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# USE OF HOMOTOPY ANALYSIS METHOD FOR SOLVING NONLINEAR CAUCHY PROBLEM

**Prepared**

**by**

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## Abstract :

In this paper, by the structure for the Homotopy Analysis Method (HAM), the frameworks was of some nonlinear. Arithmetical topology is a champion among the most pivotal signs in science which utilizes numerical instruments to take a gander at topological spaces. The HAM is a skilled and a sensible orderly mechanical gathering to deal with nonlinear issues and does not require little parameters in the directing conditions and purpose of repression/beginning conditions.[1]The outcome of this examination presents the utility and ampleness of HAM strategy.The fundamental goal is to find arithmetical invariants that gathering topological spaces up to homeomorphism (however generally bunch up to homotopy likeness). The most essential of these invariants are homotopy, homology, and cohomology get-togethers.[2] This subject is an exchange among topology and polynomial math and concentrates numerical invariants gave by homotopy and homology hypotheses. The twentieth century saw its most basic change.The Homotopy Analysis Method (HAM) is autonomous of little/vast physical parameters. Furthermore, it gives incredible flexibility to pick equation compose and arrangement articulation of related direct high-arrange guess equations. The HAM gives a basic method to ensure the merging of arrangement. Such uniqueness separates the HAM from all other explanatory estimation methods. Likewise, the HAM can be connected to take care of some trying issues with high nonlinearity.[3]

The courses of action of some nonlinear Cauchy issue of illustrative hyperbolic compose are accurately obtained as joined Taylor game plan. The HAM gives an advantageous strategy for merging and controlling of arrangement. Indicative procedure used unwind straight cases to procure the right game plans. The results reveal that the proposed procedure is greatly effective and fundamental.

**Keywords**: Cauchy, Homotopy Analysis Method (HAM), Perturbation Theory (PT), Linear Expansion Method(LDE), variational Perturbation Theory(VPT)

## Introduction

Most problem for debate in science and building are nonlinear. Thus, it is basic to make profitable techniques to comprehend them. In the earlier decades, with the fast headway of brilliant significant figuring programming, for instance, Maple, Mathematical and Matlab, logical and numerical strategies for nonlinear differential conditions have been created quickly.[4] It is a champion among the best techniques to manufacture indicatively evaluated courses of action of nonlinear differential conditions. This strategy has been linked with a broad spectrum of assortment of nonlinear differential conditions.

Lately, significant enthusiasm for fragmentary differential equations has been animated because of their various applications in material science and designing. For example for the engendering of waves through a fractal medium or diffusion in a confused framework it is sensible to plan the structure of the nonlinear development equations as far as partial subsidiaries as opposed to in the traditional shape. Moreover, we realized that numerous nonlinear differential equations show bizarre attractors and their answers have been found to advance toward peculiar attractors . Such odd attractors are fractals by definition. A few cases are utilized to outline the viability of this method. It is demonstrated that the HAM is effective for traditional differential equations as well as for differential equations with partial subordinates.[5]

Perturbation Theory (PT) Augmentation around the unsettling influence parameter is consolidated. Obviously, non perturbative systems have been produced to investigate physical issues which don't a little physical parameter to be utilized as the unsettling influence parameter. The non-parameter enlargement technique, the Optimised Perturbation Theory (OPT), the variational PerturbationTheory(VPT) and the Linear Expansion Method(LDE), are ordinary nonperturbative strategies, and have been made as an outstanding contraptions in quantum field hypothesis and in fluctuates physical settings amidst the prior years.[6]

These systems do avoid bothering strategy in forces of physical parameters, and the social affair of actuated is controlled by some delivered parameters which don't exists in the key issues. The HAM was attainably associated with oversee diverse nonlinear issues, for instance, nonlinear Riccati differential condition with fragmentary demand, nonlinear Vakhnenko condition, the Glauert-fly issue halfway KdV-Burgers-Kuramoto Equation, a summed up Hirota-Satsuma coupled KdV condition, nonlinear warmth trade, to shot headway with the quadratic law, to constrain layer stream of nanofluid past a broadening sheet, to the Poisson-Boltzmann state of semiconductor contraptions, solitary methodology of discrete MKdV condition, to the strategy of Fractional differential conditions, to the Oldroyd 6-driving forward fluid with engaging field, MHD-stream of an Oldroyd 8-steady fluid, to the nonlinear streams with slip confine condition et cetera.

Cauchy nonlinear hyperbolic condition=

$$\left(\frac{\partial}{\partial t}-\Delta\right)\left(\frac{\partial^2}{\partial t^2}-\Delta\right)u=F(u), \quad\ldots\ldots(1,1)$$

with the underlying conditions

$$\frac{\partial^k}{\partial t^k}(X,0)=\phi_k(X), X=(x_1,x_2.........x_i), k=0,1,2$$

$F(u)$ and $\Delta=$ Laplace in $R^n$ where equation is in homotopy perturbation method is the exceptional instance of homotopy examination technique at , acquired by A.Roozi et.al in

## Homotopy analysis method

Keeping in mind the end goal with differential eqn condition

$$N[u(x,t)]=0,$$

Where N - nonlinear administrator, x and t = free factors, u = limit.

