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EFFECTS OF VARIABLE VISCOSITY AND VARIABLE THERMAL CONDUCTIVITY ON UNSTEADY MHD DUSTY FLUID FLOW PAST AN EXPONENTIALLY ACCELERATED PLATE

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Abstract

A numerical model is presented to study the effects of temperature dependent viscosity and thermal conductivity on unsteady flow of a viscous incompressible dusty fluid past an exponentially accelerated vertical plate with viscous dissipation. A uniform magnetic field is applied in the transverse direction of the flow. The viscosity and thermal conductivity of the fluid are assumed to be varying with respect to temperature. Saffman model of dusty fluid is considered for the investigation. The non-linear partial differential equations with prescribed boundary conditions governing the flow are discretized using Crank-Nicolson formula and the resulting finite difference equations are solved by developing programming codes for MATLAB software. Numerical computations are carried out for various values of the physical parameters and the effects over the velocity, temperature and species concentration are analyzed with the help of graphs. Numerical values of skin friction coefficient, rate of heat and mass transfer are also obtained and presented in tabulated form.

Keywords:

Dusty fluid;
Variable viscosity;
Variable thermal conductivity;
Viscous dissipation;
Finite difference method.

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1. Introduction

In recent years the study of momentum, heat and mass transfer of dusty fluids has great practical importance due to tremendous applications in sciences and engineering. In the past few decades, researchers have been focusing their research work on analyzing the heat and mass transfer characteristics of dusty fluids through different channels. In the present study, we are taking initiation to discuss the momentum, and heat and mass transfer characteristics of a dusty fluid flows over an exponentially stretching surface. The convective flow of dusty viscous fluids has a variety of applications like wastewater treatment, combustion and petroleum transport, power plant piping etc. Heat transport in dusty fluids plays a major role in heat transfer enhancement in the renewable energy systems, material processing and industrial thermal management like aerodynamic extrusion of plastic sheets, manufacturing and rolling of plastic films, cooling of metal sheets etc.

Sakiadis [1] analyzed the pioneering work on the flow past a continuous moving surface with a constant velocity in boundary layer. He formulated the equations governing the two-dimensional flow problems.

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Magyari and Keller [2] were the first authors who investigate the boundary layer flow due to an exponentially stretching continuous surface. They analyzed the problem both analytically and numerically. The effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface was studied by Partha *et al*. [3]. Khan [4] also presented the visco-elastic boundary layer flow and heat transfer characteristics over an exponentially stretching sheet. Al-odat *et al*. [5] investigated the effect of magnetic field on the flow and heat transfer over an exponentially stretching continuous surface. Ishak [6] investigated the radiation effect on MHD boundary layer flow due to an exponentially stretching. Soret and Dufour effects on mixed convection flow and heat transfer from an exponentially stretching surface were studied by Srinivasacharya and RamReddy [7]. They obtained numerical solution for the problem using Kellar-box method.

Important applications of dust particles in a boundary layer include a wide range of real world applications. Initially, Saffman [8] worked on the laminar flow of a gas containing dust particles and stability, which describes the fluid-particle system. He derived the equations of motion for a flow of gas carrying the dust particles. The flow of dusty fluid in the boundary layer over a semi-infinite flat plate was studied by Datta and Mishra [9]. Vajravelu and Nayfeh [10] discussed the hydromagnetic flow of a dusty fluid over a stretching sheet with the effects of particle loading, fluid-particle interaction and suction on the flow characteristics. Also, they compared their analytical solution with numerical ones. Unsteady MHD boundary layer flow and heat transfer characteristics of dusty fluid over a stretching sheet with variable wall temperature (VWT) and variable heat flux (VHF) were observed by Gireesha *et al*. [11, 12]. In these papers, they analyzed the effect of magnetic field on the flow and heat transfer within the boundary layer of dusty fluid. Pavithra and Gireesha [13] investigated boundary layer flow problem of dusty fluid for an exponentially stretching sheet by considering the internal heat generation/absorption and viscous dissipation.

Recently, a study on a convective heat transfer characteristics of an incompressible viscous dusty fluid over an exponentially stretching surface has been carried out by Izani and Ali [14] with an exponential temperature distribution. The effect of a magnetic field on a boundary layer flow of an electrically conducting dusty fluid over a stretching surface has been investigated by Jalil *et al*. [15].

In most of the studies mentioned above are carried out under a steady-state condition. But, in many cases, flow becomes time dependent due to a sudden stretching of the flat sheet or heat flux of the sheet or by a step change of the temperature, and consequently, it becomes an unsteady flow problem. Furthermore, in all the flow problems mentioned above, the viscosity and thermal conductivity of fluid were considered as constants. However, a more accurate prediction for the flow, heat and mass transfer can be obtained by taking into account the variation of such properties with temperature. Therefore, the goal of this paper is to study the effects of temperature dependent viscosity and thermal conductivity on an unsteady hydromagnetic boundary layer flow over an exponentially stretching sheet.

