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THE BOUNDARY LAYER CONVECTION WITH UNIFORM HEAT FLUX FROM THE SIDE WALLS OF AN INCLINED RECTANGULAR ENCLOSURE

D. R. Sasi Rekha *
Dr. Veena Jawali **

Abstract

This paper is the analysis of buoyancy driven convection in an inclined fluid filled rectangular enclosure. The existence of constant heat flux heating and cooling through the walls of the vertical side. It is demonstrate analytically that of the boundary layer thickness must be independent of altitude in the boundary layer regime, that the core must be linearly stratified and motionless, and that the temperature of the vertical inclined walls must vary continuously as the core temperature with the same gradient.

Keywords:

Boundary layer Convection; Rectangular Enclosure; Inclination Cavity; Heat Flux.

Author correspondence:

D. R. Sasi Rekha,
Assistant Professor, Department of Mathematics,
Dayananda Sagar College of Engineering, Bangalore, India

1. Introduction

Many engineering and environmental applications of natural convection in enclosures, today heat transfer research is the most effective fields. Most of the activity has been abstract in this references and reviews, by Catton [4], Jaluria [6], and Yang [12]. The two-dimensional model with vertical isothermal walls at various temperatures of a tiled rectangular enclosure which is filled with fluid with one side wall is heated and the other side is cooled with uniform heat fluxes. If the research is conducted analytically this model may give definite advantages. The problem of natural convection with isothermal or heat flux conditions in a rectangular enclosure has been studied analytically by Gills [5], Kimura and Bejan [7]. Numerical study of this type of problem was done by Wilkes and Churchill (1966), de Vahl Davies (1968), Newell and Schmidt (1970), Quon (1972), Patterson and Imberger (1980) and Chen and Ko (1991).

* Assistant Professor, Department of Mathematics, Dayananda Sagar College of Engineering, Bangalore, India.
** Professor, Department of Mathematics, BMS College of Engineering, Bangalore, India.

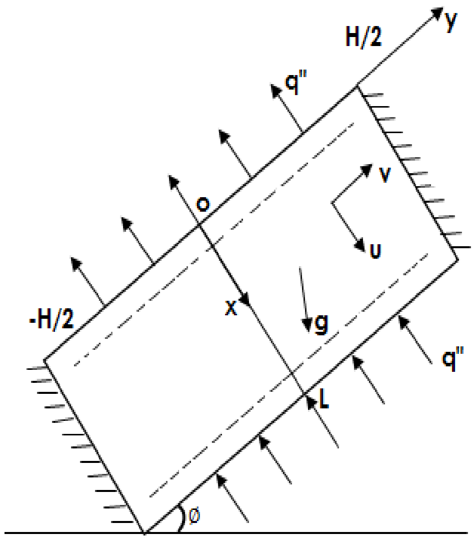

Fig. 1 Schematic illustration of the coordinate system
(the broken lines indicates the extent of the boundary layer)

The object of this study is to find the effect of various parameters on the steady conduct of heat transfer and fluid flow features. Analytical result gives the velocity profile and temperature profile are remarkable changed and it is found that the consequence of that heat transfer rate and there by restraining the convection. The study of this natural convection in a tiled rectangular enclosures is to enrich the understanding of how the energy and air are carried by air which flow through the buildings, the isothermal walls model is clearly inadequate: the temperature of the walls experienced in solar and architectural applications is not uniformly maintained, rather, it is the effect of the heat flux conducted to the wall. Two fluid chambers are separated due to the temperature of a wall at various temperatures which moves such that the wall becomes progressively warmer with altitude and the wall is essentially uniform through heat flux.

Among the previous studies on free convection flow by vertical plate in terms of boundary layer analysis. It seems that a many suitable model for the study of convection in cavity is the tiled rectangular enclosure along the two vertical sides with uniform heat flux (Fig. 1). This fact was indicated before by Balvanz and Kuehn [2] in their study of convection in a vertical slot with one isothermal wall facing a conducting wall with internal heat generation. The object of this paper is to record the heat transfer and the flow aspects in a tiled rectangular vertical enclosure where both side walls are subjected to the uniform heat flux condition. The following demonstration by an analytical result for the boundary layer regime is developed along the outlines of the solution framed by Gill [5] and Kimura and Bejan [7] for the same regime in a tiled rectangular box with isothermal side walls.

Nomenclature

a_n = coefficient in equation
c_p = specific heat at constant pressure
g = gravitational acceleration
H = enclosure height
k = thermal conductivity
l = thermal boundary layer thickness
L = enclosure thickness
Nu = overall Nusselt number
P = total pressure
Pr = Prandtl number = $\frac{\nu}{\alpha}$
$p(y)$ = even function
$q(y)$ = odd function
$\vec{q}$ = velocity vector
q" = heat flux from the side
Ra = Rayleigh number = $g\beta q''H^4/(\alpha\nu k)$
T = temperature
T_0 = reference temperature
ΔT = average side to side temperature difference
u, v = horizontal and vertical velocity components
x, y = horizontal and vertical coordinates
α = thermal diffusivity
β = thermal expansion coefficient
δ = boundary layer thickness
λ_n = function of y
ν = $\frac{\mu_f}{\rho_0}$ = kinematic viscosity
μ_f = viscosity of the fluid
ρ_0 = reference density
ρ = density
ψ = stream function
ξ = vorticity function
$\emptyset$ = inclination angle

