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QUASI-IDEALS AND BI-IDEALS OF A NEAR-ALGEBRA

P. Narasimha Swamy[1], B. Jyothi[2], Rakshita Deshmukh[2] and T. Srinivas[3]

[1]Department of Mathematics, GITAM University, Hyderabad Campus-502 329, Telangana State, INDIA
[2]Department of Mathematics, University College for Women, Osmania University, Hyderabad -500 095, Telangana State, INDIA
[3]Department of Mathematics, Kakatiya University, Warangal-506 009, Telangana State, INDIA

ABSTRACT

The purpose of this paper is to study the notion of a quasi-ideal and bi-ideal of a near-algebra. It also shown that the kernel of a near-algebra homomorphism is also quasi-ideal.

Key words: Near-algebra, Quasi-ideal, Bi-ideal, Kernel of a near-algebra.

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INTRODUCTION

O. Steinfield [9] had introduced the notion of quasi-ideal in ring theory and that has proved very useful. I. Yakabe [11]had introduced the notion of quasi-ideals in near-rings. Lajos and Szasz [4] had introduced the concept of quasi-ideals in associate near-rings. Good and Hughes [2] had introduced the concept of bi-ideals for semi groups. T. Tamizhchelvam and N. Ganesan [5] had introduced the concept of bi-ideals in near-ring. A near-algebra is a near ring which admits a field as a right operator domain. H. Brown [1], T. Srinivas [8], Irish [3], Narasimha Swamy [11] have studied certain properties of near-algebra. In 1933, P. Jordan proposed a quantum mechanical formalism in which in the operators form only a near algebra. In this regard, not only as an axiomatic purpose, the investigation of near-algebras has been found interesting for physical reasons also. In this paper, we introduce the concept of quasi-ideal and bi-ideal in a near-algebras and applied this notion of quasi-ideal to the kernel of homomorphism of near-algebras.

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PRELIMINARIES

Definition: *A right near-algebra* Y over a field X is a linear space Y over X on which multiplication is defined such that

i. Y forms a semi-group under multiplication.
ii. Multiplication is right distributive over addition, that is $(a+b)c = ac+bc$ for every a,b,c $\in$ Y and
iii. $\lambda(ab) = (\lambda a)b$ for every $a,b \in Y$ and $\lambda \in X$.

Definition: *A left near algebra*Y over a field$X$ is a linear space Yover Xon which multiplication is defined such that

i. Y forms a semi-group under multiplication.
ii. Multiplication is left distributive over addition that is $a(b+c) = ab+ac$ for every $a,b,c \in Y$.
iii. $\lambda(ab) = a(\lambda b)$ for every $a,b \in Y$ and $\lambda \in X$.

Everywhere in this paper near-algebra Y means right near-algebra Y.

Definition: A subset M of a near-algebra Y over a field X i s said to be a *sub near-algebra* of Y, if it satisfies the following conditions.

i. M is a linear subspace of Y.
ii. (M, ˙) is a semi group.

Definition: Let I be a non-empty subset of near algebra Y over a field X. I is said to be an *ideal of near-algebra* if

i. I is a linear subspace of the linear space Y.
ii. $ia \in I$ for every $a \in Y, i \in I$ and,
iii. $b(a+i) - ba \in I$ for every $a,b \in Y, i \in I$.

Definition: Let Y and Y' be two near-algebras over a field X. A mapping $f: Y \to Y'$ is called a near-algebra homomorphism if

i. $f(x+y) = f(x) + f(y)$.
ii. $f(xy) = f(x)f(y) \forall x,y \in Y and \lambda \in X$.
iii. $f(\lambda x) = \lambda f(x)$.

Definition: Let Y and Y' be two near-algebras over afield X. Let $f: Y \to Y'$ be a near - algebra homomorphism. Then the *kernel* of f is denoted by kernel f and is defined $Kerf = \{x \in Y / f(x) = 0'\}, 0'$ is the additive identity in Y' .

Definition: Let Y be a near-algebra over a field X. Then the set $Y_0 = \{a \in Y / a0 = 0\}$ is called the zero-symmetric part of Y, $Y_c = \{a \in Y / a0 = a\}$ is called the constant part of Y.Y is called *zero-symmetric near-algebra* if $Y = Y_0$,Y is called constant near-algebra if $Y = Y_c$.

Definition: An element $a \in Y$ is called *distributive element* if $a(b+c) = ab + ac, \forall b, c \in Y$.