2. Mathematical Formulation of the Problem

Explaining Consider an unsteady flow, heat and mass transfer flow of an incompressible viscous electrically conducting and radiating dusty fluid past an exponentially accelerated infinite isothermal vertical plate. It is assumed that a temperature dependent heat source present in the flow and dust particles are assumed to be electrically nonconductive, spherical in shape having the same radius and mass, and un-deformable. Also, the fluid is supposed to be gray, absorbing-emitting but non-scattering. At the beginning, the fluid is considered to be at rest. The x'-axis is taken along the plate in the vertically upward direction and the *y*-axis is taken normal to the plate as shown in Fig. 1. A magnetic field of strength $\vec{B}(0, B_0)$ is applied perpendicular to the plate. Reynolds number is supposed to be so small that the induced magnetic field can be neglected (Sutton [16]).

At the time $t' = 0$, the temperature of the plate and species concentration of the fluid are T'_∞ and C'_∞, respectively. At time $t' > 0$, the plate is exponentially accelerated in its own plane with a velocity $u = u_0 e^{a't'}$ and the plate temperature and species concentration of the fluid upstanding to T'_W and C'_W, and are maintained constantly thereafter.

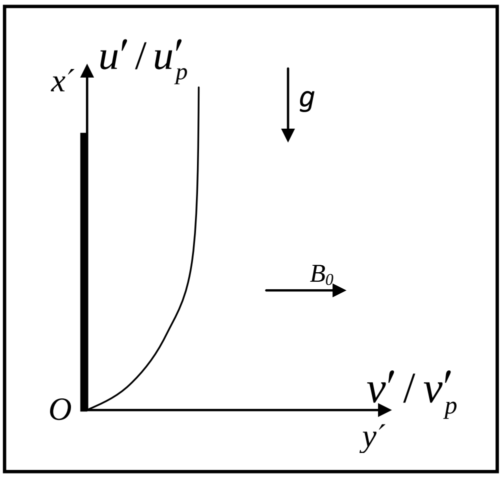

Fig. 1: Geometry of the problem

Under the above assumptions, using the usual Boussinesq's approximation and the Saffman [8] model, the governing equations for the two-phase flow are:

For the fluid phase:

$$\frac{\partial u'}{\partial t'} = \frac{\partial}{\partial y'}\left(\upsilon \frac{\partial u'}{\partial y'}\right) + g\beta^*(T' - T'_\infty) + g\beta^{**}(C' - C'_\infty) - \frac{\sigma B_0^2}{\rho}u' + \frac{KN}{\rho}(u'_p - u') \tag{1}$$

$$\rho c_p \frac{\partial T'}{\partial t'} = \frac{\partial}{\partial y'}\left(\lambda \frac{\partial T'}{\partial y'}\right) - \frac{\partial q_r}{\partial y'} + Q_0(T' - T'_\infty) + \frac{N c_p}{\tau_T}(T'_p - T') + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

$$\frac{\partial C'}{\partial t'} = \frac{\partial}{\partial y'}\left(D_m \frac{\partial C'}{\partial y'}\right) + \frac{mN}{\rho \tau_c}(C'_p - C') \tag{3}$$

For the dust phase:

$$\frac{\partial u'_p}{\partial t'} = -\frac{K}{m}(u'_p - u') \tag{4}$$

$$\frac{\partial T'_p}{\partial t'} = -\frac{c_p}{c_m \tau_T}(T'_p - T') \tag{5}$$

$$\frac{\partial C'_p}{\partial t'} = -\frac{mN}{\rho \tau_c}(C'_p - C') \tag{6}$$

The initial and boundary conditions are:

$$\left.\begin{aligned} & t' = 0 : u' = 0, T' = T'_\infty, C' = C'_\infty \text{ for } y' \geq 0 \\ & t' > 0 : u' = u_0 e^{a't'}, T' = T'_w, C' = C'_w \text{ at } y' = 0 \\ & u' \to 0, u'_p \to u', T' \to T'_\infty, T'_p \to T'_\infty, C' \to C'_\infty, C'_p \to C'_\infty \text{ as } y' \to \infty. \end{aligned}\right\} \tag{7}$$

Where T' and C' are the temperature and species concentration of fluid, respectively. T'_p and C'_p are the temperature and species concentration of dust phase, respectively. u' and u'_p are the velocities of fluid and dust phases, respectively.