Subscript

∞ = indicating condition in the core

2. Mathematical Formulation

Consider the inclined rectangular cavity of two-dimension which is filled with the fluid shown in Fig. 1. The both side walls are specified with heat flux, and the top and bottom walls are both impermeable and insulated

$$q^{"} = k\left(\frac{\partial T}{\partial x}\right)_{x=0,\ L} \quad , \text{constant} \tag{1}$$

The following set of governing equation by [7]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial^3 v}{\partial x^3} + \sin\phi\ \frac{\partial T}{\partial x} = 0 \tag{3}$$

$$u\ \frac{\partial T}{\partial x} + v\ \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} \tag{4}$$

where the non-dimensional variables are

$$x^* = \frac{x}{HRa^{-\frac{1}{5}}} \ , \ y^* = \frac{y}{H} \ , \ u^* = \frac{uH}{\alpha Ra^{\frac{1}{5}}}, \quad v^* = \frac{vH}{\alpha Ra^{\frac{2}{5}}} \ , \ T^* = \frac{(T-T_0)}{\frac{\alpha v}{g\beta H^3} Ra^{\frac{4}{5}}} \ , \ \xi^* = \frac{H^2 \xi}{\alpha Ra^{\frac{2}{5}}} \tag{5}$$

For the left boundary layer the dimensionless boundary conditions are

$$(i)\ u = v = 0\ ,\ \frac{\partial T}{\partial x} = 1 \ \text{at} \quad x = 0$$

$$\left.\begin{array}{l}(ii)\ u \to u_\infty(y) \\ \quad\ v \to 0 \\ \quad\ T \to T_\infty(y)\end{array}\right\} \text{as } x \to \infty \tag{6}$$

Where $T_\infty(y)$ and $u_\infty(y)$ are the temperature and flow in the core (at x >> 1). Equations (2 - 4) are the same equations without the inclination angle that Gill [5] solved in a slot with isothermal walls for the boundary layer regime. The linearized solution of Gill's can be adopted without any change as

$$v = \sum_{n=0}^{4} a_n(y)\ \exp[-\lambda_n(y)\ x] \tag{7}$$

where $\lambda_n(y)$ are imaginary numbers with positive real parts [5]. Subjecting this solution to the existing conditions, equations (6), gives

$$v = \frac{\sin\phi}{\lambda_1{}^3 - \lambda_2{}^3}\left(e^{-\lambda_1 x} - e^{-\lambda_2 x}\right) \tag{8}$$

$$T = T_\infty(y) - \frac{(\lambda_1{}^2 e^{-\lambda_1 x} - \lambda_2{}^2 e^{-\lambda_2 x})}{\lambda_1{}^3 - \lambda_2{}^3} \tag{9}$$

where $\lambda_1(y)$, $\lambda_2(y)$ are unknown functions. To find the unknown functions and $T_\infty(y)$, use the additional three statements

(a) Mass conservation integral:

$$\psi_\infty = -\int_0^\infty v\ dx \tag{10}$$

(b) Energy conservation integral:

$$|\,u\,T\,|_0^\infty + \frac{d}{dy}\int_0^\infty v\ T\ dx = \left|\frac{\partial T}{\partial x}\right|_0^\infty \tag{11}$$

(c) Core centrosymmetry about x = L/2, y = 0 in Fig. 1, which means that

$$\begin{array}{l}(i)\ T_\infty(y) = -T_\infty(-y) \\ (ii)\ T_\infty{}'(y) = T_\infty{}'(-y) \\ (iii)\ \psi_\infty(y) = \psi_\infty(-y)\end{array} \tag{12}$$

The two integral conditions (a, b) gives

$$\psi_\infty = \frac{sin\emptyset}{\lambda_1\ \lambda_2\,(\lambda_1{}^2 + \lambda_1\,\lambda_2 + \lambda_2{}^2)} \quad (13)$$

$$\frac{d}{dy}\left[\frac{sin\emptyset}{2(\lambda_1 + \lambda_2)(\lambda_1{}^2 + \lambda_1\,\lambda_2 + \lambda_2{}^2)^2}\right] - \psi_\infty\, T_\infty{}'(y) = -1 \quad (14)$$

The centrosymmetry conditions (c), it is suitable to express equations (13, 4) in terms of Gill's functions [5]

$$p(y) = \text{even function}$$
$$q(y) = \text{odd function}$$

where

$$\lambda_{1,2} = \frac{p}{4}(1-q)(1 \pm i\sqrt{1+2q} \quad (15)$$

Thus, equations (13, 14) become

$$\psi_\infty = \frac{64\ sin\emptyset}{p^4\,(1-q)^5\,(1+q)} \quad (16)$$

$$\frac{d}{dy}\left[\frac{64\ sin\emptyset}{p^5\,(1-q)^7}\right] - \psi_\infty\, T_\infty{}'(y) = -1 \quad (17)$$

The principle of the complete analysis is concealed in these two equations: it is clear that in order for both ψ_∞ and the left hand side of equation (17) are to be even functions of y, at the same time

$$p = \text{constant}$$
$$q = 0 \quad (18)$$
$$T_\infty{}'(y) = \text{constant}$$

In conclusion, in the boundary layer regime of a tiled vertical box with uniform heat flux, the core must be linearly stratified and motionless (u , v → 0). Moreover, since the λ's are constant, equation (15).

