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Neutrosophic Vague Generalized Pre-Continuous and Irresolute Mappings

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Abstract: This paper is to investigate and introduce a new class of continuous mappings in neutrosophic vague topological spaces namely neutrosophic vague generalized pre continuous mapping and neutrosophic vague generalized pre irresolute mapping by suitable examples and also their properties are deliberated.

Keywords: Neutrosophic Vague topology, neutrosophic vague generalized pre continuous mapping, neutrosophic vague generalized pre irresolute mapping.

1. Introduction:

The concept of fuzzy sets and intuitionistic fuzzy sets was introduced by Zadeh [24] in 1965 and Atanassov [4] in 1986. In 1970, Levine [11] initiated the study of generalized closed sets. The theory of fuzzy topology was introduced by C.L.Chang [6] in 1967. The theory of vague sets was first proposed by Gau and Buehrer [9] as an extension of fuzzy set theory. Then, Smarandache[23] introduces the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where]−0,1+[is the non-standard unit interval in 1995 (published in 1998). Shawkat Alkhazaleh[22] in 2015 introduced the concept of neutrosophic vague set as a combination of neutrosophic set and vague set. In this paper we introduce the concept of neutrosophic vague generalized pre-continuous mapping and neutrosophic vague generalized pre-irresolute mappings and also compare with the other existing functions with counter examples. Also its properties are discussed.

2. Preliminaries

Definition 2.1:[22] A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV} = \left\{\left\langle x; \hat{T}_{A_{NV}}(x); \hat{I}_{A_{NV}}(x); \hat{F}_{A_{NV}}(x)\right\rangle; x \in X\right\}$, whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_{A_{NV}}(x) = [T^-, T^+], \hat{I}_{A_{NV}}(x) = [I^-, I^+], \hat{F}_{A_{NV}}(x) = [F^-, F^+]$$

where,

1) $T^+ = 1 - F^-$
2) $F^+ = 1 - T^-$ and
3) $^-0 \leq T^- + I^- + F^- \leq 2^+$.

Definition 2.2:[22] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, $\hat{T}_{A_{NV}}(u_i) \leq \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i) \geq \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i) \geq \hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \leq i \leq n$.

Definition 2.3:[22] The complement of NVS A_{NV} is denoted by A_{NV}^c and is defined by

$$\hat{T}^c_{A_{NV}}(x) = [1 - T^+, 1 - T^-], \hat{I}^c_{A_{NV}}(x) = [1 - I^+, 1 - I^-], \hat{F}^c_{A_{NV}}(x) = [1 - F^+, 1 - F^-].$$

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Definition 2.4:[22] Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x)=[1,1]$; $\hat{I}_{A_{NV}}(x)=[0,0]$; $\hat{F}_{A_{NV}}(x)=[0,0]$. Then A_{NV} is called a unit NVS(1_{NV} in short), where $1 \le i \le n$.

Definition 2.5:[22] Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x)=[0,0]$; $\hat{I}_{A_{NV}}(x)=[1,1]$; $\hat{F}_{A_{NV}}(x)=[1,1]$. Then A_{NV} is called a zero NVS(0_{NV} in short), where $1 \le i \le n$.

Definition 2.6:[22] The union of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x)=\left[\max\left(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}\right), \max\left(T^+_{A_{NV_x}}, T^+_{B_{NV_x}}\right)\right]$$

$$\hat{I}_{C_{NV}}(x)=\left[\min\left(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}\right), \min\left(I^+_{A_{NV_x}}, I^+_{B_{NV_x}}\right)\right]$$

$$\hat{F}_{C_{NV}}(x)=\left[\min\left(F^-_{A_{NV_x}}, F^-_{B_{NV_x}}\right), \min\left(F^+_{A_{NV_x}}, F^+_{B_{NV_x}}\right)\right].$$

Definition 2.7:[22] The intersection of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x)=\left[\min\left(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}\right), \min\left(T^+_{A_{NV_x}}, T^+_{B_{NV_x}}\right)\right]$$

$$\hat{I}_{C_{NV}}(x)=\left[\max\left(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}\right), \max\left(I^+_{A_{NV_x}}, I^+_{B_{NV_x}}\right)\right]$$

$$\hat{F}_{C_{NV}}(x)=\left[\max\left(F^-_{A_{NV_x}}, F^-_{B_{NV_x}}\right), \max\left(F^+_{A_{NV_x}}, F^+_{B_{NV_x}}\right)\right].$$

Definition 2.8:[22] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, $\hat{T}_{A_{NV}}(u_i)=\hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i)=\hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i)=\hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} and B_{NV}, are called equal, where $1 \le i \le n$.

Definition 2.9: Let (X, τ) be a topological space. A subset A of X is called:

i) *semi closed set* (SCS in short)[12] if $\operatorname{int}(cl(A)) \subseteq A$,

ii) *pre- closed set* (PCS in short)[17] if $cl(\operatorname{int}(A)) \subseteq A$,

iii) *semi-pre closed set* (SPCS in short)[1] if $\operatorname{int}(cl(\operatorname{int}(A))) \subseteq A$,

iv) α *-closed set* (α CS in short)[20] if $cl(\operatorname{int}(cl(A))) \subseteq A$,

v) *regular closed set* (RCS in short)[24] if $A = cl(\operatorname{int}(A))$.

Definition 2.10: Let (X, τ) be a topological space. A subset A of X is called:

i) *generalized closed* (briefly, *g-closed*) [11] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X.

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ii) *generalized semi closed* (briefly, *gs-closed*)[3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

iii) *α-generalized closed* (briefly, *αg-closed*) [13] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

iv) *generalized pre-closed* (briefly, *gp-closed*) [14] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

v) *generalized semi-pre closed* (briefly, *gsp-closed*) [8] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Definition 2.11: Let (X,τ) and (Y,σ) be any two topological spaces. A map $f:(X,\tau)\to(Y,\sigma)$ is said to be

- *semi-continuous* [12] if $f^{-1}(V)$ is semi-closed set in (X,τ) for every closed set V of (Y,σ).
- *pre-continuous* [17] if $f^{-1}(V)$ is pre-closed set in (X,τ) for every closed set V of (Y,σ).
- *semi pre-continuous* [19] if $f^{-1}(V)$ is semi pre-closed set in (X,τ) for every closed set V of (Y,σ).
- α *-continuous* [16] if $f^{-1}(V)$ is α -closed set in (X,τ) for every closed set V of (Y,σ).
- *generalized continuous* [5] if $f^{-1}(V)$ is generalized closed set in (X,τ) for every closed set V of (Y,σ).
- *generalized semi-continuous* [7] if $f^{-1}(V)$ is generalized semi-closed set in (X,τ) for every closed set V of (Y,σ).
- *generalized pre-continuous* [21] if $f^{-1}(V)$ is generalized pre-closed set in (X,τ) for every closed set V of (Y,σ).
- *generalized semi pre-continuous* [18] if $f^{-1}(V)$ is generalized semi pre-closed set in (X,τ) for every closed set V of (Y,σ).
- α *-generalized continuous* [10] if $f^{-1}(V)$ is α -generalized closed set in (X,τ) for every closed set V of (Y,σ).
- *generalized pre-irresolute* [2] if $f^{-1}(V)$ is generalized pre-closed set in (X,τ) for every generalized pre-closed set V of (Y,σ).