Using Rosseland approximation for radiation we can write radiative heat flux as (Necati [17]):

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'}, \tag{8}$$

where σ^* and k^* are the Stefan-Boltzmann constant and mean absorption coefficient, respectively.

It is assumed that temperature difference within the flow such that the term ${T'_\infty}^4$ can be expressed as a linear function of temperature. This is accomplished by expanding ${T'_\infty}^4$ in a Taylor series about T'_∞ and neglecting the second and higher order terms, we get:

$$T'^4 \cong 4{T'_\infty}^3 T' - 3{T'_\infty}^4 . \tag{9}$$

Now we introduce the following non-dimensional quantities to make the governing equations dimensionless:

$$\left.\begin{aligned} & u=\frac{u'}{u_0},\ u_p=\frac{u'_p}{u_0},\ t=\frac{t'u_0^2}{\upsilon_\infty},\ y=\frac{y'u_0}{\upsilon_\infty},\ a=\frac{a'\upsilon_\infty}{u_0^2}, \\ & \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ \theta_p=\frac{T'_p-T'_\infty}{T'_w-T'_\infty},\ \phi=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ \phi_p=\frac{C'_p-C'_\infty}{C'_w-C'_\infty}. \end{aligned}\right\} \tag{10}$$

Viscosity of the fluid is assumed to be an inverse linear function of temperature, and it can be expressed as (following Lai and Kulacki [18]):

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\delta\left(T'-T'_\infty\right)\right] \quad \text{or,} \quad \frac{1}{\mu}=\alpha\left(T'-T'_r\right), \tag{11}$$

where $\alpha=\frac{\delta}{\mu_\infty}$ and $T'_r=T'_\infty-\frac{1}{\delta}$.

Moreover, thermal conductivity of the fluid varies with temperature. Following Choudhury and Hazarika [19], we assumed thermal conductivity of the fluid as:

$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}\left[1+\xi\left(T'-T'_\infty\right)\right] \quad \text{or,} \quad \frac{1}{\lambda}=\zeta\left(T'-T'_c\right), \tag{12}$$

where $\zeta=\frac{\xi}{\lambda_\infty}$ and $T'_c=T'_\infty-\frac{1}{\xi}$.

Here, $\alpha, \delta, \xi, \zeta, T'_r$ and T'_c are constants and their values depend on the reference state and thermal properties of the fluid i.e., υ (kinematic viscosity) and λ (thermal conductivity).
Substituting equations (8) to (12) into equations (1) to (6), we get the following dimensionless equations:

For the fluid phase:

$$\frac{\partial u}{\partial t}=\frac{\theta_r}{(\theta-\theta_r)^2}\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y}-\frac{\theta_r}{\theta-\theta_r}\frac{\partial^2 u}{\partial y^2}+Gr\,\theta+Gm\,\phi-M\,u+R(u_p-u) \tag{13}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\theta_c}{(\theta-\theta_c)^2}\left(\frac{\partial\theta}{\partial y}\right)^2-\frac{1}{\Pr}\frac{\theta_c}{\theta-\theta_c}\frac{\partial^2\theta}{\partial y^2}-\frac{R}{\Pr}\theta+Q\theta+\frac{2R}{3\Pr}(\theta_p-\theta)-Ec\frac{\theta_r}{\theta-\theta_r}\left(\frac{\partial u}{\partial y}\right)^2 \tag{14}$$

$$\frac{\partial\phi}{\partial t}=\frac{1}{Sc}\frac{\theta_r}{(\theta-\theta_r)^2}\frac{\partial\theta}{\partial y}\frac{\partial\phi}{\partial y}-\frac{1}{Sc}\frac{\theta_r}{\theta-\theta_r}\frac{\partial^2\phi}{\partial y^2}-Ra_1(\phi_p-\phi) \tag{15}$$

For the dust phase:

$$\frac{\partial u_p}{\partial t}=-G(u_p-u) \tag{16}$$

$$\frac{\partial\theta_p}{\partial t}=-L_T(\theta_p-\theta) \tag{17}$$

$$\frac{\partial\phi_p}{\partial t}=-L_c(\phi_p-\phi) \tag{18}$$

Corresponding initial and boundary conditions, equation (7.2.7), are reduced to:

$$\left.\begin{array}{l} t=0: u=0, \theta=0, \phi=0 \text{ for all } y \\ t>0: u=e^{at}, \theta=1, \phi=1 \text{ at } y=0 \\ \quad u\to 0, u_p\to 0, \theta\to 0, \theta_p\to 0, \phi\to 0, \phi_p\to 0 \text{ as } y\to\infty. \end{array}\right\} \quad (19)$$

where the dimensionless parameters are defined as follows:

$\tau_v = m/K$ is the velocity equilibration time, $\rho_r=\rho_p/\rho$ is the relative density, $R=\dfrac{KN\upsilon_\infty}{\rho u_0^2}$ is the particle concentration parameter, $a_1=\dfrac{\tau_v}{\tau_c}$ is a constant, $Gr=\dfrac{g\beta^*(T'_w-T'_\infty)\upsilon_\infty}{u_0^3}$ is the Grashof number; $Gm=\dfrac{g\beta^{**}(C'_w-C'_\infty)\upsilon_\infty}{u_0^3}$ is the local mass Grashof number, $Ec=\dfrac{u_0^2}{c_p(T'_w-T'_\infty)}$ is the Eckert number, $M=\dfrac{\sigma B_0^2\upsilon_\infty}{u_0^2}$ is the magnetic parameter, $\Pr=\dfrac{\mu_\infty c_p}{\lambda_\infty}$ is the Prandtl number, $Q=\dfrac{Q_0\upsilon_\infty}{\rho c_p u_0^2}$ is the heat generation/absorption coefficient, $G=\dfrac{K\upsilon_\infty}{mu_0^2}$ is the particle mass parameter, $L_T=\dfrac{c_p\upsilon_\infty}{c_m\tau_T u_0^2}$ is the temperature relaxation time parameter, $L_c=\dfrac{mN\upsilon_\infty}{\rho\tau_c u_0^2}$ is the species concentration relaxation time parameter. $\theta_r=\dfrac{T'_r-T'_\infty}{T'_w-T'_\infty}$ and $\theta_c=\dfrac{T'_c-T'_\infty}{T'_w-T'_\infty}$ are the viscosity variation parameter and thermal conductivity variation parameter, respectively. It is also important to note that θ_r and θ_c are negative for liquids and positive for gases (Kuppalapalle *et al.* [20]).

2.1 Skin friction coefficient, Nusselt Number and Sherwood Number

Skin friction coefficient (C_f), Nusselt number (Nu) and Sherwood number (Sh) are the parameters of physical and engineering interest for the present problem, which physically indicate the wall shear stress, rate of heat and mass transfer, respectively.

In this problem, dimensionless skin friction coefficient, Nusselt number and Sherwood number are given respectively by:

$$C_f=-\frac{\theta_r}{1-\theta_r}\left.\frac{\partial u}{\partial y}\right)_{y=0},\quad Nu=\frac{\theta_c}{1-\theta_c}\left.\frac{\partial \theta}{\partial y}\right)_{y=0}\quad \text{and}\quad Sh=\frac{\theta_r}{1-\theta_r}\left.\frac{\partial \phi}{\partial y}\right)_{y=0}.$$

3. Numerical Technique

The system of equations (13) to (18) governing the flow represents a system of coupled nonlinear partial differential equations, which are solved numerically under the boundary conditions, equation (19), by adopting Crank-Nicolson implicit finite difference scheme, which is always unconditionally stable. To obtain the difference equations, the region of the flow is divided into a grid or mesh lines parallel to *y* and *t* axes.

The region of integration is considered as a rectangle with sides y_{max}(=5) and t_{max}(=1), where y_{max} corresponds to $y\to\infty$, which lies very well outside the momentum, energy and species concentration boundary layers. The maximum of y has been chosen as 5 after some preliminary investigations so that the last two of the boundary conditions, equation (19), are satisfied within the tolerance limit 10^{-5}. After experimenting with a few set of mesh sizes, the mesh sizes have been fixed at the level $\Delta y=0.17$ with time step $\Delta t=0.0031$. In this case, the computations are carried out first by reducing the spatial mesh sizes by 50 % in one direction, and later in both directions by 50 %. The results are compared. It is observed that, in all cases, the results differ only in the fifth decimal place. Hence, the choice of the mesh sizes seems to be appropriate.

Solutions of difference equations are obtained at the intersection of these mesh lines called nodes. In this problem, the values of the dependent variables u, θ and ϕ at the nodal points along the $y=0$ are given by $u(0,t), \theta(0,t)$ and $\phi(0,t)$, hence are known from the boundary conditions. But, $u_p(0,t), \theta_p(0,t)$ and $\phi_p(0,t)$ are unknown, which are estimated by applying trial and error, in such a way that, for those values, all the boundary conditions are satisfied at the other boundary with a good accuracy (error less than 10^{-5}). Δy and Δt are taken as the constant mesh sizes along *y* and *t* directions respectively. We need the scheme to find single values at next time level in terms of known values at an earlier time level.

In order to obtain the finite difference equations for the equations (13) to (18), we have adopted forward difference approximation for the first order partial derivatives of $u, \theta, \phi, u_p, \theta_p$ and ϕ_p with respect to *t* and *y* and a central difference approximation for the second order partial derivatives of u, θ and ϕ with respect to y.

