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## TOTALLY FINE SG CONTINUOUS FUNCTIONS AND SLIGHTLY FINE SG CONTINUOUS FUNCTIONS IN FINE TOPOLOGICAL SPACES

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### Abstract

Powar P. L. and Rajak K. have introduced fine-topological space which is a special case of generalized topological space. Aim of this paper is we introduced in fine sg open sets in fine topological space. and also we investigate properties of Totally Fine Sg Continuous Functions and Slightly Fine Sg Continuous Functions in Fine Topological Spaces

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## 1. INTRODUCTION

Powar P. L. and Rajak K. have investigated a special case of generalized topological space called fine topological space[11]. Continuous functions are the most important and most researched points in the whole of the Mathematical Science. Many different forms of continuous functions have been introduced over the years Levine [9] initiated the study about g-closed sets and he generalized the concept of closedness. Following this, in 1987, Bhattacharyya[2] and Lahiri introduced the notion of semi-generalized closed sets in topological spaces by means of semi-open sets of Levine . In continuation of this work, many authors studied and investigated semi-generalized continuous maps and semi- $T_{1/2}$ -spaces. In this paper, Totally Fine Sg Continuous Functions and Slightly Fine Sg Continuous Functions in Fine Topological Spaces

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## 2. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

### Definition: 2.1[10,11]

Let  $(X, \tau)$  be a topological space we define,  $\tau(A_\alpha) = \tau_\alpha = \{G_\alpha (\neq X) : G_\alpha \cap A_\alpha \neq \emptyset, \text{ for } A_\alpha \in \tau \text{ and } A_\alpha \neq X, \emptyset \text{ for some } \alpha \in J, \text{ where } J \text{ is the index set}\}$ . Now, define  $\tau_f = \{\{\emptyset, X\} \cup \tau_\alpha\}$ . The above collection  $\tau_f$  of subsets of  $X$  is called the fine collection of subsets of  $X$  and  $(X, \tau, \tau_f)$  is said to be the fine topological space  $X$  and generated by the topology  $\tau$  on  $X$ .

The element of  $\tau_f$  are called fine open sets in  $(X, \tau, \tau_f)$  and the complement of fine open set is called fine closed sets and it is denoted by  $\tau_f^c$ .

### Example: 2.2 [10,11]

Consider a topological space  $X = \{p, q, r\}$  with the topology

$$\tau = \{X, \emptyset, \{p\}\} \cong \{X, \emptyset, A_\alpha\} \text{ where } A_\alpha = \{p\}. \text{ In view of Definition 2.1 we have,}$$

$$\tau_\alpha = \tau(A_\alpha) = \tau\{p\} = \{\{p\}, \{p, q\}, \{p, r\}\}$$

then the fine collection is  $\tau_f = \{\emptyset, X\} \cup \{\tau_\alpha\} = \{\emptyset, X, \{p\}, \{p, q\}, \{p, r\}\}$ .

We quote some important properties of fine topological spaces.

### Lemma: 2.3[10]

Let  $(X, \tau, \tau_f)$  be a fine space then arbitrary union of fine open set in  $X$  is fine-open in  $X$ .

### Lemma: 2.4 [10,11]

The intersection of two fine-open sets need not be a fine-open set as the following example shows.

### Example: 2.5 [10,11]

Let  $X = \{i, j, k\}$  be a topological space with the topology

$$\tau = \{X, \emptyset, \{i\}, \{j\}, \{i, j\}\}, \tau_f = \{X, \emptyset, \{i\}, \{j\}, \{i, j\}, \{j, k\}, \{i, k\}\}.$$

It is easy to see that, the above collection  $\tau_f$  is not a topology. Since,  $\{i, k\} \cap \{j, k\} = \{k\} \notin \tau_f$ . Hence, the

collection of fine open sets in a fine space  $X$  does not form a topology on  $X$ , but it is a generalized topology on  $X$ .

### Remark: 2.6 [10,11]

In view of Definition 2.1 of generalized topological space and above Lemmas 2.1 and 2.2 it is apparent that  $(X, \tau, \tau_f)$  is a special case of generalized topological space. It may be noted specifically that the topological space plays a key role while defining the fine space as it is based on the topology of  $X$  but there is no topology in the back of generalized topological space.