Definition 2.12:[15] A neutrosophic vague topology (NVT in short) on X is a family τ of neutrosophic vague sets (NVS in short) in X satisfying the following axioms:

- $0_{NV}, 1_{NV} \in \tau$
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- $\cup G_i \in \tau, \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair (X,τ) is called a neutrosophic vague topological space (NVTS in short) and any NVS in τ is known as a neutrosophic vague open set (NVOS in short) in X .

The complement A^c of a NVOS in a NVTS (X,τ) is called neutrosophic vague closed set (NVCS in short) in X .

Definition 2.13:[15] A NVTS (X,τ) is said to be neutrosophic vague ${}_{gp}\mathrm{T}_{1/2}$ space (NV${}_{gp}\mathrm{T}_{1/2}$ in short) if every NVGPCS in X is a NVCS in X .

Definition 2.14:[15] A NVTS (X,τ) is said to be neutrosophic vague ${}_{gp}\mathrm{T}_{p}$ space (NV ${}_{gp}\mathrm{T}_{p}$ in short) if every NVGPCS in X is a NVPCS in X .

3. Neutrosophic Vague Continuous mapping:

Definition 3.1: Let (X,τ) and (Y,σ) be any two neutrosophic vague topological spaces. A map $f:(X,\tau)\to(Y,\sigma)$ is said to be,

- *Neutrosophic vague continuous* (NV continuous) if $f^{-1}(V)$ is neutrosophic vague closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).
- *neutrosophic vague semi-continuous* (NVS continuous) if $f^{-1}(V)$ is neutrosophic vague semi-closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).
- *neutrosophic vague pre-continuous* (NVP continuous) if $f^{-1}(V)$ is neutrosophic vague pre-closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).
- *neutrosophic vague semi pre-continuous* (NVP continuous) if $f^{-1}(V)$ is neutrosophic vague semi pre-closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).
- *neutrosophic vague α -continuous* (NV α -continuous) if $f^{-1}(V)$ is neutrosophic vague α -closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).
- *neutrosophic vague regular continuous* (NVR continuous) if $f^{-1}(V)$ is neutrosophic vague regular closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).
- *neutrosophic vague generalized continuous* (NVG continuous) if $f^{-1}(V)$ is neutrosophic vague generalized closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).
- *neutrosophic vague generalized semi-continuous* (NVGS continuous) if $f^{-1}(V)$ is neutrosophic vague generalized semi-closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).
- *neutrosophic vague generalized semi pre-continuous* (NVGSP continuous) if $f^{-1}(V)$ is neutrosophic vague generalized semi-closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).
- *neutrosophic vague α -generalized continuous* (NV α G continuous) if $f^{-1}(V)$ is neutrosophic vague α -generalized closed set in (X,τ) for every neutrosophic vague closed set V of (Y,σ).

4. Neutrosophic Vague Generalized Pre-Continuous Mappings:

Definition 4.1: A map $f:(X,\tau)\to(Y,\sigma)$ is said to be *neutrosophic vague generalized pre-continuous* (NVGP continuous in short) mapping if $f^{-1}(A)$ is NVGPCS in (X,τ) for every neutrosohpic vague closed set A of (Y,σ).

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Example 4.2: Let $X = \{a,b\}$, $Y = \{u,v\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.5,0.7];[0.2,0.4];[0.3,0.5]\rangle}, \frac{b}{\langle[0.6,0.9];[0.3,0.4];[0.1,0.4]\rangle} \right\},$$

$$G_2 = \left\{ x, \frac{a}{\langle[0.2,0.5];[0.5,0.6];[0.5,0.8]\rangle}, \frac{b}{\langle[0.3,0.4];[0.5,0.7];[0.6,0.7]\rangle} \right\},$$

$$G_3 = \left\{ y, \frac{u}{\langle[0.1,0.3];[0.6,0.8];[0.7,0.9]\rangle}, \frac{v}{\langle[0.2,0.3];[0.8,0.9];[0.7,0.8]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, G_2, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_3, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a NVGP continuous mapping.

Theorem 4.3: Let (X,τ) and (Y,σ) be any two vague topological spaces. For any vague continuous function $f:(X,\tau)\to(Y,\sigma)$ we have the following results.

i) Every NV continuous mapping is a NVG continuous mapping.
ii) Every NV continuous mapping is a NVα continuous mapping.
iii) Every NV continuous mapping is a NVP continuous mapping.
iv) Every NVα continuous mapping is a NVP continuous mapping.
v) Every NVR continuous mapping is a NV continuous mapping.
vi) Every NVα continuous mapping is a NVS continuous mapping.
vii) Every NVP continuous mapping is a NVSP continuous mapping.
viii) Every NV continuous mapping is a NVGP continuous mapping.
ix) Every NVG continuous mapping is a NVGP continuous mapping.
x) Every NVP continuous mapping is a NVGP continuous mapping.
xi) Every NVα continuous mapping is a NVGP continuous mapping.
xii) Every NVR continuous mapping is a NVGP continuous mapping.
xiii) Every NVαG continuous mapping is a NVGP continuous mapping.
xiv) Every NVGP continuous mapping is a NVSP continuous mapping.
xv) Every NVGP continuous mapping is a NVGSP continuous mapping.

Proof: (i) Let $f:(X,\tau)\to(Y,\sigma)$ be NV continuous mapping. Let A be NVCS in Y. Then $f^{-1}(A)$ is NVCS in X. Since every NVCS is NVGCS, $f^{-1}(A)$ is NVGCS in X. Hence f is NVG continuous mapping.

(ii) Let $f:(X,\tau)\to(Y,\sigma)$ be NV continuous mapping. Let A be NVCS in Y. Then $f^{-1}(A)$ is NVCS in X. Since every NVCS is NVα CS, $f^{-1}(A)$ is NVα CS in X. Hence f is NVα continuous mapping.

(iii) Let $f:(X,\tau)\to(Y,\sigma)$ be NV continuous mapping. Let A be NVCS in Y. Then $f^{-1}(A)$ is NVPCS in X. Since every NVCS is NVPCS, $f^{-1}(A)$ is NVPCS in X. Hence f is NVP continuous mapping.

The proof of (iv) to (xv) are similar.

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Remark 4.4: The converse of the above Theorem 4.3 need not be true as shown by the following examples.