Definition: Let Y be a near-algebra. Given two sub sets K and D of Y. We define an operation * on them as $K * D = \{a(a'+b) - aa' \in D / a, a' \in K, b \in D\}$.

QUASI-IDEAL OF A NEAR-ALGEBRA

In this section, we introduce the concept of quasi-ideals for near-algebra and study its elementary properties.

Definition: A non-empty sub set K of a near algebra Y is Quasi-ideal of Y if

i. K is a linear sub space of near-algebra Y.

ii. $KY \bigcap YK \cap Y * K \subseteq K$

Example: Let $X = \{0,1\}_{\oplus_2 \otimes_2}$ is a field. Let $Y = \{0, a, b, c\}$ be a set with two binary operations + and $\cdot$ defined as

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

·	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Scalar multiplication on Y is defined by $0.x = 0, 1.x = x$ for each $x \in X$. Then Y is a near algebra over the field X. Let $K = \{0, b\} \subset Y$. Then it is clear that K is a linear sub space of Y and $KY = \{0\}$, $YK = \{0\}$, $Y * K = \{0\}$. There fore $KY \bigcap YK \bigcap Y * K = \{0\} \subset K$. Hence K is a quasi-ideal of Y.

Theorem1: Arbitrary intersection of quasi-ideals is a quasi-ideal.

Proof: Let $\{K_i\}_{i \in I}$ be a set of quasi-ideals of near algebra Y over a field X. Then it is clear that $\bigcap_{i \in I} K_i$ is a linear sub space of Y. Further for every $K_j = \bigcap_{i \in I} K_i$, we have $A = (\bigcap_{i \in I} K_i)Y \bigcap Y(\bigcap_{i \in I} K_i) \bigcap Y * (\bigcap_{i \in I} K_i) \subseteq K_j Y \bigcap YK_j \bigcap Y * K_j \subseteq K_j$ hence $A \subseteq \bigcap_{i \in I} K_i$. Thus $\bigcap_{i \in I} K_i$ is a quasi-ideal of Y.

Theorem2: The intersection of a quasi-ideal K and a subnear algebra M of a near algebra Y is a quasi-ideal of M.

Proof: Let Y be a near-algebra over a field X. Let K be a quasi-ideal of Y, and M be a sub near-algebra of Y. Since K is a quasi-ideal of Y, it is a linear sub space of Y. Let

$a,b \in K \cap M$ and $\lambda \in X$, then $a,b \in K$ and $a,b \in M$. This implies that $a-b, ab, \lambda a \in K$, $(\lambda a)b = \lambda(ab)$ for all $a,b \in K$, $\lambda \in X$ also $a-b, ab, \lambda a \in M$ and $(\lambda a)b = \lambda(ab)$ $\forall a,b \in M$, $\lambda \in X$. Thus we get $a-b, ab, \lambda a \in K \cap M$, $(\lambda a)b = \lambda(ab)$ for every $a,b \in K \cap M$, $\lambda \in X$. Hence $K \cap M$ is a sub near-algebra of M. More over $(K \cap M)M \cap M(K \cap M) \cap M*(K \cap M) \subseteq (K \cap M)M \cap M(K \cap M) \subseteq MM \subset M$ and $(K \cap M)M \cap M(K \cap M) \cap M*(K \cap M) \subseteq KY \cap YK \cap Y*K \subseteq K$. Hence $K \cap M$ is a Quasi-ideal of M.

Theorem3: Let Y be a near-algebra. Then a sub near-algebra M of Y is a Quasi-ideal of Y if and only if $MY \cap YM \subseteq M$.

Proof: Let $\alpha \in Y$ and $\beta \in M$. We have $\alpha\beta = \alpha(0+\beta) - \alpha 0$. Since Y is zero-symmetric hence $YM \subseteq Y*M$. By this property, we have $MY \cap YM \cap Y*M = MY \cap YM$.

From the above theorem, we can see that $MY \cap YM \cap Y*M = MY \cap YM \subseteq M$

Theorem4: If $f: Y \to Y'$ is a near-algebra homomorphism, then the kernel f is a quasi-ideal of Y.

Prof: Let $f: Y \to Y'$ be near-algebra homomorphism. Then $Kerf = \{x \in Y / f(x) = 0'\}$, $0'$ is the additive identity in Y'.We know that kernel f is a linear subspace of Y. Thus, from theorem3 we have $KY \cap YK \cap Y*K \subseteq KY \cap YK \subseteq K$ this implies that $KY \cap YK \cap Y*K \subseteq K$ .Therefore, K is a quasi -ideal of Y.