### Definition: 2.7[10,11]

A subset  $A$  of a Fine space  $(X, \tau, \tau_f)$  is called

- (i) Fine semi open if  $A \subset \text{Fcl}(\text{Fint}(A))$ .
- (ii) Fine regular open if  $A = \text{Fint}(\text{Fcl}(A))$ .

The complement of above Fine semi open, Fine regular open sets are called Fine closed.

Fine semi closed ,Fine regular closed sets

The Fine semi-closure of a subset  $A$  of Fine space  $X$ , denoted by  $Fscl(A)$ , is defined to be the intersection of all Fine semi-closed sets containing  $A$  in Fine space  $X$ .

**Definition:2.8**

A subset  $A$  of a Fine space  $(X, \tau, \tau_f)$  is called Fsg-closed if  $Fscl(A) \subset X$  whenever  $A \subset X$  and  $X$  is Fine semi-open in Fine space  $(X, \tau, \tau_f)$

The complement of Fsg-closed set is called Fsg-open.

**Definition:2.9**

The union of all Fsg-open sets, each contained in a set  $A$  in a Fine space  $(X, \tau, \tau_f)$  is called the Fsg-interior of  $A$  and is denoted by  $Fsg-int(A)$ .

**Definition :2.10**

The intersection of all Fsg-closed sets containing a set  $A$  in a space  $(X, \tau, \tau_f)$  is called the Fsg-closure of  $A$  and is denoted by  $Fsg-cl(A)$ .

**Definition:2.11**

A subset  $A$  of a Fine space  $(X, \tau, \tau_f)$  is said to be

- (i) Fine semi-open[10] if  $A \subseteq Fcl(Fint(A))$ .
- (ii) Fine regular open[10] if  $A = Fint(Fcl(A))$ .

The complement of the above mentioned Fine open sets are called their respective Fine closed sets.

The intersection of all Fine semi-closed sets containing  $A$  is called the Fine semi-closure of  $A$  and is denoted by  $Fscl(A)$ .

**Definition:2.12**

Let  $A$  be a subset of Fine space  $(X, \tau, \tau_f)$ .,Then  $A$  is called F-sg-closed if  $Fscl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is a Fine semi-open set.The complement of a F-sg-closed set is called an F-sg-open set.

The intersection of all F-sg-closed sets containing a set  $A$  is called the semi-generalized closure of  $A$  and is denoted by  $F-sgcl(A)$ .

**Definition: 2.13**

A Fine topological space  $(X, \tau, \tau_f)$  is said to be connected if  $(X, \tau, \tau_f)$  cannot be expressed as

the union two disjoint non empty Fine open sets in  $X$ .

**Definition: 2.14**

A Fine topological space  $(X, \tau, \tau_f)$  is said to be Fine-sg connected if  $(X, \tau, \tau_f)$  cannot be expressed as a disjoint union of two non empty Fine-sg open sets

**Definition: 2.15**

A space  $(X, \tau, \tau_f)$  is called a locally indiscrete space if every Fine open set of  $(X, \tau, \tau_f)$  is Fine closed in  $(X, \tau, \tau_f)$ .

**Definition: 2.16**

Every Fine open set is Fine-sg open and every Fine closed set is Fine-sg closed.

### 3. TOTALLY FINE-SG-CONTINUOUS FUNCTIONS

In this section, we introduce a continuous functions is called totally Fine-sg-continuous functions

and investigate some of the basic properties .

**Definition: 3.1**

A function  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  is called totally Fine continuous if the inverse image of every Fine open subset of  $(Y, \sigma, \sigma_f)$  is a Fine clopen subset of  $(X, \tau, \tau_f)$ .

**Definition: 3.2**

A function  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  is said to be totally Fine-sg-continuous, if the inverse image of every Fine open subset of  $(Y, \sigma, \sigma_f)$  is a Fine-sg clopen subset of  $(X, \tau, \tau_f)$ .