Knowing the values of $u, \theta, \phi, u_p, \theta_p$ and ϕ_p at time *t* we can calculate the values at time $t+\Delta t$. Using initial and boundary conditions, the system can be solved based on an iterative scheme and developing suitable programming codes for the method in MATLAB software.

The truncation error in the finite difference approximation is $O(\Delta t^2 + \Delta y^2)$ and it tends to zero as Δt and Δy tend to zero. Hence the scheme is compatible. Thus, stability and compatibility ensure convergence.

4. Results and Discussion

In order to analyze the problem physically, numerical computations are carried out to explain the effects of different parameters governing the flow upon the nature of the flow, heat and mass transport phenomenon. The numerical values of the velocity, temperature, species concentration, skin-friction coefficient, Nusselt number and Sherwood number are obtained for different physical parameters like the viscosity variation parameter θ_r, thermal conductivity variation parameter θ_c, magnetic parameter *M* and time *t*. A representative set of numerical results is presented graphically in Figs. 2 to 15 and in Tables 1 and 2. Numerical values of the parameters used for simulation are: *M*=1, *Pr*=0.71, *Ec*=0.05, a_1=1, *Q*=0.5, *Gr*=5, *Gm*=5, *R*=1, *Sc*=0.22, $\theta_r=3$ and $\theta_c=5$, unless otherwise stated.

The effect of the viscosity variation parameter θ_r on the velocity, temperature, and species concentration profiles is shown in Figs. 2 to 4. It is observed from these figures that the velocity and species concentration decreases with the increase of the viscosity variation parameter. Physically, as the values of viscosity variation parameter θ_r increases, the resistance to the relative motion of the different layers of fluid increases due to the increase of viscous force, as a result, velocity of fluid phase decreases. Thus, the increase of θ_r decelerates the fluid motion and reduces the species concentration profile of the fluid and dust phases along the wall. Also, one can see that the temperature of both phases is almost not affected by the increase of the viscosity variation parameter θ_r (Fig. 3).

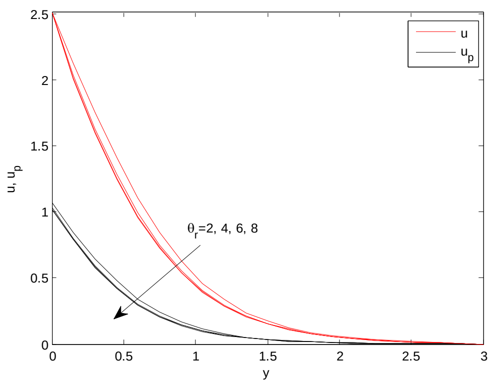

Fig. 2: Velocity profiles for different θ_r

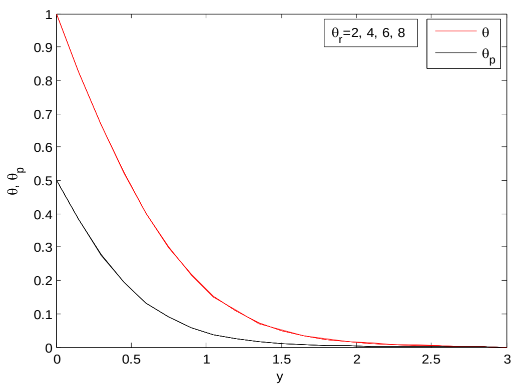

Fig. 3: Temperature profiles for different θ_r

The effect of thermal conductivity variation parameter θ_c on velocity, temperature, and species concentration profiles is shown in Figs. 5 to 7. Figure 5 has shown that viscosity increases for both the fluid and dust phases with increasing values of θ_c. On the other hand, a reverse trend is noticed with the temperature profile (Fig. 6). It is due to the reason that thermal conductivity is an inverse linear function of temperature, hence the result. Thermal conductivity variation parameter has no significant effect on species concentration profile (Fig. 7).