Notation: Let Y_1 and Y_2 be two near-algebras over a field X . And let Q_1 and Q_2 be two quasi-ideals of Y_1 and Y_2 respectively. Then

i. $Q_1 \times Q_2 = \{(x_1, y_1) / x_1 \in Q_1, y_1 \in Q_2\}$
ii. $(Q_1 \times Q_2)(Y_1 \times Y_2) = \{(x_1, y_1)(x, y) / (x_1, y_1) \in Q_1 \times Q_2, (x, y) \in Y_1 \times Y_2\}$
   Where $(x_1, y_1)(x, y) = (x_1 x, y_1 y)$
iii. $(Y_1 \times Y_2)*(Q_1 \times Q_2) = \{(x, y)((x', y') + (x_1, y_1)) - (x, y)(x', y') / (x, y), (x', y') \in Y_1 \times Y_2, (x_1, y_1) \in Q_1 \times Q_2\}$

Theorem5: Let Y_1 and Y_2 be two near-algebras over a field X . And let Q_1 and Q_2 be two quasi-ideals of Y_1 and Y_2 respectively. Then the direct product $Q_1 \times Q_2 = \{(x_1, y_1) / x_1 \in Q_1, y_1 \in Q_2\}$ is a quasi-ideal of $Y_1 \times Y_2$. Where $Y_1 \times Y_2$ is a near-algebra over a field X .

Proof: Let Y_1 and Y_2 be two near algebras. We know that $Y_1 \times Y_2$ is a near-algebra over a field X . It is clear that $Q_1 \times Q_2$ is a non-empty subset of $Y_1 \times Y_2$. Now

i. Let $(x_1, y_1), (x_2, y_2) \in Q_1 \times Q_2$ and $\lambda_1, \lambda_2 \in X$ then

$$\lambda_1(x_1, y_1) + \lambda_2(x_2, y_2) = (\lambda_1 x_1, \lambda_1 y_1) + (\lambda_2 x_2, \lambda_2 y_2)$$
$$= (\lambda_1 x_1 + \lambda_2 x_2, \lambda_1 y_1 + \lambda_2 y_2) \in Q_1 \times Q_2$$

Hence $Q_1 \times Q_2$ is a linear sub space of the near-algebra of $Y_1 \times Y_2$.

ii. Let $(x_1, y_1) \in Q_1 \times Q_2$ and $(x, y) \in Y_1 \times Y_2$ then

$$(Q_1 \times Q_2)(Y_1 \times Y_2) = \{(x_1, y_1)(x, y) / x_1 \in Q_1, y_1 \in Q_2, x \in, Y_1, y \in Y_2\}$$
$$= \{(x_1 x, y_1 y) / x_1 x \in Q_1, y_1 y \in Q_2\} \subseteq Q_1 \times Q_2$$
$$(Y_1 \times Y_2)(Q_1 \times Q_2) = \{(x, y)(x_1, y_1) / x_1 \in Q_1, y_1 \in Q_2, x \in, Y_1, y \in Y_2\}$$
$$= \{(xx_1, yy_1) / xx_1 \in Q_1, yy_1 \in Q_2\} \subseteq Q_1 \times Q_2$$
$$(Y_1 \times Y_2) * (Q_1 \times Q_2) = \{(x, y)((x', y') + (x_1, y_1)) - (x, y)(x', y') / (x, y), (x', y') \in Y_1 \times Y_2, (x_1, y_1) \in Q_1 \times Q_2\}$$
$$= \{(x, y)(x' + x\ , y' + y_1) - (xx', yy') / x, x' \in Y_1, y, y' \in Y_2, x_1 \in Q_1, y_1 \in Q_2\}$$
$$= \{(x(x' + x_1), y(y' + y_1)) - (xx', yy') / x, x' \in Y_1, y, y' \in Y_2, x_1 \in Q_1 y_1 \in Q_2\}$$
$$= \{(xx' + xx_1, yy' + yy_1) - (xx', yy') / x, x' \in Y_1, y, y' \in Y_2, x_1 \in Q_1 y_1 \in Q_2\}$$
$$= \{(xx' + xx_1 - xx', yy' + yy_1 - yy') / x, x' \in Y_1, y, y' \in Y_2, x_1 \in Q_1 y_1 \in Q_2\}$$
$$= \{(xx_1, yy_1) / x, x' \in Y_1, y, y' \in Y_2, x_1 \in Q_1 y_1 \in Q_2\} \subseteq Q_1 \times Q_2$$

Therefore $(Q_1 \times Q_2)(Y_1 \times Y_2) \bigcap (Y_1 \times Y_2)(Q_1 \times Q_2) \bigcap (Y_1 \times Y_2) * (Q_1 \times Q_2) \subseteq Q_1 \times Q_2$

Hence $Q_1 \times Q_2$ is a quasi-ideal of $Y_1 \times Y_2$.

