International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 3, March 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, **Email: ijesmj@gmail.com**
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Process Control and Process Capability Indices in the Presence of Autocorrelation

Anuj Kumar Singh
Research Scholar, Department of Statistics, University of Allahabad, Allahabad (UP)

Abstract:
In the present world of competition all the players in the market are very much concerned about the customer satisfaction and process capability indices are important tools to measure the customer satisfaction. In this paper, process capability indices in the presence of auto correlation for a process with autocorrelation have been dealt with. Their statistical properties along with a few estimation results have also been studied.

Introduction:

In Statistics Process Control procedures are being used to control and maintain a satisfactory quality level and ensure that the proportion of defective items in the manufactured product is not too large this termed as process control and is achieved through the technique of control charts, and the process capability indices are introduced to give a clear indication of the capability of a manufacturing process. They are formulated to quantify the relation between the desired engineering specifications and the actual performance of the process. The process control indices are organised to determine whether the process is capable of meeting specification limits on the quality features. The quantitative measure of process capability indices indicates the amount of customer's requirements that are obtained from quality characteristics. Generally a large value of process capability shows a better process.

In general products with multiple features could usually contain huge non central specification and central specification. In fact when ever all process capabilities of each characteristic satisfy present specification, customers will not reject products. It is clear that a single process capability indices is not able to visit the consumer requirements. In fact those process capabilities indices are predominantly define under the independence assumptions. If in process capability when one of its three normal assumptions are not meet. We are calculating process capability indices when data display on inner dependent behaviour. We explore the process capability estimation in AR(1) process. There are few studies dealing with process capability indices estimation for auto correlated process. Shore (1997) described some of the undesirable effects that auto correlated. May have on sampling distribution of estimates of the mean and the standard deviation when autocorrelation is present and therefore critical values and confidence intervals extracted under the assumption of independent data should not be used as the rate of type one and type two errors may be high. Jing (2009) also used the Taguchi method in order to estimate the process capability indices of auto correlated observations. They evaluated the impacts of autocorrelation on mean and standard deviation and probability density function for modes for order one. Jing (2010) developed a comparison method for five different estimation strategies of process capability when the observations are not independent. Vannman and Kulahci (2008) devised a new called the iterative skipping strategy to perform process capability. Analysis when observations are auto correlated. In this method the data set was separated into sub samples by skipping a predetermined number of observations.

Method for Calculate Capability Indices:

In auto correlated data are treated independently during capability analysis the conclusions may lead to incorrect decision. The present work will focus on studying the impact of presence of autocorrelation structure in data on different process capability indices. The first process capability index Cp, defined by Kane (1986) as: $Cp = \frac{\text{USL} - \text{LSL}}{6\,\sigma}$

Where USL is the upper specification limit, LSL is the lower specification limit, and σ is the process standard deviation. The numerator of Cp provides the range over which the process measurements are acceptable. The denominator gives the range over which the process is actually varying. The index C p was designed to measure the magnitude of the overall process variation relative to the manufacturing tolerance, which is to be used for processes based on data that are normal, independent, and in the statistical control Clearly, the index measures only the potential of a process to provide an acceptable product and does not take into account whether the process is cantered or not. In the deviations of process mean from the target value, several indices, similar in nature to Cp, have been proposed. These indices attempt to account for the magnitude of process variance as well as for the process departures from the target value. One of such indices is Cpk defined as:

$$Cpk = \min\{\frac{\mathrm{USL}-\mu}{3\sigma}, \frac{\mathrm{LSL}-\mu}{3\sigma}\}$$

Where μ is the process mean, USL, LSL upper specification limit, lower specification limit and σ is standard deviation. The relationships between Cp and Cpk are discussed by Barnett (1990), Kotz & Johnson (1999). The Cpk can alternatively be written as

$$Cpk = \frac{\mathrm{d}-\mathrm{mod}(\mu-\mathrm{m})}{3\sigma} = Cp(1-\frac{\mathrm{mod}(\mu-\mathrm{m}}{\mathrm{d}})$$

where d=(USL LSL)/2 is half of the length of the specification interval and m=(USL LSL)/2 is the mid-point between the lower and the upper specification limit.