**Example :3.3**

Let  $X = Y = \{a, b, c\}$  and  $\tau = \{ \emptyset, X, \{a\} \}$ ,  $\tau_f = \{ \emptyset, X, \{a\}, \{a, b\}, \{a, c\} \}$

Fine-sg open set =  $\{ \emptyset, X, \{c\}, \{b, c\}, \{a, c\} \}$

Fine-sg closed set =  $\{ \emptyset, X, \{a\}, \{b\}, \{a, b\} \}$

$\sigma = \{ \emptyset, Y, \{a\}, \{a, b\}, \{a, c\} \}$ ,  $\sigma_f = \{ \emptyset, Y, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\} \}$

Then the identity function  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$

Thus the inverse image of every Fine open set in Fine space Y is Fine sg-clopen in Fine space X.

**Theorem: 3.4**

Every totally Fine continuous functions is totally Fine-sg continuous.

**Proof:**

Let N be an Fine open set of  $(Y, \sigma, \sigma_f)$  .Since f is totally Fine continuous functions,  $f^{-1}(N)$  is both Fine open and Fine closed in  $(X, \tau, \tau_f)$  Since every Fine open set is Fine-sg open and every Fine closed set is Fine-sg closed.  $f^{-1}(N)$  is both Fine-sg open and Fine-sg closed in  $(X, \tau, \tau_f)$ . Therefore f is totally Fine-sg continuous.

**Remark: 3.5**

The converse of the above theorem need not be true.

**Example:3.6**

Let  $X = Y = \{a, b, c\}$

$\tau = \{ \emptyset, X, \{d\} \}$ ,  $\tau_f = \{ \emptyset, X, \{b\}, \{a, b\}, \{b, c\} \}$ .

Fine-sg open set =  $\{ \emptyset, X, \{b\}, \{a, b\}, \{b, c\} \}$

Fine-sg closed set =  $\{ \emptyset, X, \{a\}, \{c\}, \{a, c\} \}$

$\sigma = \{ \emptyset, Y, \{a, c\} \}$ ,  $\sigma_f = \{ \emptyset, Y, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\} \}$

Then the identity function  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  is totally Fine-sg continuous but not totally Fine continuous function

**Theorem 3.7**

If f is a totally Fine-sg-continuous function from a Fine-sg-connected Fine space X onto any Fine space Y, then Y is an indiscrete space.

**Proof**

Suppose that Fine space Y is not indiscrete. Let A be a proper non-empty Fine open subset of Fine space Y. Then  $f^{-1}(A)$  is a proper non-empty Fine-sg-clopen subset of  $(X, \tau, \tau_f)$  which is a contradiction to the fact that X is Fine-sg-connected.

**Definition 3.8**

A Fine space X is said to be Fine-sg- $T_2$  if for any pair of distinct points x, y of Fine space X, there exist disjoint Fine-sg-open sets M and N such that  $x \in M$  and  $y \in N$ .

**Lemma 3.9**

The Fine-sg closure of every Fine-sg-open set is Fine-sg-open.

**Proof**

Every Fine regular open set is Fine open and every Fine open set is Fine-sg-open. Thus, every Fine regular closed set is Fine - sgclosed. Now let  $A$  be any Fine-sg-open set. There exists an Fine open set  $M$  such that  $M \subset A \subset \text{cl}(M)$ . Hence, we have  $M \subset \text{Fine-sg-cl}(M) \subset \text{Fine-sg-cl}(A) \subset \text{Fine-sg-cl}(\text{cl}(M)) = \text{cl}(M)$  since  $\text{cl}(M)$  is Fine regular closed. Therefore,  $\text{Fine-sg-cl}(A)$  is Fine-sg-open.

**Theorem 3.10**

A space  $(X, \tau, \tau_f)$  is Fine-sg- $T_2$  if and only if for any pair of distinct points  $x, y$  of  $X$  there exist Fine-sg-open sets  $M$  and  $N$  such that  $x \in M$ , and  $y \in N$  and  $\text{Fine-sgcl}(M) \cap \text{Fine-sgcl}(N) = \phi$ .