By strategies for the initially build up the affirmed zeroth-arrange misshapening condition.[7]

$$(1-q)L\left[\phi(x,t;q)-u_0[x,t]\right]=q\hbar H(x,t)N\left(\phi(x,t'q)\right)..................(2.2)$$

where $q \in [0,1]$ s = installing parameter, $h \neq 0$ =auxiliary parameter, we can conlude that L=Secondary linear operator, $\phi(x,t;q)$ is an unknown capacity, $u_0(x,t)$ = speculation of and means and $H(x,t)$ = nonzero auxiliary function. Clearly when the installing parameter q = 0 and q = 1, condition (2.2) progresses toward becoming

$$\phi(x,t;0)=u_0(x,t),\phi(x,t;1)=u(x,t)$$

separately . Along these lines as q increments from 0 to 1, the arrangement changes from the underlying

figure $u_0(x,t)$ to the arrangement $u(x,t)$ . Growing in $\phi(x,t;q)$ in Taylor series with regard to q , one has

$$\phi(x,t;q)=u_0(x,t)+\sum_{m=1}^{\infty}u_m(x,t)q^m$$

Where

$$u_m(x,t)=\frac{1}{m!}\frac{\partial^m\phi(x,t;q)}{\partial q^m}\mid q=0$$

The convergence of the series (2.3) relies on the auxiliary parameter h. On the off chance convergent at q = 1, one has

$$u(x,t)=u_0(x,t)+\sum_{m=1}^{\infty}u_m(x,t),$$

which must be one of the game plans of the main nonlinear condition, as showed by Liao. Portray the vectors [8]

$$\overrightarrow{u_n} = (u_0(x,t), u_1(x,t), \ldots\ldots, u_n(x,t)).$$

Dierentiate the zeroth-arrange deformation equation (2.1) m-times as for q and after that isolating them by m! lastly setting q = 0, we get the accompanying mth-arrange deformation condition:

$$L\left[u_m(x,t) - \chi_m u_m - 1(x,t)\right] = \hbar \Re_m\left(\overrightarrow{u_m} - 1\right),$$

Where

$$\Re_m\left(\overrightarrow{u_m} - 1\right) = \frac{1}{m!}\frac{\partial^{m-1} N[\phi(x,t;q)]}{\partial q^{m-1}} \mid q = 0,$$

And

$$\chi_m = \begin{cases} 0, m \le 1 \\ 1, m > 1. \end{cases}$$

It should be underlined that um(x; t) for m 1 is spoken to by the straight condition (2.4) with coordinate point of confinement conditions that comes outline the primary issue, which can be handled by the agent computation programming, for instance, Mathematical or Maple. For the joining of the above system we suggest the peruser to Liao. In the occasion that condition (2.1) yields exceptional course of action, by then this procedure will make the novel game plan. In case condition (2.1) does not posses an intriguing extraordinary arrangement, the HAM will give an answer among various other possible arrangements.

**Applications**

Around there the relevance of HAM may be displayed by the going with representations:

3.1 Example 1

Think about the accompanying condition

$$\left(\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)u = -\left(\frac{1}{3}\frac{\partial^2 u}{\partial x^2}\right)^2 + \left(\frac{1}{6}\frac{\partial^2 u}{\partial t^2}\right)^3 - 16u, \qquad (3.1)$$

with the initial conditions

$$u(x,0) = -x^4, \frac{\partial u}{\partial t}(x,0) = 0,$$

And

$$\frac{\partial^2 u}{\partial t^2}(x,0) = 0.$$

To understand condition (3.1) by methods for the homotopy investigation technique let us consider the accompanying linear operator:

$$L[\phi(x,t;q)] = \frac{\partial^3 \phi(x,t;q)}{\partial t^3},$$

with the property that

$$L\left[c_1 + c_2 t + c_3 \frac{t^2}{2}\right],$$

which implies that

$$L^{-1}(.) = \int_0^t \int_0^t \int_0^t (.) dt dt dt,$$

So we can come to the conclusion that the nonlinear operator as

$$N[\phi(x,t;q)] = \frac{\partial^3 \phi(x,t;q)}{\partial t^3} - \frac{\partial^3 \phi(x,t;q)}{\partial t \partial x^2} - \frac{\partial^4 \phi(x,t;q)}{\partial t^2 \partial x^2} + \frac{\partial^4 \phi(x,t;q)}{\partial x^4} + \frac{1}{216}\left(\frac{\partial^2 \phi(x,t;q)}{\partial t^2}\right)^3 - \frac{1}{9}\left(\frac{\partial^2 \phi(x,t;q)}{\partial x^2}\right)^2 + 16\phi(x,t;q).$$

