CONCEPTUAL STUDY ON FUZZY NUMBERS AND TRANSPORTATION PROBLEMS

Deepak Ramchandra Patidar*

*** Research Scholar, Calorx Teachers' University, Ahmedabad (India)**

TRANSPORTATION PROBLEMS

Transportation problems play an important role in logistics and supply chain management for reducing cost and improving service. In today's highly competitive market the pressure on organizations to fmd better ways to create and deliver products and services to customers becomes stronger. How and when to send the products to the customers in the quantities they want in a cost-effective manner becomes more challenging. Transportation models provide a powerful framework to meet this challenge. They ensure efficient movement and timely availability of raw materials and finished goods.

Transportation problem is one of the earliest applications of linear programming problems. Transportation problem is a linear programming problem stenmied from a network structure consisting of a finite number of nodes and arcs attached to them. Efficient algorithms have been developed for solving transportation problems when the cost coefficients and the demand and supply quantities are known precisely.

The basic transportation problem was originally developed by Hitchcock [18]. Efficient methods for solution derived from the simplex algorithm were introduced in 1947 by Dantzig [12], and then later by Chames et al. [11]. Transportation problem is a special type of linear programming Problem where special mathematical structures of restrictions are used, which can then be solved using the simplex method.

An initial basic feasible solution for transportation problems can be obtained by using North-West Comer Rule, Row Minima method, Column Minima method, Matrix Minima method, or Vogel's Approximation Method. To obtain an optimal solution for transportation problems, the MODI (Modified Distribution) method is used. Chames and Cooper [11] developed the Stepping Stone Method (SSM), which is an alternative way of determining the optimal solution. The LINDO (Linear Interactive and Discrete Optimization) package handles transportation problems in an explicit equation form, and thus solves the problem as a standard linear programming problem.

Isermann [21] introduced an algorithm for solving this problem, which provides effective solutions. Ringuest and Rinks [36] proposed two iterative algorithms for solving linear, multicriterial transportation problems. Similar solutions were proposed in [6, 7, and 43].

However, there are cases where these parameters may not be presented in a precise manner. Examples for this are, the unit shipping cost may vary in a time trame or the demand and supply quantities may be uncertain due to some uncontrollable factors. To quantitatively deal with imprecise information in making decisions, Bellman and Zadeh [3] and Zadeh [47] introduced the concept of fuzziness.

Since transportation problems are essentially linear programs, a straightforward idea is to apply the existing fuzzy linear programming techniques [16, 22, 28, 34, 37, and 45] to fuzzy transportation problems. Unfortunately, most of the existing techniques [22, 28, 37, and 45] provide only crisp solutions. Chanas et al. [10] investigated transportation problems with fuzzy supply and demand quantities and solved them using the parametric programming technique in terms of the Bellman-Zadeh criterion. Their method derived the solution which simultaneously satisfied the constraints and the goal to a maximum degree. Chanas and Kuchta [9] discussed a type of transportation problem with fuzzy cost coefficients and transformed the problem into a bicriterial transportation problem with crisp objective function. Their method is determined efficient solutions for the transformed problem; nevertheless, only crisp solutions were provided.

If the cost coefficients or the demand and supply quantities are fuzzy numbers, the total transportation cost will be fuzzy as well.

FUZZY SET THEORY

Fuzzy set theory has been studied extensively over the past 40 years. Most of the early research in fuzzy set theory pertained to representing uncertainty in human cognitive processes (Zadeh (1965)). Fuzzy set theory is now applied to problems in the fields of engineering, business, medical and related health sciences, and the natural sciences. The theory proposes a mathematical technique for handling imprecise concepts and problems that have many possible solutions. The concept of fuzzy mathematical programming in a general level was introduced by Tanaka et al (1974) in the framework of the fuzzy decision of Bellman and Zadeh [3]. Relevant terms are defined in the following sections [25, 50]

FUZZY SETS

A fuzzy set A is defined as, $\tilde{A} = \{(x, \mu_A(X)): X \epsilon A, \mu_A(x) \epsilon [0, 1]\}$. In the pair $(x, \mu_A(X))$, the first element x belongs to the classical set A, and the second element $\mu_A(X)$ belongs to the interval [0, 1], called *membership function.*

A fuzzy set can also be denoted by, $\tilde{A} = \{ \mu_A(X)/X: x \epsilon A, \mu_A(x) \epsilon [0, 1]\}$. Here the symbol '/' does not represent the division sign. It indicates that the top number $\mu_A(X)$ is the membership value of the element x on the bottom.