Example 4.5: Let $X = \{a,b,c\}$, $Y = \{u,v,w\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.5,0.6];[0.3,0.5];[0.4,0.5]\rangle}, \frac{b}{\langle[0.6,0.9];[0.2,0.5];[0.1,0.4]\rangle}, \frac{c}{\langle[0.5,0.7];[0.4,0.5];[0.3,0.5]\rangle} \right\},$$

$$G_2 = \left\{ x, \frac{a}{\langle[0.8,0.9];[0.1,0.3];[0.1,0.2]\rangle}, \frac{b}{\langle[0.7,0.9];[0.1,0.2];[0.1,0.3]\rangle}, \frac{c}{\langle[0.6,0.8];[0.2,0.5];[0.2,0.4]\rangle} \right\},$$

$$G_3 = \left\{ y, \frac{u}{\langle[0.8,0.9];[0.2,0.4];[0.1,0.2]\rangle}, \frac{v}{\langle[0.7,0.9];[0.1,0.5];[0.1,0.3]\rangle}, \frac{w}{\langle[0.6,0.8];[0.2,0.3];[0.2,0.4]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, G_2, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_3, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$, $f(b) = v$ and $f(c) = w$. Then f is NVG continuous mapping but not NV continuous mapping, since

$$G_3^c = \left\{ y, \frac{u}{\langle[0.1,0.2];[0.6,0.8];[0.8,0.9]\rangle}, \frac{v}{\langle[0.1,0.3];[0.5,0.9];[0.7,0.9]\rangle}, \frac{w}{\langle[0.2,0.4];[0.7,0.8];[0.6,0.8]\rangle} \right\}$$

is NVCS in Y, but $f^{-1}(G_3^c)$ is not NVCS in X.

Example 4.6: Let $X = \{a,b\}$, $Y = \{u,v\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.1,0.3];[0.5,0.7];[0.7,0.9]\rangle}, \frac{b}{\langle[0.2,0.5];[0.3,0.5];[0.5,0.8]\rangle} \right\},$$

$$G_2 = \left\{ x, \frac{a}{\langle[0.2,0.6];[0.1,0.4];[0.4,0.8]\rangle}, \frac{b}{\langle[0.3,0.5];[0.2,0.5];[0.5,0.7]\rangle} \right\},$$

$$G_3 = \left\{ y, \frac{u}{\langle[0.6,0.8];[0.2,0.5];[0.2,0.4]\rangle}, \frac{v}{\langle[0.7,0.9];[0.1,0.4];[0.1,0.3]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, G_2, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_3, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is NV α continuous mapping but not NV continuous mapping, since $G_3^c = \left\{ y, \frac{u}{\langle[0.2,0.4];[0.5,0.8];[0.6,0.8]\rangle}, \frac{v}{\langle[0.1,0.3];[0.6,0.9];[0.7,0.9]\rangle} \right\}$ is NVCS in Y, but $f^{-1}(G_3^c)$ is not NVCS in X.

Example 4.7: Let $X = \{a,b,c\}$, $Y = \{u,v,w\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.3,0.7];[0.5,0.6];[0.3,0.7]\rangle}, \frac{b}{\langle[0.4,0.5];[0.6,0.8];[0.5,0.6]\rangle}, \frac{c}{\langle[0.3,0.5];[0.4,0.7];[0.5,0.7]\rangle} \right\},$$

$$G_2 = \left\{ y, \frac{u}{\langle[0.1,0.2];[0.6,0.9];[0.8,0.9]\rangle}, \frac{v}{\langle[0.2,0.4];[0.7,0.8];[0.6,0.8]\rangle}, \frac{w}{\langle[0.1,0.3];[0.7,0.9];[0.7,0.9]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_2, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$, $f(b)=v$ and $f(c)=w$. Then f is NVP continuous mapping but not NV continuous mapping and NV α continuous mapping, since

$$G_2^c = \left\{ y, \frac{u}{\langle[0.8,0.9];[0.1,0.4];[0.1,0.2]\rangle}, \frac{v}{\langle[0.6,0.8];[0.2,0.3];[0.2,0.4]\rangle}, \frac{w}{\langle[0.7,0.9];[0.1,0.3];[0.1,0.3]\rangle} \right\}$$

is NVCS in Y, but $f^{-1}(G_2^c)$ is not NVCS and NV α CS in X.

Example 4.8: Let $X = \{a,b,c\}$, $Y = \{u,v,w\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.7,0.8];[0.1,0.3];[0.2,0.3]\rangle}, \frac{b}{\langle[0.6,0.9];[0.2,0.4];[0.1,0.4]\rangle}, \frac{c}{\langle[0.5,0.6];[0.3,0.4];[0.4,0.5]\rangle} \right\},$$

$$G_2 = \left\{ y, \frac{u}{\langle[0.7,0.8];[0.1,0.3];[0.2,0.3]\rangle}, \frac{v}{\langle[0.6,0.9];[0.2,0.4];[0.1,0.4]\rangle}, \frac{w}{\langle[0.5,0.6];[0.3,0.4];[0.4,0.5]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_2, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is NV continuous mapping but not NVR continuous mapping, since

$$G_2^c = \left\{ y, \frac{u}{\langle[0.2,0.3];[0.7,0.9];[0.7,0.8]\rangle}, \frac{v}{\langle[0.1,0.4];[0.6,0.8];[0.6,0.9]\rangle}, \frac{w}{\langle[0.4,0.5];[0.6,0.7];[0.5,0.6]\rangle} \right\}$$

is NVCS in Y, but $f^{-1}(G_2^c)$ is not NVRCS in X.

Example 4.9: Let $X = \{a,b,c\}$, $Y = \{u,v,w\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.2,0.4];[0.8,0.9];[0.6,0.8]\rangle}, \frac{b}{\langle[0.1,0.2];[0.7,0.9];[0.8,0.9]\rangle}, \frac{c}{\langle[0.2,0.3];[0.5,0.9];[0.7,0.8]\rangle} \right\},$$

$$G_2 = \left\{ y, \frac{u}{\langle[0.5,0.6];[0.1,0.2];[0.4,0.5]\rangle}, \frac{v}{\langle[0.7,0.8];[0.2,0.4];[0.2,0.3]\rangle}, \frac{w}{\langle[0.5,0.7];[0.1,0.6];[0.3,0.5]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_2, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is NVS continuous mapping but not NV α continuous mapping, since

$$G_2^c = \left\{ y, \frac{u}{\langle[0.4,0.5];[0.8,0.9];[0.5,0.6]\rangle}, \frac{v}{\langle[0.2,0.3];[0.6,0.8];[0.7,0.8]\rangle}, \frac{w}{\langle[0.3,0.5];[0.4,0.9];[0.5,0.7]\rangle} \right\}$$

is NVCS in Y, but $f^{-1}(G_2^c)$ is not NV α CS in X.