In summary, the solution is

$$\lambda_{1,2} = \frac{p}{4}(1 \pm i)$$

$$v = -\frac{32}{p^3}\, e^{-\frac{p}{4}x} \sin\left(\frac{p}{4}x\right)\, sin\emptyset$$

$$T = T_\infty(y) - \frac{4}{p}\, e^{-\frac{p}{4}x} \cos\left(\frac{p}{4}x\right)$$

if it is independent of y then

$$T = -\frac{4}{p}\, e^{-\frac{p}{4}x} \cos\left(\frac{p}{4}x\right)$$

$$\psi_\infty = \frac{64\ sin\emptyset}{p^4}\ ,\quad T_\infty(y) = \frac{y}{\psi_\infty} \quad (19)$$

where p is an unknown constant. The solution will get complete by determining p. The vertical enthalpy flow through the flow must be balanced exactly by the downward heat conduction

$$\int_0^L \rho\, c_p\, v\, T\, dx = \int_0^L k\left(\frac{\partial T}{\partial y}\right) dx \quad (20)$$

and substituting these along with equations (19) into equation (20) yields

$$p^9 = 8192\left(\frac{H}{L}\right) Ra^{\frac{1}{5}}\, sin^2\emptyset \quad (21)$$

The analytical solution for the boundary layer regime is now complete and represented by equations (19) with p given by equation (21).

The Nusselt number estimated by this study can be found by first evaluating the dimensionless temperature difference of wall-to-wall

$$\Delta T = T_{x=L} - T_{x=0} = 8\left(8192\left(\frac{H}{L}\right) Ra^{\frac{1}{5}} \sin^2\emptyset\right)^{-\frac{1}{9}} \quad (22)$$

The Nusselt number follow

$$Nu = \frac{q''}{k\left(\frac{\Delta T}{H}\right)} = \frac{(8192)^{\frac{1}{9}}}{8}\left(\frac{H}{L}\right)^{\frac{1}{9}} Ra^{\frac{2}{9}} (\sin\emptyset)^{\frac{2}{9}}$$

$$= 0.34\left(\frac{H}{L}\right)^{\frac{1}{9}} Ra^{\frac{2}{9}} (\sin\emptyset)^{\frac{2}{9}} \quad (23)$$

where ΔT is the actual wall-to-wall temperature difference (note that the temperature of both walls increases linearly with y, at the same rate, so that ΔT is independent of y).

For validity of the analytical result, for the inclination angle $\emptyset = 90^{\circ}$, equation (19) and (23) reduces to

$$\lambda_{1,2} = \frac{p}{4}(1 \pm i)$$

$$v = -\frac{32}{p^3} e^{-\frac{p}{4}x} \sin\left(\frac{p}{4}x\right)$$

$$T = -\frac{4}{p} e^{-\frac{p}{4}x} \cos\left(\frac{p}{4}x\right)$$

$$\psi_\infty = \frac{64}{p^4} \quad , \; T_\infty(y) = \frac{y}{\psi_\infty}$$

$$Nu = 0.34\left(\frac{H}{L}\right)^{\frac{1}{9}} Ra^{\frac{2}{9}}$$

which is exactly the same as given by Kimura and Bejan (1984).

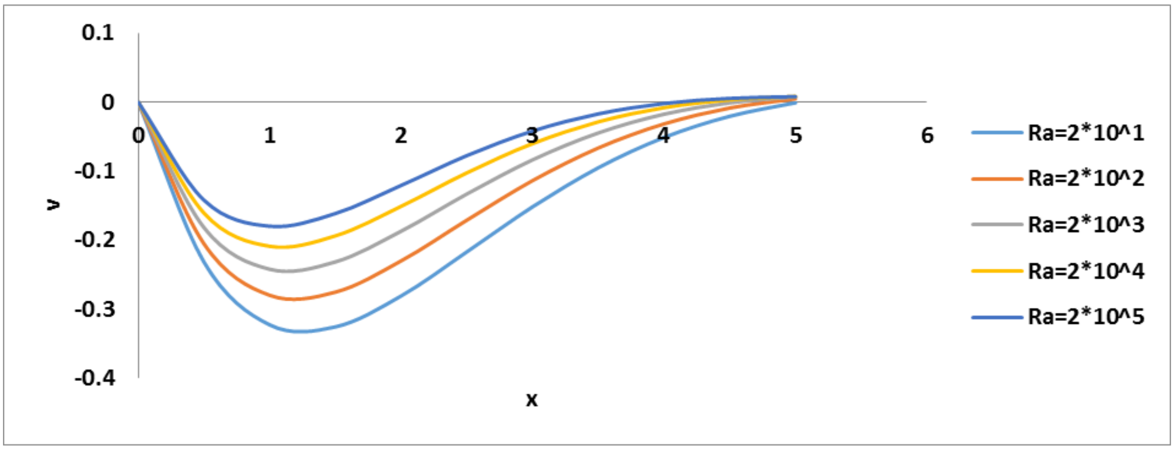

Fig. 2 Velocity profiles for Ø = 30 and for different values of ' Ra '

