196 (0.0121)	0.3365 (0.0112)	0.2985 (0.0087)	0.3925 (0.0124)
		(0*20,5*5)	0.3517 (0.0105)	0.3853 (0.0097)	0.3571 (0.0087)	0.4624 (0.0104)	0.3686 (0.0086)	0.3152 (0.0084)	0.4246 (0.0092)
		(0*24,25)	0.3744 (0.0092)	0.4185 (0.0086)	0.3786 (0.0075)	0.4663 (0.0093)	0.3935 (0.0075)	0.3436 (0.0076)	0.4421 (0.0084)
	40	(10,0*39)	0.2674 (0.0184)	0.3326 (0.0173)	0.2717 (0.0132)	0.3527 (0.0181)	0.2853 (0.0121)	0.2125 (0.0105)	0.3014 (0.0162)
		(2*5,0*35)	0.2984 (0.0156)	0.3617 (0.0128)	0.3125 (0.0105)	0.3876 (0.0162)	0.3214 (0.0102)	0.2763 (0.0090)	0.3458 (0.0135)
		(0*35,2*5)	0.3217 (0.0125)	0.4125 (0.0102)	0.3516 (0.0087)	0.4514 (0.0138)	0.3681 (0.0089)	0.3141 (0.0083)	0.3724 (0.0118)
		(0*39,10)	0.3944 (0.0097)	0.4437 (0.0092)	0.4053 (0.0081)	0.4758 (0.0112)	0.3725 (0.0815)	0.3616 (0.0075)	0.4325 (0.0097)
	50	(0*50)	0.2796 (0.0172)	0.3257 (0.0147)	0.2919 (0.0112)	0.3525 (0.0162)	0.3127 (0.0123)	0.2636 (0.0104)	0.3271 (0.0140)

From Tables 1-2, the Bayes estimates based on progressive Type-II censored data relative to SELF and GELF are better than their corresponding maximum likelihood, for most cases of *n* and *m*. When the effective sample sizes (*n*, *m*) increases, the MSEs of the all estimates based on progressive Type-II censored data decreases.

5. Conclusion

In this paper, we proposed the bayes estimates of the unknown parameter and reliability function of the ISB p-dim Rayleigh distribution under progressive type-II censored data. The Bayes estimates of the parameter and reliability function are computed under SELF and GELF with respect to Hartigan prior using L-Approximation method and MCMC technique. It is observed that the approximation techniques works well and we noticed that the performances of Bayes estimates obtained under asymmetric invariant prior using L-Approximation have smaller mean square error as compared to rest of the methods, while the MSEs of the Bayes estimators are quite similar under MCMC methods.

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