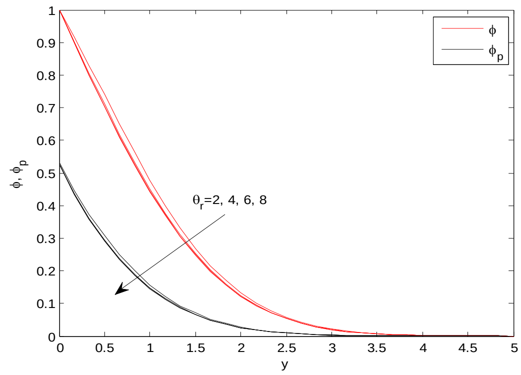

Fig. 4: Species concentration profiles for different θ_r

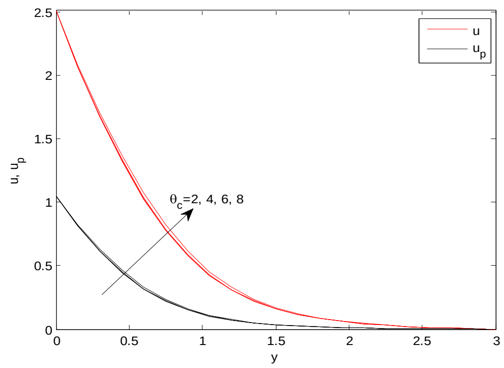

Fig. 5: Velocity profiles for different θ_c

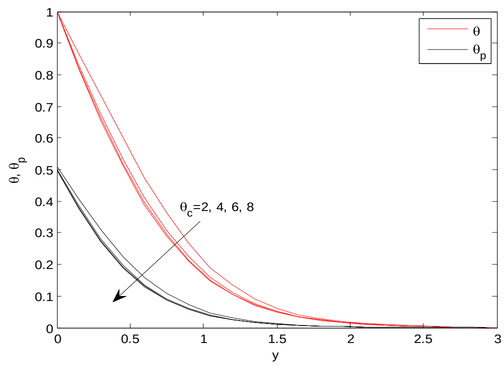

Fig. 6: Temperature profiles for different θ_c

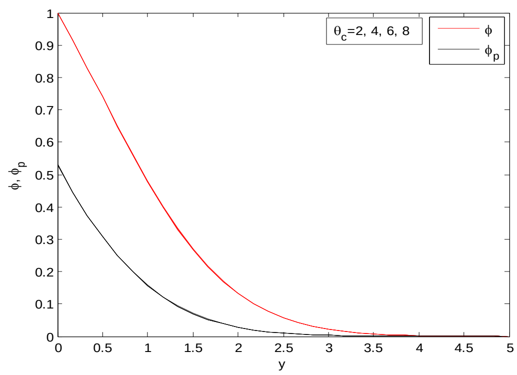

Fig. 7: Species concentration profiles for different θ_c

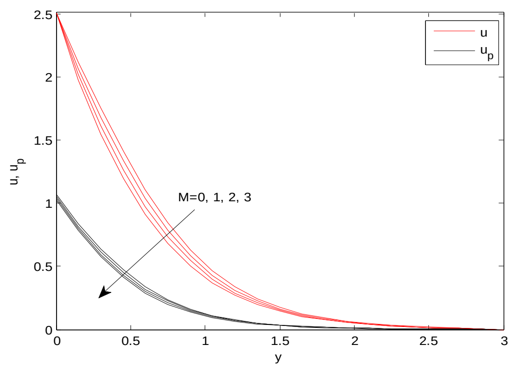

Fig. 8: Velocity profiles for different *M*

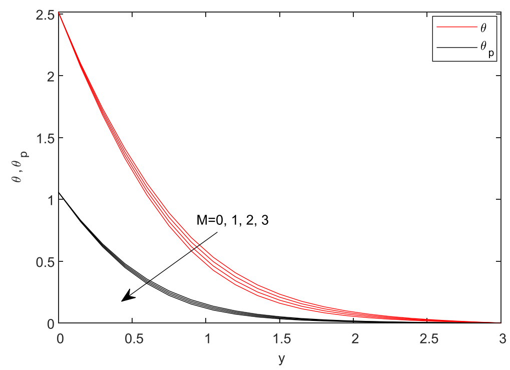

Fig. 9: Temperature profiles for different *M*

Figures 8 and 9 illustrate the effect of magnetic field on velocity and temperature profiles through the magnetic parameter (*M*). As the increasing values of magnetic parameter *M*, velocity decreases for both the fluid and dust phases. The presence of magnetic field sets a resistive force, called Lorentz force, which opposes the velocity field and also causes increase in its temperature (Fig. 9). Therefore, as the values of M increase so does the retarding force and hence the velocity decreases.