**Proof**

Necessity. Suppose that Fine space  $X$  is Fine-sg- $T_2$ . Let  $x$  and  $y$  be distinct points of Fine space  $X$ . There exist Fine-sg-open sets Fine space  $X$  and Fine space  $Y$  such that  $x \in M$ ,  $y \in N$  and  $M \cap N = \phi$ . Hence  $\text{Fine-sgcl}(M) \cap \text{Fine-sgcl}(N) = \phi$  and by Lemma 3.9,  $\text{Fine-sgcl}(M)$  is Fine-sg-open. Therefore, we obtain  $\text{Fine-sgcl}(X) \cap \text{Fine-sgcl}(N) = \phi$ . Sufficiency. This is obvious.

**Theorem 3.11**

If  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  is a totally Fine-sg continuous injection and  $Y$  is  $T_0$  then Fine space  $X$  is Fine-sg- $T_2$ .

**Proof**

Let  $x$  and  $y$  be any pair of distinct points of Fine space  $X$ . Then  $f(x) \neq f(y)$ . Since Fine space  $Y$  is  $T_0$ , there exists an open set  $M$  containing say,  $f(x)$  but not  $f(y)$ . Then  $x \in f^{-1}(M)$  and  $y \notin f^{-1}(M)$ . Since  $f$  is totally Fine-sg-continuous,  $f^{-1}(M)$  is a Fine-sg-clopen subset of  $X$ . Also,  $x \in f^{-1}(M)$  and  $y \in X - f^{-1}(M)$ . By Theorem 3.10, it follows that  $X$  is Fine-sg- $T_2$ .

**Theorem 3.12**

Let  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  be a totally Fine-sg-continuous function and  $Y$  be a  $T_1$ -space. If  $A$  is a non-empty Fine-sg-connected subset of  $X$ , then  $f(A)$  is a single point.

**Definition 3.13**

Let  $(X, \tau, \tau_f)$  be a Fine topological space. Then the set of all points  $y$  in Fine space  $X$  such that  $x$  and  $y$  cannot be separated by a Fine-sg-separation of Fine space  $X$  is said to be the quasi Fine-sg-component of Fine space  $X$ .

**Theorem 3.14**

Let  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  be a totally Fine-sg-continuous function from a Fine topological space  $(X, \tau, \tau_f)$  into a  $T_1$ -space  $Y$ . Then  $f$  is constant on each quasi Fine-sg-component of Fine space  $X$ .

**Proof**

Let  $x$  and  $y$  be two points of  $X$  that lie in the same quasi-Fine-sg-component of Fine space  $X$ . Assume that  $f(x) = \alpha \neq \beta = f(y)$ . Since Fine space  $Y$  is  $T_1$ ,  $\{\alpha\}$  is Fine closed

in Fine space  $Y$  and so  $Y - \{\alpha\}$  is an Fine open set. Since  $f$  is totally Fine-sg-continuous, therefore  $f^{-1}(\{\alpha\})$  and  $f^{-1}(Y - \{\alpha\})$  are disjoint Fine-sg-clopen subsets of Fine space  $X$ . Further,  $x \in f^{-1}(\{\alpha\})$  and  $y \in f^{-1}(Y - \{\alpha\})$ , which is a contradiction in view of the fact that  $y$  belongs to the quasi Fine-sg component of  $x$  and hence  $y$  must belong to every Fine-sg-open set containing  $x$ .

#### 4. SLIGHTLY FINE-sg CONTINUOUS FUNCTIONS

In this section, we introduce a continuous functions is called slightly Fine-sg continuous functions

and investigate some of the basic properties .

**Definition: 4.1**

A function  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  is called slightly Fine continuous if the inverse image of every Fine clopen set in  $(Y, \sigma, \sigma_f)$  is Fine open in  $(X, \tau, \tau_f)$

**Definition: 4.2**

A function  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  is said to be slightly Fine-sg continuous if the inverse image of every Fine clopen set in  $(Y, \sigma, \sigma_f)$  is Fine-sg-open in  $(X, \tau, \tau_f)$

**Example: 4.3**

Let  $X = Y = \{a, b, c\}$

$\tau = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}$ ,  $\tau_f = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\} \}$ .