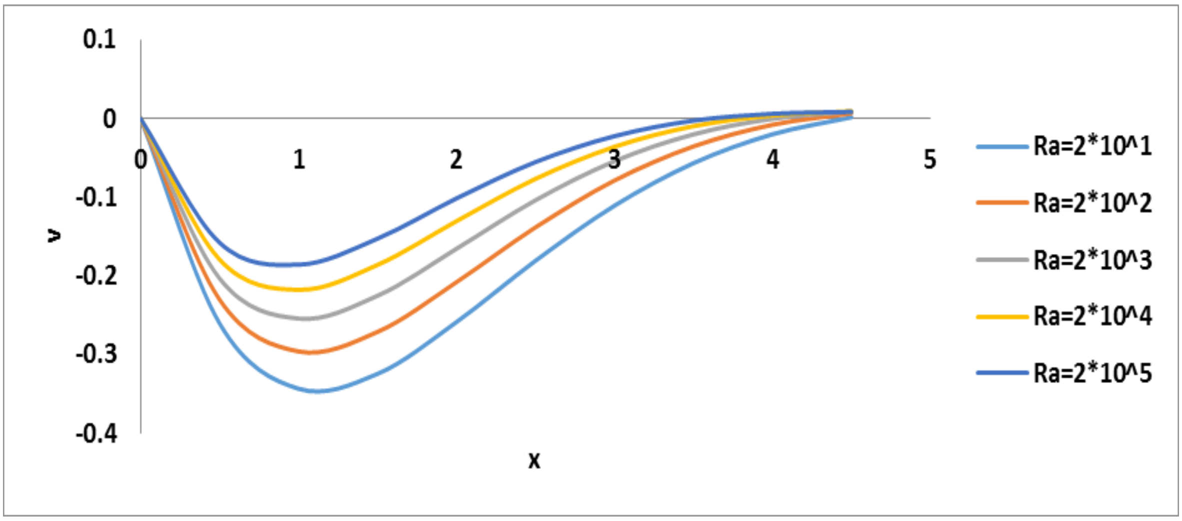

Fig. 3 Velocity profiles for ∅ = 120 and for different values of ' Ra '

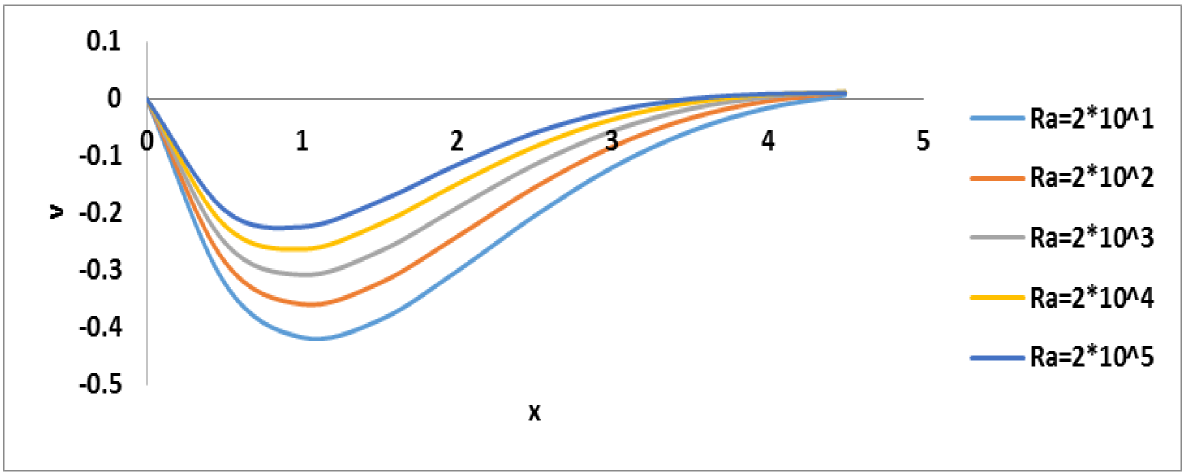

Fig. 4 Velocity profiles for ∅ = 90 and for different values of ' Ra '

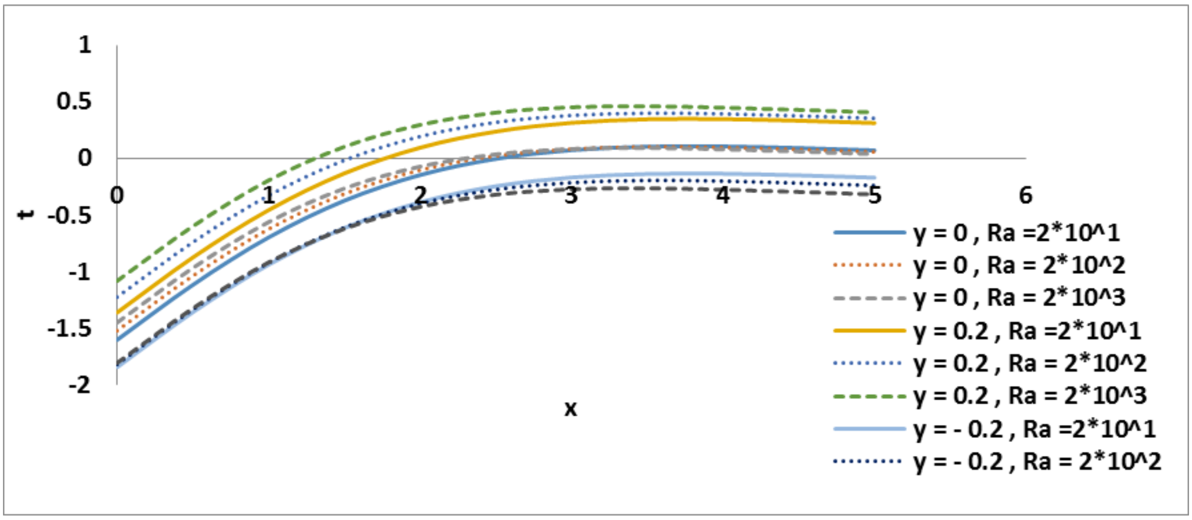

Fig. 5 The Horizontal variation of temperature 't' at the level y =0, ±0.2 and at ∅ = 30