BI-IDEALS OF NEAR ALGEBRA

In this section, we introduce the concept of Bi-ideal of a near-algebra. Also obtain some results with this notion.

Definition: A sub set D of a near algebra Y is a *bi-ideal* if

i. D is a linear subspace of Y.

ii. $DYD \bigcap (DY) * D \subseteq D$

Example: Let $X = \{0,1\}_{\oplus_2 \otimes_2}$ is a field. Let $Y = \{0, a, b, c\}$ be a set with two binary operations + and $\cdot$ defined as

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Scalar multiplication on Y is defined by $0.x = 0, 1.x = x$ for each $x \in Y$. Then the routine calculations show that Y is a near algebra over the field X. Let $D = \{0, a\} \subseteq Y = \{0, a, b, c\}$.

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It is clear that $D$is a linear sub space of Y.
$DY = \{0, b\}, DYD = \{0\}, DY * D = \{b(0 + a) - b0/b, 0 \in DY, a \in D\} = \{0\}$.
$DYD \cap (DY) * D = \{0\} \subseteq D$. This implies that$D$ is a bi-ideal of Y. Also
$DY \cap YD \cap Y * D = \{0. b\} \nsubseteq D$ which says that D is not a quasi-ideal.

Theorem6: The intersection of bi-ideals of a near-algebra Yover a field X forms a bi-ideal of Y.

Proof: Let $\{D_i\}_{i\in I}$ be the set of all bi-ideals of a near algebra Y over a field X. Let $D = \bigcap_{i\in I} D_i$. Then $DYD \bigcap (DY)*D \subseteq D_iYD_i \bigcap (D_iY)*D_i \subseteq D_i \forall i \in I$. Therefore $DYD \bigcap (DY)*D \subseteq D$. Hence D is a bi-ideal of Y.

Theorem7: Let Y be a near-algebra over a field X,Dbe a bi-ideal of $Y$and M be a sub near-algebra of Y. Then $D \bigcap M$ is a bi-ideal of M.

Proof: Since D is a bi-ideal of near- algebra Y, then $DYD \bigcap (DY)*D \subseteq D$. Let $H = D \bigcap M$. Then $HMH \bigcap (HM)*H = (D \bigcap M)M(D \bigcap M) \bigcap ((D \bigcap M)M)*(D \bigcap M)$
$\subseteq DMD \bigcap M \bigcap (DM)*D \subseteq D \bigcap M = H$. Hence H is a bi-ideal of M.

Theorem8: Let Y be a near-algebra over a field X.A sub space D of$Y$is a bi-ideal if and only if $DYD \subseteq D$.

Proof: For a subspace D of Y if $DYD \subseteq D$ then $DYD \bigcap (DY)*D \subseteq D$. Which shows that D is a bi-ideal of Y. Conversely suppose$D$ is a bi-ideal of Ythen $DYD \bigcap (DY)*D \subseteq D$. Since $Y$is zero-symmetric $YD \subseteq Y*D$ then $DYD = DYD \bigcap DYD \subseteq DYD \bigcap (DY)*D \subseteq D$. Hence $DYD \subseteq D$.

Theorem9: Let Y be a near-algebra. If D is a bi-ideal of Ythen Da and $a'D$ are bi-ideals of $Y$where $a, a' \in Y$ and a' is distributive element in Y.

Proof: Clearly Da is a linear sub-space of Y and $DaYDa \subseteq DYDa \subseteq Da$. This implies that Da is a bi-ideal of Y. Also $a'D$ is a linear space of Y, a' is distributive in $Y$and $a'DYa'D \subseteq a'DYD \subseteq a'D$. Thus $a'D$ is a bi-ideal of Y.

Corollary: If D is a bi-ideal of a near-algebra $Y$and a is a distributive element in Y, then aDb is a bi-ideal of $Y$where $b \in Y$.

Theorem10: kernel of near-algebra homomorphism from Y *to* $Y'$is a bi-ideal of Y.

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