Example 4.10: Let $X = \{a,b,c\}$, $Y = \{u,v,w\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.8,0.9];[0.1,0.3];[0.1,0.2]\rangle}, \frac{b}{\langle[0.7,0.9];[0.2,0.4];[0.1,0.3]\rangle}, \frac{c}{\langle[0.8,1];[0.1,0.4];[0,0.2]\rangle} \right\},$$

$$G_2 = \left\{ x, \frac{a}{\langle[0.2,0.4];[0.7,0.9];[0.6,0.8]\rangle}, \frac{b}{\langle[0.1,0.5];[0.6,0.8];[0.5,0.9]\rangle}, \frac{c}{\langle[0.4,0.5];[0.7,0.8];[0.5,0.6]\rangle} \right\},$$

$$G_3 = \left\{ y, \frac{u}{\langle[0.5,0.7];[0.3,0.4];[0.3,0.5]\rangle}, \frac{v}{\langle[0.4,0.8];[0.3,0.5];[0.2,0.6]\rangle}, \frac{w}{\langle[0.3,0.5];[0.5,0.6];[0.5,0.7]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, G_2, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_3, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$, $f(b)=v$ and $f(c)=w$. Then f is NVSP continuous mapping but not NVP continuous mapping, since

$$G_3^c = \left\{ y, \frac{u}{\langle[0.3,0.5];[0.6,0.7];[0.5,0.7]\rangle}, \frac{v}{\langle[0.2,0.6];[0.5,0.7];[0.4,0.8]\rangle}, \frac{w}{\langle[0.5,0.7];[0.4,0.5];[0.3,0.5]\rangle} \right\}$$

is NVCS in Y, but $f^{-1}(G_3^c)$ is not NVPCS in X.

Example 4.11: Let $X = \{a,b,c\}$, $Y = \{u,v,w\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.5,0.6];[0.1,0.3];[0.4,0.5]\rangle}, \frac{b}{\langle[0.7,0.9];[0.1,0.2];[0.1,0.3]\rangle}, \frac{c}{\langle[0.7,0.8];[0.1,0.4];[0.2,0.3]\rangle} \right\},$$

$$G_2 = \left\{ x, \frac{a}{\langle[0.3,0.5];[0.4,0.6];[0.5,0.7]\rangle}, \frac{b}{\langle[0.4,0.8];[0.1,0.3];[0.2,0.6]\rangle}, \frac{c}{\langle[0.5,0.6];[0.4,0.5];[0.4,0.5]\rangle} \right\},$$

$$G_3 = \left\{ y, \frac{u}{\langle[0.8,0.9];[0.2,0.4];[0.1,0.2]\rangle}, \frac{v}{\langle[0.7,0.9];[0.1,0.4];[0.1,0.3]\rangle}, \frac{w}{\langle[0.6,0.8];[0.2,0.6];[0.2,0.4]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, G_2, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_3, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$, $f(b)=v$ and $f(c)=w$. Then f is NVGP continuous mapping but not NV continuous mapping and NVG continuous mapping, since

$$G_3^c = \left\{ y, \frac{u}{\langle[0.1,0.2];[0.6,0.8];[0.8,0.9]\rangle}, \frac{v}{\langle[0.1,0.3];[0.6,0.9];[0.7,0.9]\rangle}, \frac{w}{\langle[0.2,0.4];[0.4,0.8];[0.6,0.8]\rangle} \right\}$$

is NVCS in Y, but $f^{-1}(G_3^c)$ is not NVCS and NVGCS in X.

Example 4.12: Let $X = \{a,b,c\}$, $Y = \{u,v,w\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.7,0.9];[0.2,0.5];[0.1,0.3]\rangle}, \frac{b}{\langle[0.8,0.9];[0.3,0.4];[0.1,0.2]\rangle}, \frac{c}{\langle[0.6,0.8];[0.2,0.4];[0.2,0.4]\rangle} \right\},$$

$$G_2 = \left\{ y, \frac{u}{\langle[0.1,0.2];[0.6,0.8];[0.8,0.9]\rangle}, \frac{v}{\langle[0.1,0.2];[0.7,0.9];[0.8,0.9]\rangle}, \frac{w}{\langle[0.2,0.3];[0.7,0.9];[0.7,0.8]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_2, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping

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$f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$, $f(b)=v$ and $f(c)=w$. Then f is NVGP continuous mapping but not NVP continuous mapping, since

$$G_2^c=\left\{y,\frac{u}{\langle[0.8,0.9];[0.2,0.4];[0.1,0.2]\rangle},\frac{v}{\langle[0.8,0.9];[0.1,0.3];[0.1,0.2]\rangle},\frac{w}{\langle[0.7,0.8];[0.1,0.3];[0.1,0.2]\rangle}\right\}$$

is NVCS in Y, but $f^{-1}(G_2^c)$ is not NVPCS in X.

Example 4.13: Let $X=\{a,b,c\}$, $Y=\{u,v,w\}$ and

$$G_1=\left\{x,\frac{a}{\langle[0.2,0.3];[0.6,0.7];[0.7,0.8]\rangle},\frac{b}{\langle[0.4,0.5];[0.6,0.8];[0.5,0.6]\rangle},\frac{c}{\langle[0.3,0.5];[0.5,0.6];[0.5,0.7]\rangle}\right\},$$

$$G_2=\left\{x,\frac{a}{\langle[0.1,0.3];[0.6,0.9];[0.7,0.9]\rangle},\frac{b}{\langle[0.2,0.4];[0.7,0.8];[0.6,0.8]\rangle},\frac{c}{\langle[0.3,0.4];[0.5,0.6];[0.6,0.7]\rangle}\right\},$$

$$G_3=\left\{y,\frac{u}{\langle[0.8,0.9];[0.1,0.3];[0.1,0.2]\rangle},\frac{v}{\langle[0.8,1];[0.2,0.3];[0,0.2]\rangle},\frac{w}{\langle[0.7,0.9];[0.3,0.4];[0.1,0.3]\rangle}\right\}.$$

Then $\tau=\{0_{NV},G_1,G_2,1_{NV}\}$ and $\sigma=\{0_{NV},G_3,1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$, $f(b)=v$ and $f(c)=w$. Then f is NVGP continuous mapping but not NV α continuous mapping, NVR continuous mapping and NV α G continuous mapping since

$$G_3^c=\left\{y,\frac{u}{\langle[0.1,0.2];[0.7,0.9];[0.8,0.9]\rangle},\frac{v}{\langle[0,0.2];[0.7,0.8];[0.8,1]\rangle},\frac{w}{\langle[0.1,0.3];[0.6,0.7];[0.7,0.9]\rangle}\right\}$$

is NVCS in Y, but $f^{-1}(G_3^c)$ is not NV α CS, NVRCS and NV α GCS in X.