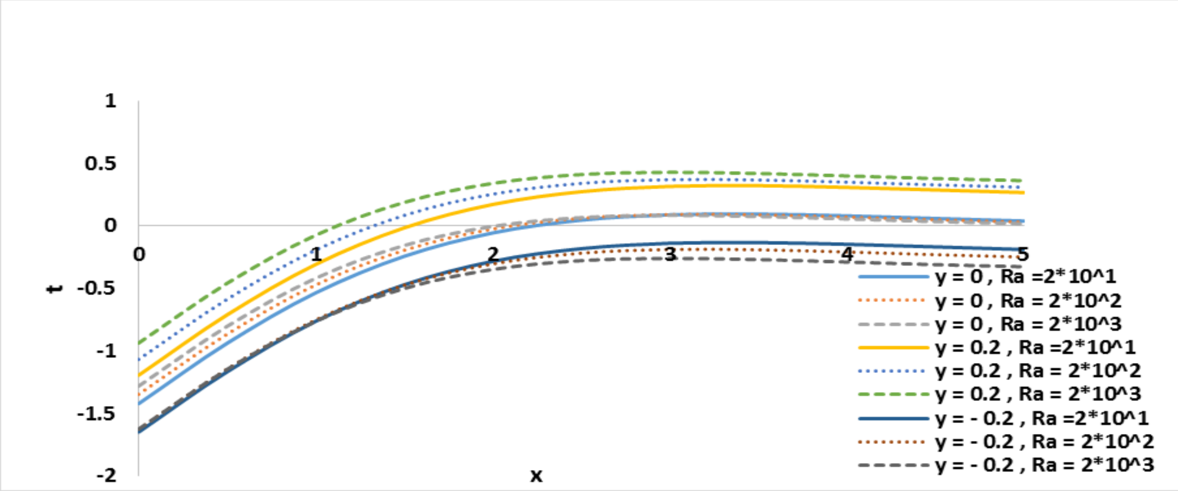

Fig. 6 The Horizontal variation of temperature 't' at the level y =0, ±0.2 and at Ø = 120

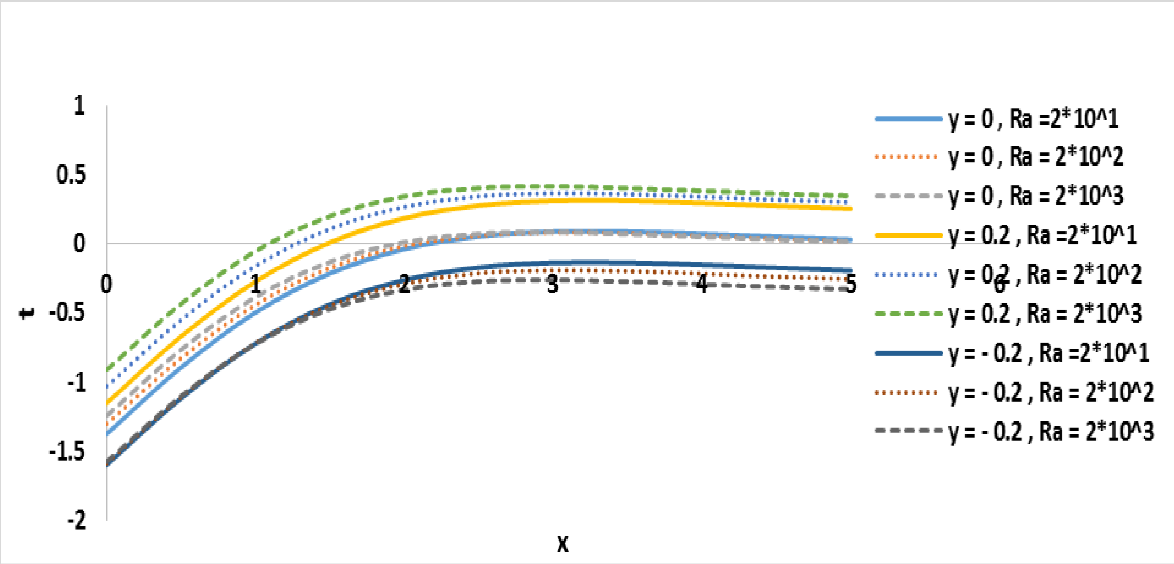

Fig. 7 The Horizontal variation of temperature 't' at the level y =0, ±0.2 and at Ø = 90