Fine-sg open set =  $\{ X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\} \}$

$\sigma = \{ Y, \phi, \{c\} \}$ ,  $\sigma_f = \{ Y, \phi, \{c\}, \{a, c\}, \{b, c\} \}$

Then the identity function  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  slightly Fine-sg continuous

**Theorem: 4.4**

For a function  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  the following statements are equivalent.

- (i)  $f$  is slightly Fine-sg-continuous.
- (ii) The inverse image of every Fine clopen set  $N$  of  $Y$  is Fine-sg- open in Fine space  $X$ .
- (iii) The inverse image of every Fine clopen set  $N$  of  $Y$  is Fine-sg- closed in Fine space  $X$ .
- (iv) The inverse image of every Fine clopen set  $N$  of  $Y$  is Fine-sg- clopen in Fine space  $X$ .

**Proof:**

(i) $\Rightarrow$ (ii): Follows from the Definition 4.4.

(ii) $\Rightarrow$ (iii): Let  $N$  be a Fine clopen set in Fine space  $Y$  which implies  $N^c$  is Fine clopen in Fine space  $Y$ .

By (ii),  $f^{-1}(N^c) = (f^{-1}(N))^c$  is Fine-sg-open in Fine space  $X$ . Therefore,  $f^{-1}(N)$  is Fine-sg-closed in

Fine space  $X$ .

(iii) $\Rightarrow$ (iv): By (ii) and (iii),  $f^{-1}(N)$  is Fine-sg-clopen in Fine space  $X$ .

(iv) $\Rightarrow$ (i): Let  $N$  be a Fine clopen set in Fine space  $Y$  containing  $f^{-1}(x)$ , by (iv),  $f^{-1}(N)$  is Fine-sg clopen

in Fine space  $X$  Take  $U = f^{-1}(N)$ , then  $f(U) \subseteq N$ . Hence,  $f$  is slightly Fine-sg-continuous.

**Theorem: 4.5**

Every slightly continuous function is slightly Fine-sg-continuous.

**Proof:**

Let  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  be a Fine-sg continuous function. Let  $N$  be a Fine clopen set in Fine space  $Y$ . Then,  $f^{-1}(N)$  is Fine-sg-open and Fine-sg-closed in Fine space  $X$ . Hence,  $f$  is slightly Fine-sg-continuous.

**Theorem: 4.6**

Every contra Fine-sg continuous function is slightly Fine-sg continuous.

**Proof:**

Let  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  be a contra Fine-sg continuous function. Let  $N$  be a Fine clopen set in Fine space  $Y$ . Then,  $f^{-1}(N)$  is Fine-sg- open in Fine space  $X$ . Hence,  $f$  is slightly Fine-sg continuous.

**Theorem: 4.7**

If the function  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  is slightly Fine-sg continuous and  $(X, \tau, \tau_f)$  is Fine-sg $T_{1/2}$  space, then  $f$  is slightly continuous.

**Proof:**

Let  $N$  be a Fine clopen set in  $Z$ . Since  $g$  is slightly Fine-sg- continuous,  $f^{-1}(N)$  is Fine-sg open in Fine space  $X$ . Since  $X$  is Fine-sg $T_{1/2}$  space,  $f^{-1}(N)$  is Fine open in  $X$ . Hence  $f$  is slightly continuous.

**Theorem: 4.8**

Let  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  and  $g : (Y, \sigma, \sigma_f) \rightarrow (Z, \rho, \rho_f)$  be functions. If  $f$  is surjective and pre Fine-sg-open and  $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$  is slightly Fine-sg continuous, then  $f$  is slightly Fine-sg-continuous.

**Proof:**

Let  $N$  be a Fine clopen set in  $(Z, \rho, \rho_f)$ . Since  $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$  is slightly Fine-sg -continuous,  $f^{-1}(f^{-1}(N))$  is Fine-sg-open in Fine space  $X$ . Since,  $f$  is surjective and pre Fine-sg -open  $(f^{-1}(f^{-1}(N)))=f^{-1}(N)$  is Fine-sg -open. Hence  $f$  is slightly Fine-sg-continuous.