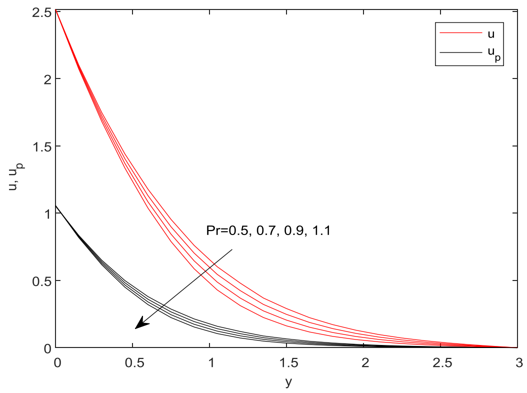

Fig. 10: Velocity profiles for different *Pr*

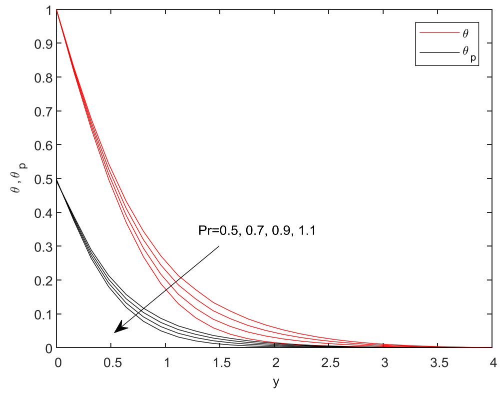

Fig. 11: Temperature profiles for different *Pr*

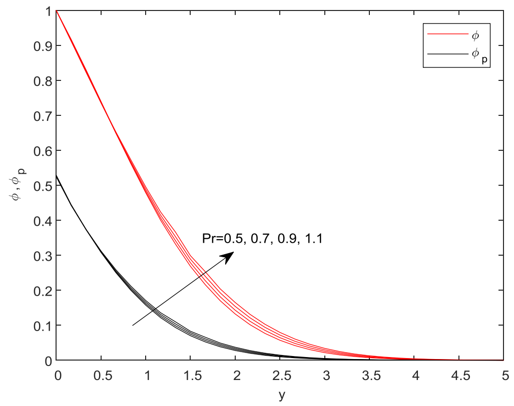

Fig. 12: Species concentration profiles for different *Pr*

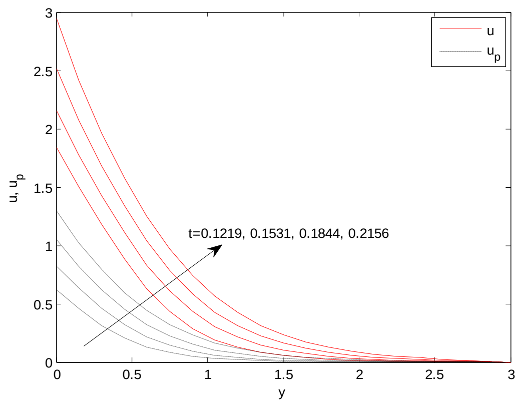

Fig. 13: Velocity profiles for different *t*

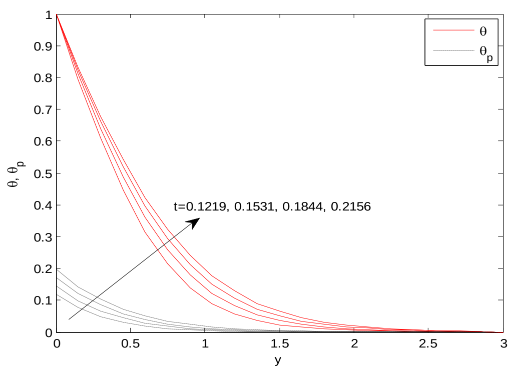

Fig. 14: Temperature profiles for different *t*

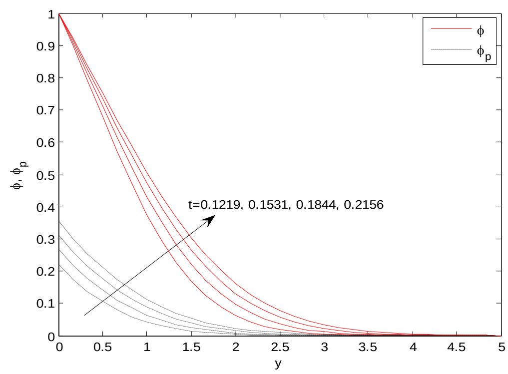

Fig. 15: Species concentration profiles for different *t*

Figures 10 to 12 indicate the effect of Prandtl number *Pr* on velocity, temperature and species concentration, respectively. Since, *Pr* is the ratio of velocity boundary layer to thermal boundary layer, velocity boundary layer and thermal boundary layer coincides when *Pr*=1. When *Pr* <1, it means that heat diffuses very quickly compared with velocity, hence velocity and temperature decrease with the increase of *Pr*, but species concentration increases with the same condition.

The behaviors of velocity, temperature and species concentration profiles for different values of the time *t* are presented in Figs. 13 to 15, respectively. It is observed that velocity, temperature, and species concentration increases with increasing values of time *t* for both the fluid and dust phases.