FUZZY NUMBERS

Fuzzy sets representing linguistic concepts such as 'low', 'medium', 'high' and so on are employed to define states of a variable. Such a variable is known as fuzzy variable. The relevance of fuzzy variables is that they facilitate gradual transitions between states and consequently possess a natural capability to express and deal with the uncertainties in observation and measurement. Traditionally, computation involves manipulation of numbers and symbols. However, nowadays humans employ mostly words in the natural language for computing and reasoning. A key aspect of computation with words is that it involves the combination of natural languages, and computation is done with fuzzy variables. The notion of a granule plays a vital role in computing with words. According to Zadeh [48], 'granulation plays a key role in human cognition. For humans, it serves as a way of achieving data comparison'.

Fuzzy sets defined on a set of real numbers R, have great importance. Membership functions $\mu: R \rightarrow [0, 1]$ possess a quantitative meaning and may be viewed as fuzzy numbers provided they satisfy certain conditions. One such condition is the initiative conceptions of approximate numbers or intervals, such as 'numbers that are close to five' or 'numbers that are around the given real numbers'. Such notions are essential for characterizing the states of fuzzy variables.

Fuzzy numbers play an important role in many applications such as fuzzy control, decision making, approximate reasoning and optimization. A fuzzy number is the fuzzy subset of the real line, where the highest membership values are clustered around a given real number. Although there are a variety of fuzzy numbers, as for as this thesis is concerned, the focus will be more towards triangular fuzzy numbers. The definition of a fuzzy number and its associated terms are furnished in the following section.

BASIC DEFINITIONS

A convex and normalized fuzzy set defined on R whose membership function is piecewise continuous is called *Fuzzy Number*.

A fuzzy set is called normal when at least one of its elements attains the maximum possible membership grade, if for all x ϵ R,

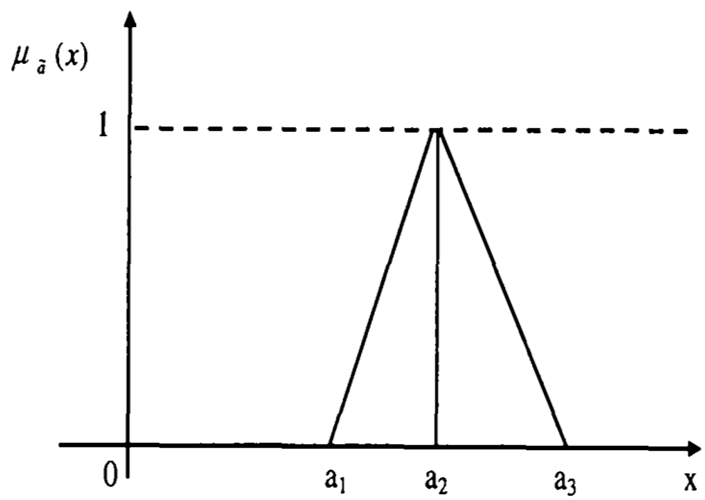
max $\mu_A(X) = 1$ where, $\tilde{A}$ is a fuzzy set.

A fuzzy set is convex if and only if each of its α-cut is a convex set, i.e., a fuzzy set A is convex if and only if $\mu_A(\lambda p + (1 - \lambda)q) \geq \min(\mu_A(p), \mu_A(q))$, for all $p, q \in R^n$ and all $\lambda \in [0, 1]$.

TRIANGULAR FUZZY NUMBER

A fuzzy number *a* is denoted as a triangular fuzzy number by (a, a J, a 3) and its membership function *ju* . (jc) is given as:

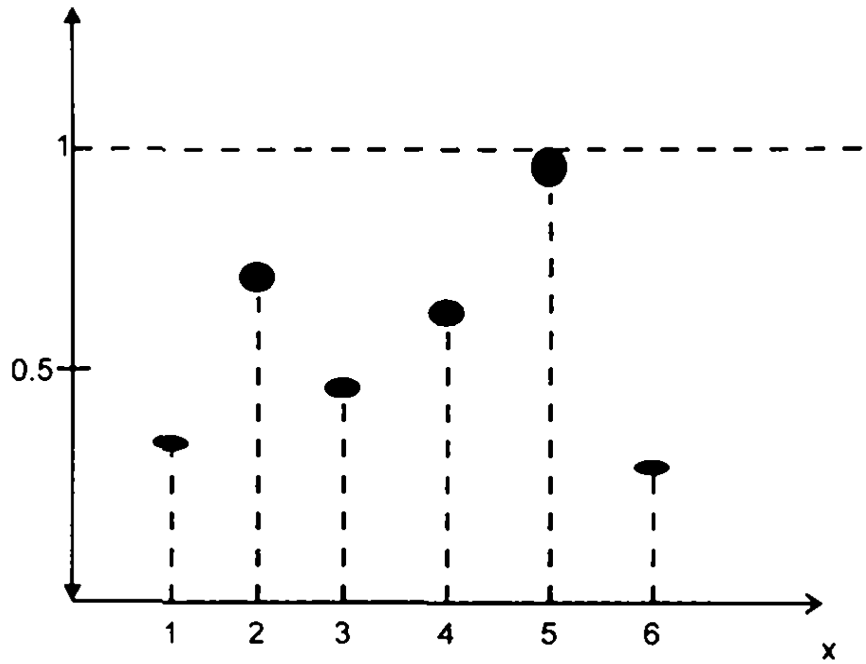
$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & \text{if } a_1\le x\le a_2\\ \dfrac{x-a_3}{a_2-a_3}, & \text{if } a_2\le x\le a_3\\ 0, & \text{otherwise.}\end{cases}$$