**Theorem: 4.9**

Let  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  and  $g : (Y, \sigma, \sigma_f) \rightarrow (Z, \rho, \rho_f)$  be functions. If  $f$  is surjective and Fine pre -sg-open and Fine-sg -irresolute, then  $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$  is slightly Fine-sg continuous if and only if  $f$  is slightly Fine-sg continuous.

**Proof:**

Let  $N$  be a Fine clopen set in  $(Z, \rho, \rho_f)$ . Since  $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$  is slightly Fine-sg continuous,  $f^{-1}(f^{-1}(N))$  is Fine-sg-open in Fine space  $X$ . Since,  $f$  is surjective and Fine-pre sg open  $(f^{-1}(f^{-1}(N)))=f^{-1}(N)$  is Fine-sg open in Fine space  $Y$ . Hence  $f$  is slightly Fine-sg-continuous.

Conversely, let  $f$  is slightly Fine-sg continuous. Let  $N$  be a Fine clopen set in  $(Z, \rho, \rho_f)$ , then  $f$  is Fine-sg open in Fine space  $Y$ . Since,  $f$  is Fine-sg irresolute,  $f^{-1}(f^{-1}(N))$  is Fine-sg-open in Fine space  $X$ .  $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$  is slightly Fine sg-continuous.

**Theorem: 4.10**

If  $f$  is slightly Fine-sg continuous from a Fine-sg- connected space  $(X, \tau, \tau_f)$  onto a Fine space  $(Y, \sigma, \sigma_f)$  then Fine space  $Y$  is not a discrete space.

**Proof:**

Suppose that Fine space  $Y$  is a discrete space. Let  $N$  be a proper non empty Fine open subset of Fine space  $Y$ . Since,  $f$  is slightly Fine-sg - continuous,  $f^{-1}(N)$  is a proper non empty Fine-sg clopen subset of Fine space  $X$  which is a contradiction to the fact that Fine space  $X$  is Fine-sg-connected.

**Theorem: 4.11**

If  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  is a slightly Fine-sg continuous surjection and Fine space  $X$  is Fine-sg connected, then Fine space  $Y$  is connected.

**Proof:**

Suppose Fine space  $Y$  is not connected, then there exists non empty disjoint Fine open sets  $M$  and  $N$  such that  $Y = M \cup N$ . Therefore,  $M$  and  $N$  are Fine clopen sets in Fine space  $Y$ . Since,  $f$  is slightly Fine-sg continuous,  $f^{-1}(M)$  and  $f^{-1}(N)$  are non empty disjoint Fine-sg open in Fine space  $X$  and  $X = f^{-1}(M) \cup f^{-1}(N)$ . This shows that Fine space  $X$  is not Fine-sg connected. This is a contradiction and hence, Fine space  $Y$  is connected.

**Theorem: 4.12**

If  $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$  is a slightly Fine-sg - continuous and  $(Y, \sigma, \sigma_f)$  is locally indiscrete space then  $f$  is Fine-sg-continuous.

**Proof:**

Let  $N$  be an Fine-open subset of Fine space  $Y$ . Since,  $(Y, \sigma, \sigma_f)$  is a locally indiscrete space,  $N$  is Fine-closed in Fine space  $Y$ . Since,  $f$  is slightly Fine-sg continuous,  $f^{-1}(N)$  is Fine-sg -open in Fine space  $X$ . Hence,  $f$  is Fine-sg continuous.