Example 4.14: Let $X=\{a,b,c\}$, $Y=\{u,v,w\}$ and

$$G_1=\left\{x,\frac{a}{\langle[0.1,0.3];[0.6,0.7];[0.7,0.9]\rangle},\frac{b}{\langle[0.2,0.4];[0.8,0.9];[0.6,0.8]\rangle},\frac{c}{\langle[0.1,0.2];[0.7,0.8];[0.8,0.9]\rangle}\right\},$$

$$G_2=\left\{y,\frac{u}{\langle[0.7,0.9];[0.3,0.4];[0.1,0.3]\rangle},\frac{v}{\langle[0.6,0.8];[0.1,0.2];[0.2,0.4]\rangle},\frac{w}{\langle[0.8,0.9];[0.2,0.3];[0.1,0.2]\rangle}\right\}.$$

Then $\tau=\{0_{NV},G_1,1_{NV}\}$ and $\sigma=\{0_{NV},G_2,1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$, $f(b)=v$ and $f(c)=w$. Then f is NVSP continuous mapping and NVGSP continuous mapping but not NVGP continuous mapping, since

$$G_2^c=\left\{y,\frac{u}{\langle[0.1,0.3];[0.6,0.7];[0.7,0.9]\rangle},\frac{v}{\langle[0.2,0.4];[0.8,0.9];[0.6,0.8]\rangle},\frac{w}{\langle[0.1,0.2];[0.7,0.8];[0.8,0.9]\rangle}\right\}$$

is NVCS in Y, but $f^{-1}(G_2^c)$ is not NVGPCS in X.

Proposition 4.15: NVS continuous mapping and NVGP continuous mapping are independent to each other.

Example 4.16: Let $X = \{a,b,c\}$, $Y = \{u,v,w\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.3,0.6];[0.8,0.9];[0.4,0.7]\rangle}, \frac{b}{\langle[0.1,0.3];[0.7,0.8];[0.7,0.9]\rangle}, \frac{c}{\langle[0.2,0.4];[0.6,0.7];[0.6,0.8]\rangle} \right\},$$

$$G_2 = \left\{ y, \frac{u}{\langle[0.4,0.7];[0.1,0.2];[0.3,0.6]\rangle}, \frac{v}{\langle[0.7,0.9];[0.2,0.3];[0.1,0.3]\rangle}, \frac{w}{\langle[0.6,0.8];[0.3,0.4];[0.2,0.4]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_2, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$, $f(b)=v$ and $f(c)=w$. Then f is NVS continuous mapping but not NVGP continuous mapping, since

$$G_2^c = \left\{ y, \frac{u}{\langle[0.3,0.6];[0.8,0.9];[0.4,0.7]\rangle}, \frac{v}{\langle[0.1,0.3];[0.7,0.8];[0.7,0.9]\rangle}, \frac{w}{\langle[0.2,0.4];[0.6,0.7];[0.6,0.8]\rangle} \right\}$$

is NVCS in Y, but $NVpcl\left(f^{-1}(G_2^c)\right) \not\subset G_1$ in X.

Example 4.17: Let $X = \{a,b\}$, $Y = \{u,v\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.6,0.7];[0.2,0.3];[0.3,0.4]\rangle}, \frac{b}{\langle[0.5,0.6];[0.1,0.2];[0.4,0.5]\rangle} \right\},$$

$$G_2 = \left\{ x, \frac{a}{\langle[0.4,0.5];[0.8,0.9];[0.5,0.6]\rangle}, \frac{b}{\langle[0.3,0.6];[0.7,0.9];[0.4,0.7]\rangle} \right\},$$

$$G_3 = \left\{ y, \frac{u}{\langle[0.7,0.8];[0.1,0.2];[0.2,0.3]\rangle}, \frac{v}{\langle[0.1,0.2];[0.6,0.7];[0.8,0.9]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, G_2, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_3, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is NVGP continuous mapping but not NVS continuous mapping, since

$$G_3^c = \left\{ y, \frac{u}{\langle[0.2,0.3];[0.8,0.9];[0.7,0.8]\rangle}, \frac{v}{\langle[0.8,0.9];[0.3,0.4];[0.1,0.2]\rangle} \right\}$$ is NVCS in Y, but $f^{-1}(G_3^c)$ is not NVSCS in X.

Proposition 4.18: NVGS continuous mapping and NVGP continuous mapping are independent to each other.

Example 4.19: Let $X = \{a,b,c\}$, $Y = \{u,v,w\}$ and

$$G_1 = \left\{ x, \frac{a}{\langle[0.2,0.4];[0.6,0.9];[0.6,0.8]\rangle}, \frac{b}{\langle[0.1,0.2];[0.7,0.8];[0.8,0.9]\rangle}, \frac{c}{\langle[0.2,0.3];[0.5,0.7];[0.7,0.8]\rangle} \right\},$$

$$G_2 = \left\{ y, \frac{u}{\langle[0.6,0.8];[0.1,0.4];[0.2,0.4]\rangle}, \frac{v}{\langle[0.8,0.9];[0.2,0.3];[0.1,0.2]\rangle}, \frac{w}{\langle[0.7,0.8];[0.3,0.5];[0.2,0.3]\rangle} \right\}.$$

Then $\tau = \{0_{NV}, G_1, 1_{NV}\}$ and $\sigma = \{0_{NV}, G_2, 1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping

$f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(a)=u$, $f(b)=v$ and $f(c)=w$. Then f is NVGS continuous mapping but not NVGP continuous mapping, since

$$G_2^c=\left\{y,\frac{u}{\langle[0.2,0.4];[0.6,0.9];[0.6,0.8]\rangle},\frac{v}{\langle[0.1,0.2];[0.7,0.8];[0.8,0.9]\rangle},\frac{w}{\langle[0.2,0.3];[0.5,0.7];[0.7,0.8]\rangle}\right\}$$

is NVCS in Y, but $NVpcl\left(f^{-1}(G_2^c)\right)\not\subset G_1$ in X.

Example 4.20: Let $X=\{a,b\}$, $Y=\{u,v\}$ and

$$G_1=\left\{x,\frac{a}{\langle[0.4,0.7];[0.5,0.8];[0.3,0.6]\rangle},\frac{b}{\langle[0.5,0.8];[0.6,0.7];[0.2,0.5]\rangle}\right\},$$

$$G_2=\left\{x,\frac{a}{\langle[0.7,0.9];[0.2,0.5];[0.1,0.3]\rangle},\frac{b}{\langle[0.8,0.9];[0.2,0.6];[0.1,0.2]\rangle}\right\},$$

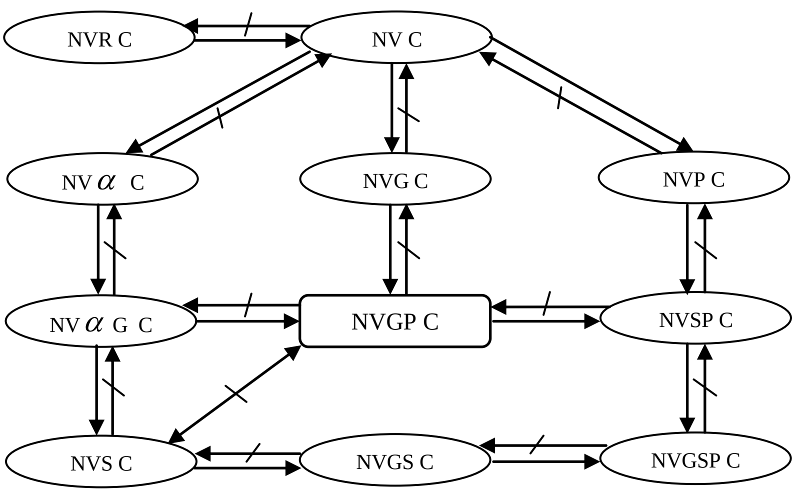
$$G_3=\left\{y,\frac{u}{\langle[0.5,0.6];[0.1,0.3];[0.4,0.5]\rangle},\frac{v}{\langle[0.8,0.9];[0.1,0.4];[0.1,0.2]\rangle}\right\}.$$

Then $\tau=\{0_{NV},G_1,G_2,1_{NV}\}$ and $\sigma=\{0_{NV},G_3,1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is NVGP continuous mapping but not NVGS continuous mapping, since

$$G_3^c=\left\{y,\frac{u}{\langle[0.4,0.5];[0.7,0.9];[0.5,0.6]\rangle},\frac{v}{\langle[0.1,0.2];[0.6,0.9];[0.8,0.9]\rangle}\right\}$$ is NVCS in Y, but

$NVscl\left(f^{-1}(G_2^c)\right)\not\subset G_1$ in X.

