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FRACTIONAL DIFFERENTIAL OPERATORS INVOLVING SPECIAL FUNCTIONS AND GENERAL CLASS OF POLYNOMIALS

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Abstract

In this paper we use fractional differential operators $D^n_{k,\alpha,x}$ to derive a certain fractional Calculus formulae for Fox's H-function by the application of fractional Calculus formulae involving a general class of polynomials.

Key words:-Fractional differential operator, special-function general class of polynomials.
AMS subject classification 2001 MSC: 26a3333c41

INTRODUCTION AND DEFINITIONS

The fractional derivative of special function of one and more variables is important such as in the evaluation of series,[10,15] the derivation of generating function [12,chap.5] and the solution of differential equations [4,14;chap-3] motivated by these and many other avenues of applications, the fractional differential operators $D^n_{k,\alpha,x}$ and ${}_{\alpha}D^{\mu}_{x}$ are much used in the theory of special function of one and more variables .

* We use the fractional derivative operator defined in the following manner [14] --/

$$D^n_{k,\alpha,x}(x^{\mu}) = \prod_{r=0}^{n-1}[\frac{\sqrt{\mu+rk+1}}{\sqrt{\mu+rk-\alpha+1}}]x^{\mu+nk} \quad .....(1.1)$$

Where α≠μ+1 and α and k are not necessarily integers
Using the following form of the binomial theorem

$$(X+\xi)^{-\lambda} = \xi^{-\lambda} \sum_{m=0}^{\infty} \frac{(\lambda)_m}{!m} \left(\frac{-x}{\xi}\right)^m$$

Raina [5] obtained a fractional differential formula for the function z^p using generalized Gauss theorem, while Ross[7] obtained the fractional integral transformation by obtained the fractional integral formula for the function$(\alpha z+\beta)^{a}$ using series expansion method .kalla et al [4] has derived the fractional integral transformation by orthogonal polynomials. Ali et al [1] generated the expansion of the Laguerre polynomials and Soni and Singh [12] obtained the **fractional** differential formulae involving a general class of polynomials.

The present work is an attempt in the direction of obtaining fractional calculus formula by utilizing series expression method, introduced by srivastava [9]. The name general class of polynomials, itself indicates the importance of the results, because we can derive a number of fractional calculus formulae for various classical orthogonal polynomials.

MAIN RESULT FIRST.

$$D^{n}_{k,\alpha,x}\left[X^{\mu}(X+\xi)^{-\lambda}\boldsymbol{S^{m}_{l}}\{X^{\rho}(X+\xi)^{-\sigma}\}\right]=$$

$$\xi^{-\lambda}X^{\mu+nk}\sum_{j=0}^{\left[\frac{l}{m}\right]}\frac{(-l)_{mj}}{!j}A_{l,j}\left(\frac{X^{\rho}}{\xi^{\sigma}}\right)^{j}\sum_{m=0}^{\infty}\frac{(\lambda+\sigma j)_{m}}{!m}\frac{(-1)^{m}}{\xi^{m}}$$

$$\prod_{g=0}^{n-1}\frac{\Gamma\mu+m+\rho j+gk+1}{\Gamma\mu+m+\rho j+gk-\alpha+1}X^{m}$$

Provided that min(k, λ , ρ , σ)> 0 $\left|\frac{x}{\xi}\right| < 1$ and Re(k+ρj-μ+1) > 0

.....(1)

MAIN RESULT FIRST.

$$D^{n}_{k,\alpha,x}\left[X^{\mu}\boldsymbol{S^{m}_{n}}\{X^{\rho}(X+\xi)^{-\sigma}\}\right]H^{M,N}_{P,Q}(X^{\mu})$$

$$=\sum_{m=0}^{\infty}\sum_{j=0}^{\left[\frac{n}{m}\right]}\frac{(-n)_{mj}}{!j}\frac{(-1)^{m}}{!m}\frac{(\sigma j)_{m}}{\xi^{m}}A_{n,j}\,\xi^{-\sigma j}\,X^{\mu+\rho j+m+nk}\,H^{M,N+1}_{P+1,Q+1}\left[X^{\mu}/\right.$$

$(-\mu-\mu s-m-\rho j,k)s=0,n-1aj,\alpha j1,Pbj,\beta j1,q(-\alpha-\mu-\mu s-m-\rho j,k)s=0,n-1$