Using the above definition, we build up the zeroth-orchestrate turning condition by

$$(1-q)L[\phi(x,t;q) - u_0(x,t)] = q\hbar \mathrm{H}(x,t)N[\phi(x,t;q)] \qquad (3.2)$$

where q 2 [0; 1] is the embedding parameter, ~ 6= 0 is a helper parameter, L is an assistant straight administrator, (x; t; q) is an obscure capacity, u0(x; t) is an underlying theory of and H(x; t) means a nonzero assistant capacity. Clearly when the inserting parameter q = 0 and q = 1, condition (3.2) advances toward getting to be [9]

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$\phi(x,t;0) = u_0(x,t), \phi(x,t;1) = u(x,t),$

at that point the outcome we acquire the accompanying mth-arrange distortion condition:

$$L\left[u_m(x,t) - \chi_m u_{m-1}(x,t)\right] = \hbar \Re_m\left(\overrightarrow{u_m} - 1\right)$$
$$\Rightarrow$$
$$u_m(x,t) = \chi_m u_m - 1(x,t) + \hbar L^{-1}(H(x,t)\Re_m(\overrightarrow{u_m} - 1)),$$

Where

$\Re_m\left(\overrightarrow{u_m} - 1\right) = \frac{\partial^3 u_{m-1}}{\partial t^3} - \frac{\partial^3 u_{m-1}}{\partial t \partial x^2} - \frac{\partial^4 u_{m-1}}{\partial t^2 \partial x^2} + \frac{\partial^4 u_{m-1}}{\partial x^4} + \frac{1}{9}\sum_{i=0}^{m-1}\frac{\partial^2 u_i}{\partial x^2}\frac{\partial^2 u_{m-1-i}}{\partial x^2} - \frac{1}{216}\sum_{i=0}^{m-1}\sum_{j=0}^{m-1-i}\frac{\partial^2 u_i}{\partial t^2}\frac{\partial^2 u_j}{\partial t^2}\frac{\partial^2 u_{m-j-i-1}}{\partial t^2} + 16u_{m-1},$ light up the above condition under the hidden conditions $u_m(x,0) = 0 \ and \ \frac{\partial u_m}{\partial t}(x,0) = 0.$

For ease let us take u0(x; t) =x4 and

$$u_m(x,t) = \chi_m u_{m-1}(x,t) + \hbar \int_0^t\int_0^t\int_0^t \left(\frac{\partial^3 u_{m-1}}{\partial t^3} - \frac{\partial^3 u_{m-1}}{\partial \xi_1 \partial x^2} - \frac{\partial^4 u_{m-1}}{\partial \xi_1^2 \partial x^2} + \frac{\partial^4 u_{m-1}}{\partial x^4}\right) d\xi_1 \, d\xi_2 \, dt$$
$$+\hbar \int_0^t\int_0^t\int_0^t \frac{1}{9}\sum_{i=0}^{m-1}\frac{\partial^2 u_i}{\partial x^2}\frac{\partial^2 u_{m-1-i}}{\partial x^2} d\xi_1 \, d\xi_2 \, dt$$
$$-\hbar \int_0^t\int_0^t\int_0^t \left(\frac{1}{216}\sum_{i=0}^{m-1}\sum_{j=0}^{m-1-i}\frac{\partial^2 u_i}{\partial t^2}\frac{\partial^2 u_j}{\partial t^2}\frac{\partial^2 u_{m-j-i-1}}{\partial t^2} - 16u_{m-1}\right) d\xi_1 \, d\xi_2 \, dt,$$

alternate segments are given by

$$u_1(x,t) = o$$
$$u_2(x,t) = o$$
$$u_3(x,t) = -\hbar 4t^3$$
$$u_4(x,t) = u_5(x,t) = ...... = 0$$

Therefore the approximate solution is given by at $\hbar = 1$

$$u(x,t) = -x^4 + 4t^3$$

Which is a correct arrangement and is same as got by and is same as got by A.Roozi et.al

## Conclusions

The homotopy examination strategy is used for figuring numerical response for nonlinear Cauchy's issues. Exceptional in connection to all other coherent methodologies, it gives us an essential technique to change and control the consolidating area of game plan by picking honest to goodness estimations of partner parameter , associate limit H(t) and aide straight chairman L. In like manner we exhibited that homotopy aggravation procedure is the uncommon occasion of homotopy examination system. There are some basic concentrations to make here. To begin with, we have unbelievable chance to pick the aide parameter ~ , colleague limit H(t) and aide coordinate executive L and the fundamental guesses. Second the HAM was gave off an impression of being direct, yet capable informative numeric arrangement for dealing with various nonlinear issues. Numerical estimation has been done by Maple 13 programming group.

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