Result 4.21: The relations between various types of neutrosophic vague continuity are given in the following diagram.



In this diagram by "A ⟶ B" we mean A implies B but not conversely and "A ⟷ B" means A and B are independent of each other. None of them is reversible "A ⟵/ B".

Theorem 4.22: A mapping $f:(X,\tau)\to(Y,\sigma)$ is NVGP continuous mapping if and only if the inverse image of each NVOS in Y is NVGPOS in X.

Proof: Necessity: Let A be NVOS in Y. This implies A^c is NVCS in Y. Since f is NVGP continuous mapping, $f^{-1}(A^c)$ is NVGPCS in X. Since $f^{-1}(A^c)=\left(f^{-1}(A)\right)^c$, $f^{-1}(A)$ is NVGPOS in X.

Sufficiency: The proof is obvious from the Definition 4.1.

Theorem 4.23: Let $f:(X,\tau)\to(Y,\sigma)$ be mapping and let $f^{-1}(A)$ is NVRCS in X for every NVCS A in Y. Then f is NVGP continuous mapping but not conversely.

Proof: Let A be NVCS in Y. Then $f^{-1}(A)$ is NVRCS in X. Since every NVRCS is NVGPCS, $f^{-1}(A)$ is NVGPCS in X. Hence f is NVGP continuous mapping.

Theorem 4.24: Let $f:(X,\tau)\to(Y,\sigma)$ is NVGP continuous mapping, then f is neutrosophic vague continuous mapping if X is NV ${}_{\mathrm{p}}\mathrm{T}_{1/2}$ space.

Proof: Let A be NVCS in Y. Then $f^{-1}(A)$ is NVGPCS in X, by hypothesis. Since X is NV ${}_{\mathrm{p}}\mathrm{T}_{1/2}$ space, $f^{-1}(A)$ is NVCS in X. Hence f is neutrosophic vague continuous mapping.

Theorem 4.25: Let $f:(X,\tau)\to(Y,\sigma)$ is NVGP continuous mapping, then f is NVP continuous mapping if X is NV ${}_{\mathrm{gp}}\mathrm{T}_{1/2}$ space.

Proof: Let A be NVCS in Y. Then $f^{-1}(A)$ is NVGPCS in X, by hypothesis. Since X is NV ${}_{\mathrm{gp}}\mathrm{T}_{1/2}$ space, $f^{-1}(A)$ is NVPCS in X. Hence f is NVP continuous mapping.

Theorem 4.26: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from NVTS X into NVTS Y. Then the following conditions are equivalent if X is NV ${}_{\mathrm{gp}}\mathrm{T}_{1/2}$ space.

i) f is NVGP continuous mapping.

ii) $f^{-1}(B)$ is NVGPCS in X for every NVCS B in Y.

iii) $NVcl\left(NV\operatorname{int}\left(f^{-1}(A)\right)\right)\subseteq f^{-1}(NVcl(A))$ for every NVS A in Y.

Proof: (i)$\Rightarrow$(ii): It is obvious from the Definition 4.1.

(ii)$\Rightarrow$(iii): Let A be NVS in Y. Then $NVcl(A)$ is NVCS in Y. By hypothesis, $f^{-1}(NVcl(A))$ is NVGPCS in X. Since X is NV ${}_{\mathrm{gp}}\mathrm{T}_{1/2}$ space, $f^{-1}(NVcl(A))$ is NVPCS. Therefore

$NVcl\left(NV\operatorname{int}\left(f^{-1}(NVcl(A))\right)\right)\subseteq f^{-1}(NVcl(A))$. Now

$NVcl\left(NV\operatorname{int}\left(f^{-1}(A)\right)\right)\subseteq NVcl\left(NV\operatorname{int}\left(f^{-1}(NVcl(A))\right)\right)\subseteq f^{-1}(NVcl(A))$.

(iii)$\Rightarrow$(i): Let A be NVCS in Y. By hypothesis $NVcl(NV\,\mathrm{int}(f^{-1}(A)))\subseteq f^{-1}(NVcl(A))=f^{-1}(A)$. This implies $f^{-1}(A)$ is NVPCS in X and hence it is NVGPCS. Thus f is NVGP continuous mapping.

Theorem 4.27: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be mapping from NVTS X into a NVTS Y. Then the following conditions are equivalent if X is a NV $_{gp}\mathrm{T}_{1/2}$ space.

i) f is NVGP continuous mapping.
ii) $f^{-1}(A)$ is NVGPOS in X for every NVOS A in Y.
iii) $f^{-1}(NV\,\mathrm{int}(A))\subseteq NV\,\mathrm{int}(NVcl(f^{-1}(A)))$ for every NVS A in Y.

Proof: (i)$\Rightarrow$(ii): It is obvious.

(ii)$\Rightarrow$(iii): Let A be NVS in Y. Then $NV\,\mathrm{int}(A)$ is NVOS in Y. By hypothesis, $f^{-1}(NV\,\mathrm{int}(A))$ is NVGPOS in X. Since X is NV $_{gp}\mathrm{T}_{1/2}$ space, $f^{-1}(NV\,\mathrm{int}(A))$ is NVPOS in X. Therefore $f^{-1}(NV\,\mathrm{int}(B))\subseteq NV\,\mathrm{int}(NVcl(f^{-1}(NV\,\mathrm{int}(B))))\subseteq NV\,\mathrm{int}(NVcl(f^{-1}(B)))$.

(iii)$\Rightarrow$(i): Let A be NVCS in Y. Then its complement, say A^c is NVOS in Y, then $NV\,\mathrm{int}(A^c)=A^c$. Now by hypothesis $f^{-1}(NV\,\mathrm{int}(A^c))\subseteq NV\,\mathrm{int}(NVcl(f^{-1}(A^c)))$. This implies $f^{-1}(A^c)\subseteq NV\,\mathrm{int}(NVcl(f^{-1}(A^c)))$. Hence $f^{-1}(A^c)$ is a NVPOS in X. Since every NVPOS is NVGPOS, $f^{-1}(A^c)$ is a NVGPOS in X. Thus $f^{-1}(A)$ is a NVGPCS in X. Hence f is NVGP continuous mapping.