Table 1 presents values of C_f, *Nu* and *Sh* for both the constant and variable cases of viscosity and thermal conductivity. The values of C_f, *Nu* and *Sh*, when variable viscosity and thermal conductivity are taken into account, are smaller than the values when viscosity and thermal conductivity are taken as constant. Thus, it shows that it is better to have viscosity and thermal conductivity as function of temperature to achieve accurate results of flow, heat and mass transfer properties.

Table 1: Comparison of skin friction coefficient (C_f), Nusselt number (*Nu*) and Sherwood number (*Sh*)

M	*Pr*	when viscosity (μ) and thermal conductivity (λ) are constants			when viscosity (μ) and thermal conductivity (λ) are variables		
		C_f	*Nu*	*Sh*	C_f	*Nu*	*Sh*
0	0.71	−3.1265	1.255177	0.550449	−3.89324	1.431015	0.772973
1		−3.48508	1.250642	0.550442	−4.36814	1.428947	0.77294
2		−3.8195	1.246162	0.550438	−4.81307	1.426889	0.772907
1	0.7	−3.48233	1.246484	0.550445	−4.36466	1.423711	0.772966
	0.9	−3.53266	1.328694	0.550338	−4.42829	1.527257	0.772586
	1.1	−3.57482	1.408184	0.550309	−4.48204	1.627533	0.772477

The variation of the skin friction coefficient, Nusselt number and Sherwood number for various flow governing parameters, viz. θ_r , θ_c and *M* is shown in Table 2. From the table, it is noticed that magnitude of skin friction coefficient decreases when viscosity variation parameter (θ_r) is increased. This is due to the reason that fluids having higher viscosity have relatively lower surface velocity gradient, for which skin friction coefficient decreases. On the contrary, the Nusselt number increases with the increasing values of the same parameter. This is due to the fact that, the friction of the fluid increases due to viscosity so the heat is generated from the friction on the surface, which leads to a rise in the magnitude of heat transfer rate, and hence Nusselt number increases. Also, Sherwood number decreases when viscosity variation parameter increases.

Table 2: Numerical values of skin friction coefficient (C_f), Nusselt number (Nu) and Sherwood number (*Sh*) for different values of θ_r , θ_c and *M*

θ_r	θ_c	*M*	C_f	*Nu*	*Sh*
3	5	1	−4.36814	1.428947	0.77294
6			−3.86887	1.44274	0.718786
9			−3.73289	1.44702	0.703404
12			−3.66932	1.449104	0.696121
3	3	1	−4.32171	1.548305	0.772223
	6		−4.37882	1.402972	0.773137
	9		−4.39593	1.362484	0.773477
	12		−4.40418	1.343432	0.773652
3	5	0	−3.89324	1.431015	0.772973
		1	−4.36814	1.428947	0.77294
		2	−4.81307	1.426889	0.772907
		3	−5.23083	1.424857	0.772875

In the same table, we have seen that the magnitude of skin friction coefficient increases and the Nusselt number decreases for increasing thermal conductivity variation parameter θ_c. The reason behind is that viscosity decreases with increasing thermal conductivity variation parameter, which enhances the magnitude of velocity gradient at the surface of the sheet and retards the magnitude of the heat transfer rate. Hence, Nusselt number decreases and skin friction coefficient increases with increasing θ_c.

Magnetic parameter leads to enhance the magnitude of skin friction coefficient because of the Lorentz force.

5. Conclusion

The effect of variable viscosity and variable thermal conductivity on the flow, heat and mass transfer characteristics of an unsteady hydromagnetic flow due to an exponentially stretching sheet with viscous dissipation has been made theoretically. The solution of this problem is obtained numerically using an implicit finite difference scheme based on Crank-Nicolson scheme, with Gauss-Seidel iteration method, by developing computer codes for MATLAB software. Graphical and tabular mode of presentation of the computed results illustrates the details of flow, heat and mass transfer characteristics and their dependence on the physical parameters involved in the problem. The important findings of this analysis are listed below:

- From the study, we have noticed that the effect of temperature on viscosity and thermal conductivity of the fluid is very significant.
- It is found that both fluid and dust phase velocity decreases with the enhancement of viscosity variation parameter and magnetic parameter.
- Thermal conductivity variation parameter has increasing effects on the velocity profile, while an opposite trend is observed for temperature profile.
- An increase in the value of viscosity variation parameter leads to decrease the concentration profile of both the fluid and dust phases.
- Velocity, temperature and species concentration raise with higher values of time *t* for both the phases.
- The magnitude of skin friction coefficient increases with increases in the value of thermal conductivity variation parameter and magnetic parameter; whereas it decreases for increasing viscosity variation parameter.
- It is noticed that rate of mass transfer decreases with increasing viscosity variation parameter and thermal conductivity variation parameter.
- An enhancement of the viscosity variation parameter leads to increase rate of heat transfer slightly.

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