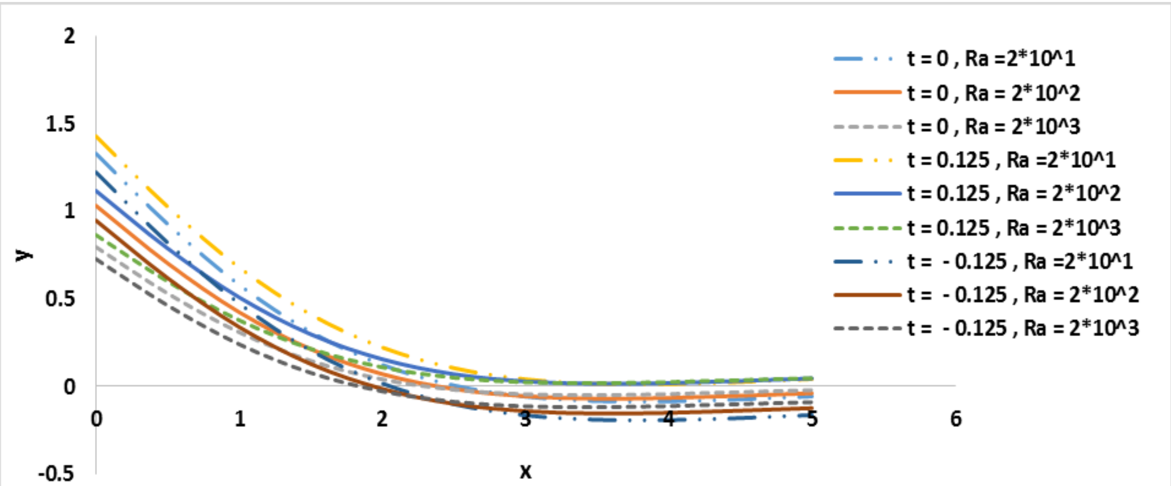

Fig. 8 Isotherms at Ø = 30

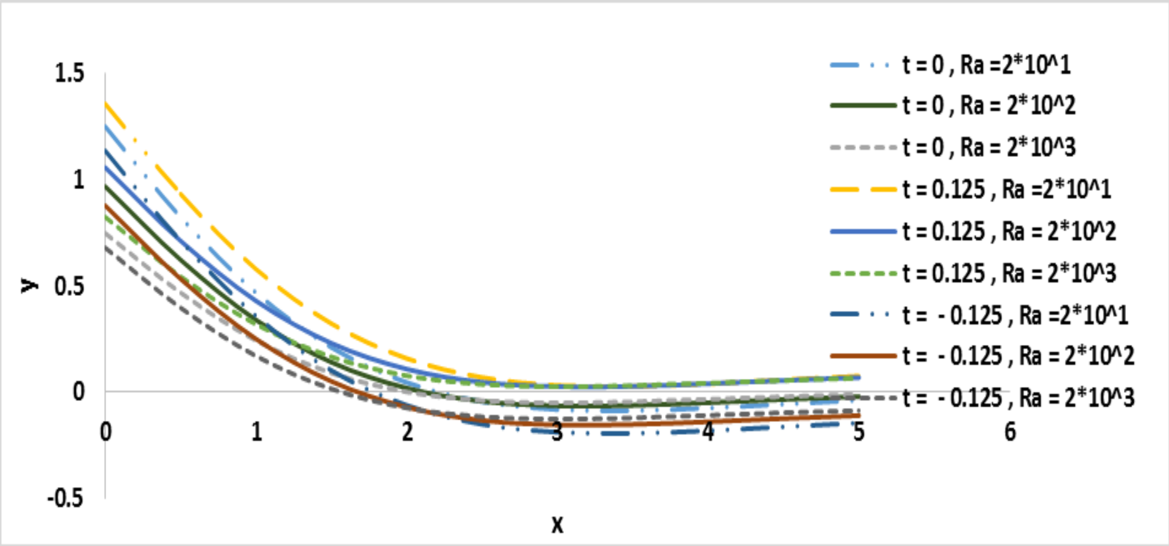

Fig. 9 Isotherms at Ø = 120

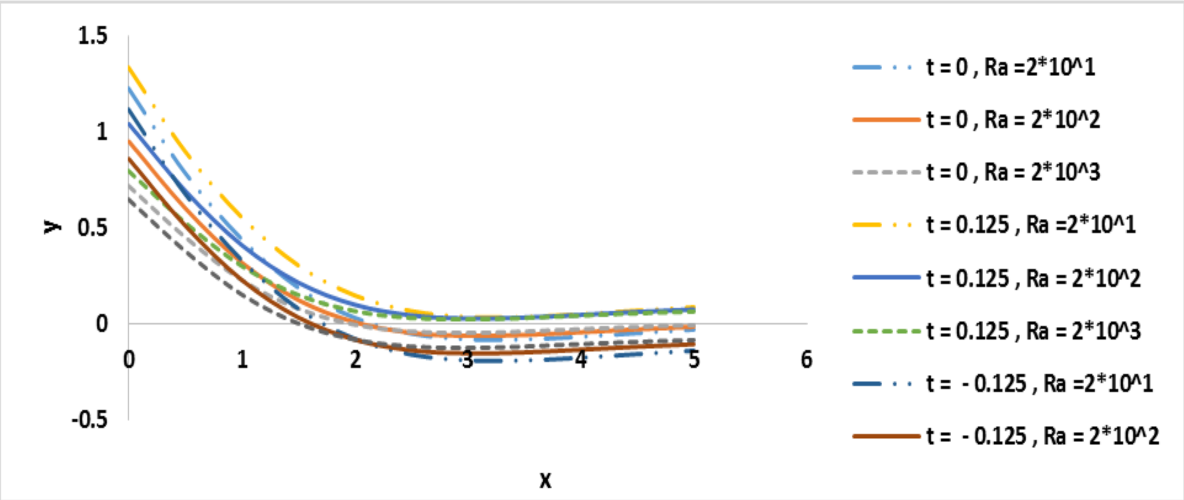

Fig. 10 Isotherms at Ø = 90

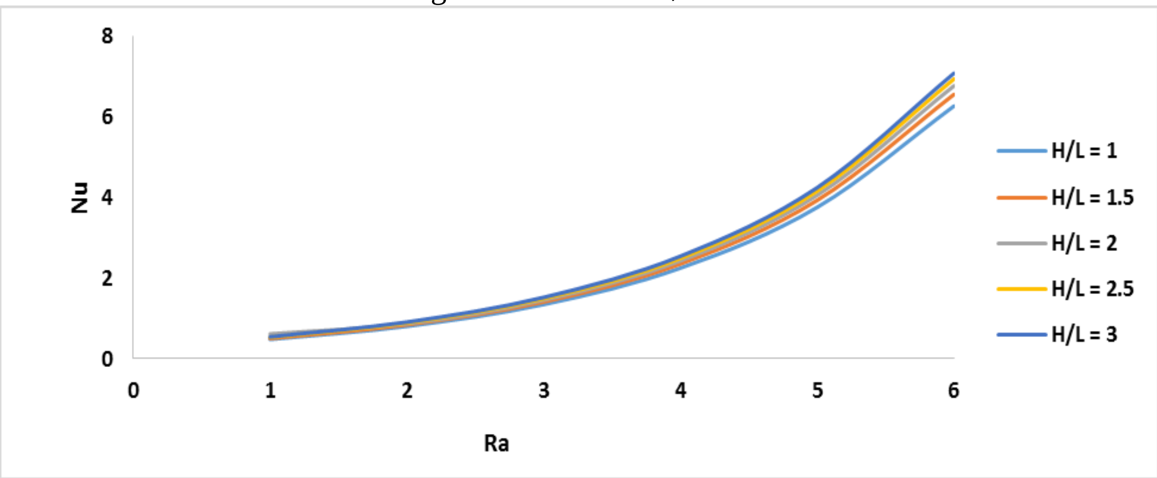

Fig. 11 Nu Vs Ra for different values of ' H/L' when Ø = 30

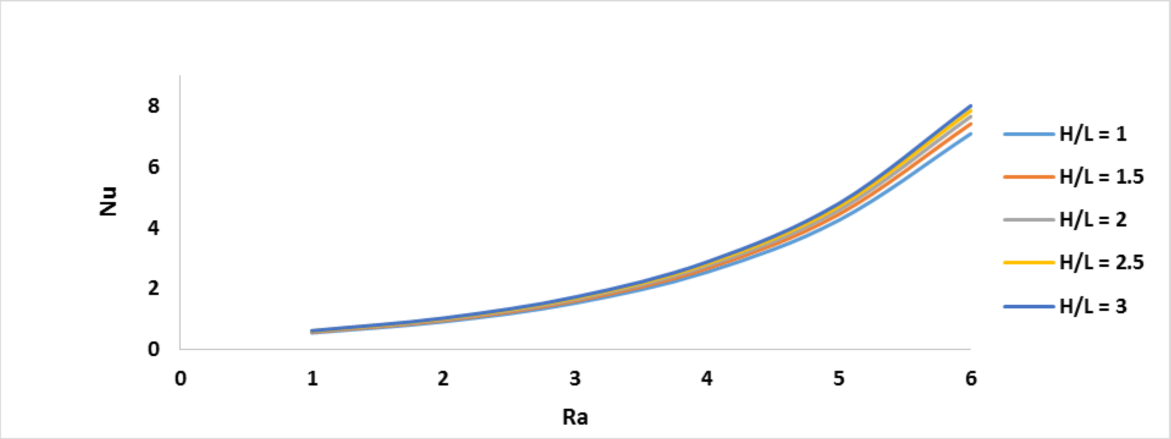

Fig. 12 Nu Vs Ra for different values of ' H/L' when Ø = 120

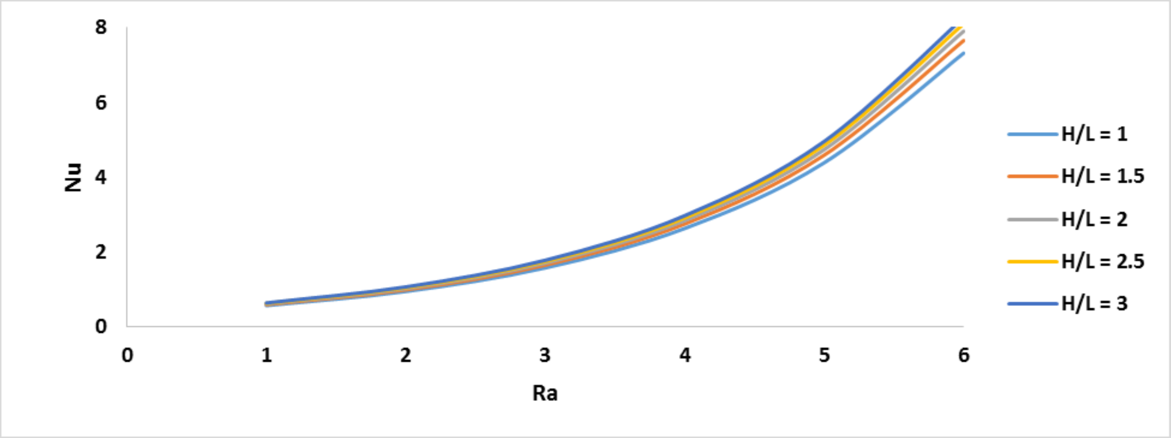

Fig. 13 Nu Vs Ra for different values of ' H/L' when Ø = 90

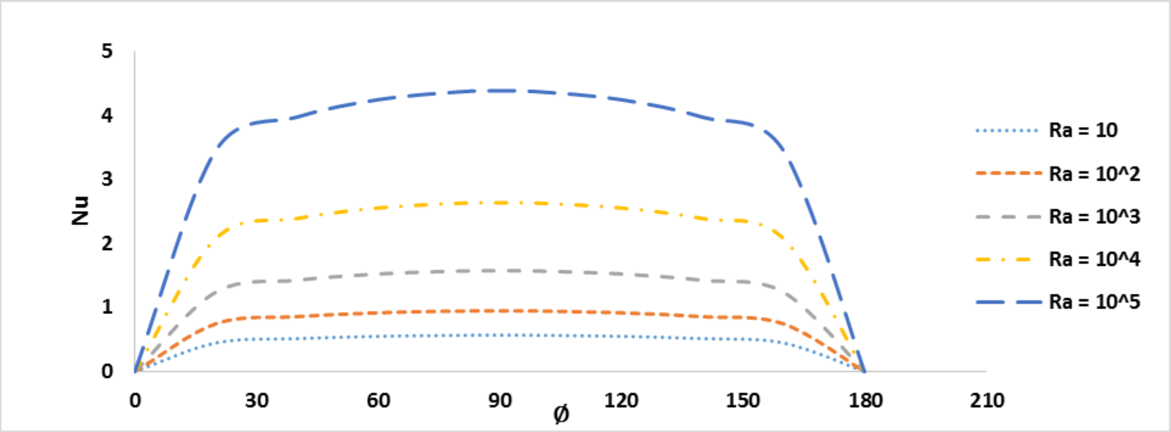

Fig. 14 Nu Vs Ø for different values of ' Ra'

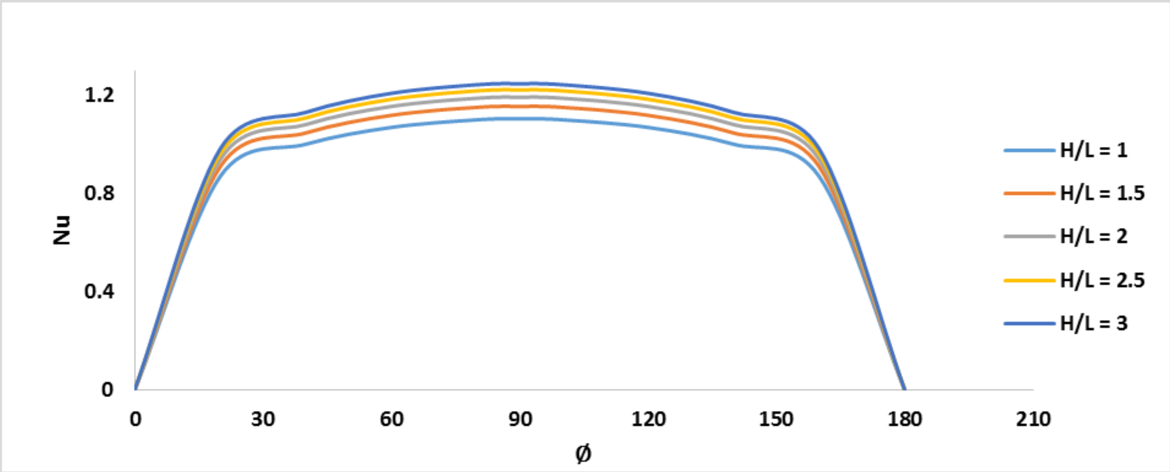

Fig. 15 Nu Vs Ø for different values of ' H/L'

3. Conclusion

This paper described an analytical study of natural convection in an inclined rectangular enclosure where the two side vertical walls are cooling and heating with uniform heat flux and the bottom and top are insulated. In this study taking the aspect ratio H/L as 1, 1.5, 2, 2.5 and 3 and the calculations are carried out for different Rayleigh number and the results are drawn graphically. The heat transfer rate is obtained by calculating the average Nusselt number.