Theorem 4.28: A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is NVGP continuous mapping if $NVcl(NV\,\mathrm{int}(NVcl(f^{-1}(A))))\subseteq f^{-1}(NVcl(A))$ for every NVS A in Y.

Proof: Let A be NVOS in Y then A^c is NVCS in Y. By hypothesis, $NVcl(NV\,\mathrm{int}(NVcl(f^{-1}(A^c))))\subseteq f^{-1}(NVcl(A^c))=f^{-1}(A^c)$, since A^c is NVCS. Now $(NV\,\mathrm{int}(NVcl(NV\,\mathrm{int}(f^{-1}(A)))))^c=NVcl(NV\,\mathrm{int}(NVcl(f^{-1}(A^c))))\subseteq f^{-1}(A^c)=(f^{-1}(A))^c$. This implies $f^{-1}(A)\subseteq NV\,\mathrm{int}(NVcl(Nv\,\mathrm{int}(f^{-1}(A))))$. Hence $f^{-1}(A)$ is NVα Os and hence it is NVGPOS. Therefore f is NVGP continuous mapping, by theorem 4.22.

Theorem 4.29: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be NVGP continuous mapping and $g:(Y,\sigma)\rightarrow(Z,\mu)$ be neutrosophic vague continuous mapping, then $g\circ f:(X,\tau)\rightarrow(Z,\mu)$ is NVGP continuous mapping.

Proof: Let A be NVCS in Z. Then $g^{-1}(A)$ is NVCS in Y, by hypothesis. Since f is NVGP continuous mapping, $f^{-1}(g^{-1}(A))$ is NVGPCS in X. Hence $g\circ f$ is NVGP continuous mapping.

Remark 4.30: The composition of two NVGP continuous mapping need not be NVGP continuous mapping and it is shown by the following example.

Example 4.31: Let $X = \{a,b,c\}, Y = \{u,v,w\}$ and $Z = \{p,q,r\}$ neutrosophic vague sets G_1, G_2 and G_3 defined as follows:

$$G_1 = \left\{ x, \frac{a}{\langle[0.2,0.4];[0.7,0.8];[0.6,0.8]\rangle}, \frac{b}{\langle[0.1,0.4];[0.6,0.7];[0.6,0.9]\rangle}, \frac{c}{\langle[0.4,0.5];[0.6,0.8];[0.5,0.6]\rangle} \right\},$$

$$G_2 = \left\{ y, \frac{u}{\langle[0.7,0.8];[0.1,0.4];[0.2,0.3]\rangle}, \frac{v}{\langle[0.8,0.9];[0.2,0.6];[0.1,0.2]\rangle}, \frac{w}{\langle[0.9,1];[0.1,0.2];[0,0.1]\rangle} \right\},$$

$$G_3 = \left\{ z, \frac{p}{\langle[0.4,0.7];[0.6,0.7];[0.3,0.6]\rangle}, \frac{q}{\langle[0.5,0.8];[0.4,0.7];[0.2,0.5]\rangle}, \frac{r}{\langle[0.3,0.4];[0.3,0.5];[0.6,0.7]\rangle} \right\}.$$

Let $\tau = \{0_{NV}, G_1, 1_{NV}\}$, $\sigma = \{0_{NV}, G_2, 1_{NV}\}$ and $\mu = \{0_{NV}, G_3, 1_{NV}\}$ be NVTs on X, Y and Z respectively. Let the mapping $f:(X,\tau)\to(Y,\sigma)$ defined by $f(a)=u$, $f(b)=v$ and $f(c)=w$, $g:(Y,\sigma)\to(Z,\mu)$ defined by $g(u)=p, g(v)=q$ and $g(w)=r$. Then the mapping f and g are NVGP continuous mapping but the mapping $g\circ f:(X,\tau)\to(Z,\mu)$ is not NVGP continuous mapping.

Definition 4.32: Let (X,τ) be a NVTS. The neutrosophic vague generalized pre closure ($NVgpcl(A)$ in short) and neutrosophic vague generalized pre interior ($NVgp\,\mathrm{int}(A)$ in short) for any NVS A is defined as follows,

$$NVgpcl(A) = \cap\{K / K \text{ is a } NVGPCS \text{ in } X \text{ and } A \subseteq K\}.$$

$$NVgp\,\mathrm{int}(A) = \cup\{G / G \text{ is a } NVGPOS \text{ in } X \text{ and } G \subseteq A\}.$$

If A is NVGPCS, then $NVgpcl(A) = A$.

Theorem 4.33: Let $f:(X,\tau)\to(Y,\sigma)$ be NVGP continuous mapping. Then the following conditions hold.

i) $f(NVgpcl(A)) \subseteq NVcl(f(A))$, for every NVS A in X.
ii) $NVgpcl(f^{-1}(B)) \subseteq f^{-1}(NVcl(B))$, for every NVS B in Y.

Proof: (i) Since $NVcl(f(A))$ is NVCS in Y and f is NVGP continuous mapping, then $f^{-1}(NVcl(f(A)))$ is NVGPCS in X. That is $NVgpcl(A) \subseteq f^{-1}(NVcl(f(A)))$. Therefore $f(NVgpcl(A)) \subseteq NVcl(f(A))$, for every NVS A in X.

(ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(NVgpcl(f^{-1}(B))) \subseteq NVcl(f(f^{-1}(B))) \subseteq NVcl(B)$. Hence $NVgpcl(f^{-1}(B)) \subseteq f^{-1}(NVcl(B))$, for every NVS B in Y.

5. Neutrosophic Vague generalized pre irresolute mapping:

Definition 5.1: A map $f:(X,\tau)\to(Y,\sigma)$ is said to be *neutrosophic vague generalized pre irresolute* (NVGP irresolute in short) mapping if $f^{-1}(A)$ is a NVGPCS in (X,τ) for every NVGPCS A in (Y,σ).

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Theorem 5.2: Let $f:(X,\tau)\to(Y,\sigma)$ be a NVGP irresolute mapping, then f is NVGP continuous mapping but not conversely.

Proof: Let f be NVGP irresolute mapping. Let A be any NVCS in Y. Since every NVCS is NVGPCS, A is NVGPCS in Y. Since f is NVGP irresolute mapping, by definition $f^{-1}(A)$ is NVGPCS in X. Hence f is NVGP continuous mapping.