α - CUT

The α - level set of the fuzzy number $\tilde{a}$ is defined as the ordinary set $L_\alpha(\tilde{\alpha})$ *for* which the degree *of their* membership *function exceeds the level* α ϵ [0,1]. The formula for $L_\alpha(\tilde{\alpha})$ is given as:

$$L_\alpha(\tilde{a})=\{a\in R^m \mid \mu_{\tilde{a}}(a_i)\ge\alpha, i=1,2,\ldots,m\}$$



ARITHMETIC OPERATIONS ON FUZZY NUMBERS

Shan Huo Chen [39], introduced the concept of Function Principle, which is used to calculate the fuzzy transportation cost. The Graded Mean Integration Representation method, used to defuzzify the fuzzy transportation cost, was also introduced by Shan Huo Chen [40].

THE FUNCTION PRINCPLE

The Function Principle was introduced by Chen (1985) for the fuzzy arithmetical operations. Here, this principle is used for the addition, subtraction, and multiplication and division operation on fuzzy numbers.

If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then the following is obtained:

(i) $\tilde{a} + \tilde{b} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(ii) $\tilde{a} - \tilde{b} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

(iii) $\tilde{a} \times \tilde{b} = (c_1, c_2, c_3)$

where, $T = \{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$, $c_1 = \min T$, $c_2 = a_2b_2$ and $c_3 = \text{Max } T$. If $a_1, a_2, a_3, b_1, b_2, b_3$ are non zero and positive real numbers, then

$$\tilde{a} \times \tilde{b} = (a_1, a_2, a_3) \times (b_1, b_2, b_3) = (a_1b_1, a_2b_2, a_3b_3)$$

(iv) $\frac{1}{\tilde{b}} = \tilde{b}^{-1} = (1/b_3, 1/b_2, 1/b_1)$ where, b_1, b_2, b_3 are non zero and positive real numbers. Therefore, $\tilde{a}/\tilde{b} = (a_1/b_3, a_2/b_2, a_3/b_1)$

(v) If $k \in R$, then $k\tilde{a} = k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$ for $k \geq 0$.

DEFUZZIFICATION

The aggregation defined by a triangular fuzzy number needs to be expressed as a crisp value that represents the corresponding average. This operation is called defuzzification.

It is found that, there is no unique way to perform defuzzification of operations. Several existing methods for defuzzification are found to take into consideration the shape of the clipped fuzzy numbers, namely length of the supporting interval, height of the clipped triangular and the closeness to central triangular fuzzy numbers. The most popular defuzzification methods are Center of Area method, Mean of Maximum method, and Height Defuzziflcation method.

In this thesis, the Graded Mean Integration Representation method is used to defuzzify triangular fuzzy numbers.

If 5 = (ai, 32, 33, a4, WA)LR is a generalized fuzzy number, then the defuzzified value p($\tilde{a}$) by the Graded Mean Integration Representation method is given by the following formula:

$$p(\tilde{a}) = \int_0^{w_A} h\left[\frac{L^{-1}(h)+R^{-1}(h)}{2}\right]dh \Big/ \int_0^{w_A} hdh \text{ where, } 0 < h \le w_A \text{ and } 0 < w_A \le 1.$$

If $\tilde{a} = (a_1, a_2, a_3)$ is a triangular number, then the Graded Mean Integration Representation of a from the above formula is calculated as follows:

p{a)=l/2

$$p(\tilde{a}) = 1/2 \frac{\int_0^1 h[a_1 + h(a_2 - a_1) + a_3 - h(a_3 - a_2)]dh}{\int_0^1 hdh} = \frac{a_1 + 4a_2 + a_3}{6}.$$

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