Provided that min(k, λ , ρ , σ)> 0 $\left|\frac{x}{\xi}\right| < 1$ and Re(k+ρj-μ+1) > 0

.....(2)

Proof:-For the proof of this result we shall utilize following definition introduced by srivastava [9] or general class of polynomials

$$\boldsymbol{S^{m}_{n}}(\boldsymbol{X})=\sum_{j=0}^{\left[\frac{n}{m}\right]}\frac{(-n)_{mj}}{!j}A_{l,j}\,X^{j} \quad ..... (2.2)$$

Where m is an arbitrary positive integer and the coefficient ($A_{l,j} > 0$) are arbitrary constant real or complex

Expressing the general class of polynomials $S_n^m\,(x)$ occuring on its left hand side in the series from given (2.2) the left hand side of (2.1) {say $\oplus$} takes the following form

$$\oplus \quad = \quad D_{k,\alpha,x}^{n}\left[X^{\mu}\,(X+\xi)^{-\lambda}\,\sum_{j=0}^{\left[\frac{n}{m}\right]}\frac{(-n)_{mj}}{!j}\,A_{l,j}\,X^{\rho j}\,\{(X+\xi)^{-\sigma j}\}\right]$$

Using the following form of the Binomial theorem

$$(X+\xi)^{-\lambda} = \xi^{-\lambda}\sum_{m=0}^{\infty}\frac{(\lambda)_m}{!m}\left(\frac{-x}{\xi}\right)^{m} \qquad \ldots\ldots \quad (2.3)$$

In the above expression we have

$$\oplus \quad = \xi^{-\lambda}\sum_{j=0}^{\left[\frac{l}{m}\right]}\frac{(-l)_{mj}}{!j}A_{l,j}\,\xi^{-\sigma j}\quad \sum_{m=0}^{\infty}\frac{(\lambda+\sigma j)_m}{!m}\,\frac{(-1)^m}{\xi^m}\,D_{k,\alpha,x}^{n}\left(\;X^{k+\rho j+m}\right)$$

We use the fractional derivative operator defined in the following manner [15]

$$D_{k,\alpha,x}^{n}(x^{\mu})=\prod_{r=0}^{n-1}\left[\frac{\sqrt{\mu+rk+1}}{\sqrt{\mu+rk-\alpha+1}}\right]x^{\mu+nk}$$

Where α≠μ+1 and α and k are not necessarily integers
and after simplification we get required result (2.1)

Proof:- First Taking as method in proof I and then using by mellin Barnes type contour integral for H-function for one variable and then simplification we get required result (2.2)

Special case I :- As special case of our main result if we take σ=0 and λ=0 we deduce the
Then the formula (2.1) we have

$$D_{k,\alpha,x}^{n}\left(X^{\mu}\boldsymbol{S_n^m}\;X^{\rho}\right) = \sum_{j=0}^{\left[\frac{n}{m}\right]}\frac{(-n)_{mj}}{!j}\,A_{n,j}\,\prod_{g=0}^{n-1}\frac{\Gamma\mu++\rho j+gk+1}{\Gamma\mu++\rho j+gk-\alpha+1}\,X^{\mu+\rho j+nk} \qquad (3.1)$$

Special case :-II if we take σ=0

$$D_{k,\alpha,x}^{n}\left(X^{\mu}\boldsymbol{S_n^m}\;X^{\rho}\,H_{P,Q}^{M,N}\,(X^{\mu})\right) = (-1)^m X^{\mu+m+nk}\quad \xi^{-\lambda}\sum_{j=0}^{\left[\frac{n}{m}\right]}\frac{(-n)_{mj}}{!j}A_{n,j}\,X^{\rho j}\;H_{P+1,Q+1}^{M,N+1}$$

$$\left[X^{\mu}\Big/\begin{matrix}(-\mu-\mu s-m-\rho j\,,k)_{s=0,n-1}\left(a_j,\alpha_j\right)_{1,P}\\ \left(b_j,\beta_j\right)_{1,q}(-\alpha-\mu-\mu s-m-\rho j\,,k)_{s=0,n-1}\end{matrix}\right] \qquad (3.2)$$

if we take λ =0 in (3.2) while this is independent from λ i.e. there is no change in (3.2)

3.Conclusion

In this paper we get fractional differential operator formulae involving special function and general class of polynomials. and their special cases.

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5.Refrence

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