Example 5.3: Let $X=\{a,b,c\}$, $Y=\{u,v,w\}$ and

$$G_1=\left\{x,\frac{a}{\langle[0.6,0.7];[0.2,0.3];[0.3,0.4]\rangle},\frac{b}{\langle[0.5,0.8];[0.1,0.2];[0.2,0.5]\rangle},\frac{c}{\langle[0.6,0.8];[0.1,0.4];[0.2,0.4]\rangle}\right\},$$

$$G_2=\left\{y,\frac{u}{\langle[0.4,0.5];[0.7,0.8];[0.5,0.6]\rangle},\frac{v}{\langle[0.2,0.6];[0.8,0.9];[0.4,0.8]\rangle},\frac{w}{\langle[0.1,0.6];[0.5,0.7];[0.4,0.9]\rangle}\right\}.$$

Then $\tau=\{0_{NV},G_1,1_{NV}\}$ and $\sigma=\{0_{NV},G_2,1_{NV}\}$ are NVTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$, $f(b)=v$ and $f(c)=w$. Then f is NVGP continuous mapping. Let

$$B=\left\{y,\frac{u}{\langle[0.7,0.9];[0.1,0.2];[0.1,0.3]\rangle},\frac{v}{\langle[0.8,0.9];[0.1,0.2];[0.1,0.2]\rangle},\frac{w}{\langle[0.6,0.8];[0.2,0.3];[0.2,0.4]\rangle}\right\}.$$

is NVGPCS in Y. But $f^{-1}(B)$ is not NVGPCS in X. Therefore f is not NVGP irresolute mapping.

Theorem 5.4: A mapping $f:(X,\tau)\to(Y,\sigma)$ is NVGP irresolute mapping if and only if the inverse image of each NVGPOS in Y is NVGPOS in X.

Proof: Necessity: Let A be NVGPOS in Y. This implies A^c is NVGPCS in Y. Since f is NVGP irresolute mapping, $f^{-1}(A^c)$ is NVGPCS in X. Since $f^{-1}(A^c)=(f^{-1}(A))^c$, $f^{-1}(A)$ is NVGPOS in X.

Sufficiency: The proof is obvious from the Definition 5.1.

Theorem 5.5: Let $f:(X,\tau)\to(Y,\sigma)$ be NVGP irresolute mapping, then f is neutrosophic vague irresolute mapping if X is NV $_pT_{1/2}$ space.

Proof: Let A be NVCS in Y. Then A is NVGPCS in Y. Since f is NVGP irresolute mapping, $f^{-1}(A)$ is NVGPCS in X, by hypothesis. Since X is NV $_pT_{1/2}$ space, $f^{-1}(A)$ is NVCS in X. Hence f is neutrosophic vague irresolute mapping.

Theorem 5.6: Let $f:(X,\tau)\to(Y,\sigma)$ be NVGP irresolute mapping, then f is NVP irresolute mapping if X is NV $_{gp}T_{1/2}$ space.

Proof: Let A be NVPCS in Y. Then A is NVGPCS in Y. Since f is NVGP irresolute mapping, $f^{-1}(A)$ is NVGPCS in X, by hypothesis. Since X is NV $_{gp}T_{1/2}$ space, $f^{-1}(A)$ is NVPCS in X. Hence f is NVP irresolute mapping.

Theorem 5.7: Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ be NVGP irresolute mapping, where X,Y and Z are NVTS, then $g\circ f$ is NVGP irresolute mapping.

Proof: Let A be NVGPCS in Z. Since g is NVGP irresolute mapping, $g^{-1}(A)$ is NVGPCS in Y. Since f is NVGP irresolute mapping, $f^{-1}(g^{-1}(A))$ is NVGPCS in X. Hence $(g\circ f)^{-1}(A)$ is NVGPCS in X. Therefore $g\circ f$ is NVGP irresolute mapping.

Theorem 5.8: Let $f:(X,\tau)\to(Y,\sigma)$ be NVGP irresolute mapping and $g:(Y,\sigma)\to(Z,\mu)$ is NVGP continuous mapping, where X,Y and Z are NVTS, then $g\circ f$ is NVGP continuous mapping.

Proof: Let A be NVCS in Z. Since g is NVGP continuous mapping, $g^{-1}(A)$ is NVGPCS in Y. Since f is NVGP irresolute mapping, $f^{-1}(g^{-1}(A))$ is NVGPCS in X. Hence $(g\circ f)^{-1}(A)$ is NVGPCS in X. Therefore $g\circ f$ is NVGP continuous mapping.

Theorem 5.9: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a NVTS X into a NVTS Y. Then the following conditions are equivalent if X and Y are NV $_{gp}T_{1/2}$ space.

i) f is NVGP irresolute mapping.
ii) $f^{-1}(B)$ is NVGPOS in X for each NVGPOS B in Y.
iii) $f^{-1}(NV\text{pint}(B))\subseteq NV\text{pint}(f^{-1}(B))$ for each NVS B of Y.
iv) $NV\text{pcl}(f^{-1}(B))\subseteq f^{-1}(NV\text{pcl}(B))$ for each NVS B of Y.

Proof: (i) $\Rightarrow$(ii): It is obviously true.

(ii) $\Rightarrow$(iii): Let B be NVS in Y and $NV\text{pint}(B)\subseteq B$. Also $f^{-1}(NV\text{pint}(B))\subseteq f^{-1}(B)$. Since $NV\text{pint}(B)$ is NVPOS in Y, it is NVGPOS in Y. Therefore $f^{-1}(NV\text{pint}(B))$ is NVGPOS in X, by hypothesis. Since X is NV $_{gp}T_{1/2}$ space $f^{-1}(NV\text{pint}(B))$ is NVPOS in X. Hence $f^{-1}(NV\text{pint}(B))=NV\text{pint}(f^{-1}(NV\text{pint}(B)))\subseteq NV\text{pint}(f^{-1}(B))$.

(iii) $\Rightarrow$(iv): It is obvious by taking complement in (iii).

(iv) $\Rightarrow$(i): Let B be NVGPCS in Y. Since Y is NV $_{gp}T_{1/2}$ space, B is NVPCS in Y and $NV\text{pcl}(B)=B$. Hence $f^{-1}(B)=f^{-1}(NV\text{pcl}(B))\supseteq NV\text{pcl}(f^{-1}(B))$. Therefore $NV\text{pcl}(f^{-1}(B))=f^{-1}(B)$. This implies $f^{-1}(B)$ is NVPCS and hence it is NVGPCS in X. Thus f is NVGP irresolute mapping.

Theorem 5.10: A mapping $f:(X,\tau)\to(Y,\sigma)$ is NVGP irresolute mapping from NVTS X into NVTS Y, then $f^{-1}(B)\subseteq NV\text{pint}(NVcl(f^{-1}(B)))$ if X is $NV_{gp}T_p$ space.

Proof: Let B be NVGPOS in Y. Then by hypothesis $f^{-1}(B)$ is a NVGPOS in X. Since X is $\mathrm{NV_{gp}T_p}$ space, $f^{-1}(B)$ is a NVPOS in X. Therefore $NV\mathrm{p\,int}\left(f^{-1}(B)\right)= f^{-1}(B)$ and $f^{-1}(B)\subseteq NV\,\mathrm{int}\left(NVcl\left(f^{-1}(B)\right)\right)$. Hence $f^{-1}(B)\subseteq NV\mathrm{p\,int}\left(NVcl\left(f^{-1}(B)\right)\right)$.

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