Figure 2 to 4 are plots of dimensionless velocity verse the stretched horizontal coordinate x for various values of Rayleigh number Ra and the inclination angle $\emptyset$. All the velocities are negative since heat is noticed in that left boundary. These figures shows that the velocity falls sharply with raising values of Rayleigh number $(2*10^1 \ to \ 2*10^5)$ and the inclination angle$(\emptyset = 30^{\circ}, 90^{\circ}, 120^{\circ})$. Figure 5 to 7 are plots of non-dimensional temperature profiles versus stretched horizontal coordinates x at various position (y = 0, $\pm 0.2)$ for various values of the inclination angle $\emptyset$ and Rayleigh number. It depicts the exponentially decreasing nature of the temperature. This is characteristic of Oseen linearized based solution

Figure 8 to 10 shows isotherms for $\emptyset = 30^{\circ}, 90^{\circ}, 120^{\circ}$ respectively for different values of Ra. As Ra raises, the thermal field in the interior core shows a linear vertical stratification. The isotherms are parallel to x-axis in the major part of configuration expect in very fine thermal layer of the boundary near the vertical rigid plates. It conclude that, for all angles of inclination, the boundary layer that exists close to the vertical walls which is very favourable to control heat transfer over the enclosure by suppressing convection.

The complete rate of heat transfer over the cavity is got by estimating the Nusselt number Nu along the perpendicular walls. Figure 11 to 15 which gives the difference of mean in the Nusselt number Nu for different values of Rayleigh number Ra, the inclination angle $\emptyset$ and aspect ratio H/L. Where the enclosure is heated from the tip, there is a change in heat transfer which will fall in the range $\emptyset < \frac{\pi}{2}$. As the angle of inclination $\emptyset$ is raised above $\frac{\pi}{2}$, it yields a situation where in the enclosure is heated from the base. The Nusselt number Nu sustain to raise with raising value of $\emptyset$ which passes through a top and then begin to fall down. The result of heating the enclosure from the tip $0 < \emptyset < \frac{\pi}{2}$, the Nusselt number Nu is observe to be huge when compare with that of the heating from base $\frac{\pi}{2} < \emptyset < \pi$. It is also observed that the result of inclination angle on Nusselt number Nu verses Rayleigh number Ra, the Nu increases with an increases in Ra and H/L. The peak of the Nu that takes place at a lesser inclination angle when the Rayleigh number are raised

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