**4. Conclusion**

Many different forms of open functions and closed functions have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, In this paper, Totally Fine Sg Continuous Functions and Slightly Fine Sg Continuous Functions in Fine Topological Spaces and investigate some of the basic properties. This shall be extended in the future Research with some applications

**REFERENCES**

- [1] D.Andrijevic, "Semi – pre open sets", Mat.Vesnik, 38(1) (1986), 24 – 32.
- [2] P. Bhattacharyya, and B. K. Lahiri, (1987), Semi Generalized Closed Sets in Topology, Indian J. Pure Appl.Math., Vol. 29, Pp. 375-382.
- [3] S.Chandrasekar,M.Suresh,T.Rajesh kannan Nano sg-interior and Nano sg-closure in Nano topological spaces Int.J.Mathematical Archive, 8(4), 2017, 94-100.
- [4] S.Chandrasekar, A Atkinswestly, M ,Sathyabama, g#b-closed sets in Topological paces. International Journal of Advanced Research (IJAR) 5(7) , 2722-2730
- [5] S Chandrasekar, M Sathyabama, A Atkinswestley g# b-Interior and g #b-Closure in Topological Spaces International Journal of Mathematics and its applications,552(B)7 (2017)
- [6] S.Chandrasekar, M.Suresh and T.Rajesh Kannan,On Quasi Soft sg-open and Quasi Soft sg-closed Functions in Soft Topological Spaces International Journal of Mathematics And its Applications (239-246) , Volume 5, Issue 2–B (2017), 239–246.



- [7] S.Chandrasekar, T Rajesh Kannan, M Suresh,, $\delta\omega\alpha$ -Closed Sets in Topological Spaces, Global Journal of Mathematical Sciences: Theory and Practical.9 (2), 103-116(2017)
- [8] S.Chandrasekar,T Rajesh Kannan,R.Selvaraj  $\delta\omega\alpha$  closed functions and  $\delta\omega\alpha$  Closed Functions in Topological Spaces, International Journal of Mathematical Archive 8(11)2017
- [9] N.Levine, (1970), Generalized closed sets in Topological Spaces, Rend. Circ. Mat. Palermo, Vol 19, Pp.89-96.
- [10] P.L. Powar and Pratibha Dubey A Concise form of Continuity in Fine Topological Space Advances in Computational Sciences and Technology Vol 10, N. 6 (2017), pp. 1785–1805,
- [11] P.L. Powar and K. Rajak, Fine irresolute mapping, Journal of Advanced Studied in Topology, 3(4) (2012): 125–139.
- [12] O.Ravi, S. Ganesan and S. Chandrasekar On Totally sg-Continuity, Strongly sg-Continuity and Contra sg-Continuity Genral Mathematical notes.Notes, Vol.7,No 1,Nov (2011), pp. 13-24
- [13] O.Ravi, S. Ganesan and S. Chandrasekar ,Almost  $\alpha$ gs-closed functions and separation axioms. Bulletin of Mathematical Analysis and Applications, Vol.3 (2011),165-177.
- [14] O. Ravi, G.Ramkumar and S. Chandrasekar On Quasi sg-open and Quasi sg-Closed FunctionsAdvances in Applied Mathematical Analysis.Vol 4 No1 (2009), pp. 73-78
- [15] O.Ravi, S.Chandrasekar, S.Ganesan( $\tilde{g},s$ ) continuous functions between topological spaces. Kyungpook Mathematical Journal, 51 (2011), 323-338.
- [16] O.Ravi ,S.Chandrasekar, S.Ganesan On weakly  $\pi$ g-closed sets in topological spaces. Italian journal of pure and applied mathematics (36), 656-666(2016)
- [17] Sakkraiveeranan Chandrasekar, Velusamy Banupriya, Jeyaraman Suresh Kumar,Properties And Applications Of  $\theta g^*\alpha$ -Closed Sets In Topological Spaces,Journal of New Theory(18) 1-11(2017)
- [18] Selvaraj Ganesan,SakkariVeeranan Chandrasekar Another quasi  $\mu S$  -open and quasi  $\mu S$ -closed functions Journal of New Theory ,15(75-80)
- [19] K.SafinaBegum,S.Chandarasekar quasi fine sg-open and quasi fine sg-closed functions infine topological spaces In Fine Topological Spaces Journal Of Computer And Mathematical Sciences Vol 9 No4 APR(2018),
- [20] S. Jeyashri , S. Chandrasekar,and M. Sathyabama , Soft  $b\#$ -open Sets and Soft  $b\#$ -continuous Functions in Soft Topological Spaces International Journal of Mathematics And its Applications., 6(1–D)(2018), 651–658