Taguchi, focuses on the loss in a product's worth when one of its characteristics departs from customers' ideal value. To handle this situation, Hsiang and Taguchi (1985) introduced the index Cpm , independently proposed by Chan et al. (1988). It concentrates on measuring the ability of the process to cluster around the target value T, which reflects the degrees of process targeting (centring). It is defined as $Cp = \frac{\mathrm{USL}-\mathrm{LSL}}{6\sigma_T} = \quad Cp = \frac{\mathrm{USL}-\mathrm{LSL}}{\sqrt{\sigma^2+(\mu-T)^2}}$

Where $\sigma_T{}^2 = \sigma^2 + (\mu - T)^2 = E[(X-T)^2]$

Incorporates two variation components variation with respect to the process mean and deviation of the process mean from the value T. Obviously, these indices could be used for processes based on data that are normal, independent, and in the statistical control.

One of the most essential assumptions is that observations are statistically independent. However, there are many processes, particularly in chemical industries, where the data are inherently correlated. With the development of measurement technology and data acquisition technology in recent years, sampling frequency is getting higher, and the existence of autocorrelation cannot be ignored. Therefore, these indices may indicate inappropriate conclusions if the correlation effort is not taken into account, because the variance of subgroup mean is larger for autocorrelation observations than for no correlated ones and the expected value of variance is smaller than the actual process variance. Zhang (1998) studied the indices Cp and Cpk for auto correlated data. There are two methods, a model-based and a model-free approach, which deal with autocorrelations in process control. As the assessment of process capability begins after the evaluation of the process in a state of statistical control, process capability indices Cp, Cpk and Cpm for auto correlated data will be discussed in the way similar to process monitoring with autocorrelation.

<u>Using an AR(1) model</u>

For many industrial processes, like oil refinery, paper production etc, it is well known that the level of individual quality characteristics often varies with a wave-like pattern. Observations on such a characteristic, made at equal time intervals, are then supposed to be dependent and the outcome of such a process can be modelled in many ways. We choose the well-known AR(1) model,

$X_t = \mu + \phi(X_{t-1} - \mu) + Z_t$ Where X_t is the time series with parameters μ and ϕ , and Zt is white noise with variance σ^2 . It is also assumed that -1 < ϕ < 1. For the simplicity of the formula, let $Z_t = (X_{t-1} - \mu)$

Then $Z_t = \phi Z_{t-1} + E_t$ For AR(1) model, the autocorrelation coefficient between X_t and X_{t-1} is $\rho_k = \phi^k$, k=1,2,3,……………………………….

Parameters of the model are unknown under most circumstances. It is necessary to estimate these parameters. AR(1) model, sometimes known as a low pass filter or Markov dependency, is probably the most commonly used single ARIMA(p,q) model in industry. With the commonly accepted 'keep it simple'-attitude to time series analysis, $\hat{\rho}_1$ the AR(1) is often a reasonable model and furthermore easy to estimate. Common methods for parameter estimation include Yule Walker equations, Least Squares Method (LSM) and Maximum Likelihood Estimation (MLE). Using LSM for AR(1) model, sample

Statistics γ_k and $\hat{\rho}_1$ are often used to estimate the auto covariance ρ_1:

$$\gamma_k = \gamma_{k-1} = \frac{1}{n-1}\sum_{t=1}^{n-1}[(X_t - \overline{X})(X_{t-1} - \overline{X})], \quad \text{K=0,1}$$

$$\hat{\rho}_1 = \hat{\rho}_{-1} = \hat{\gamma}_1 / \hat{\gamma}_0$$

Than the estimator of $\sigma^2{}_e$ is $\sigma^2{}_e$ =[OLS/n-1]

Where OLS= $OLS = \sum_{t=2}^{n}[Z_t - \phi Z_{t-1}]^2$, For a large n, there is little difference between LSM and Yule-Walker estimation but LSM is more precise.

Conclusion:

If process capability indices are lower than customers' expectation, try to reduce mean shift or process variation in order to improve process capability. If the indices are met with customers' expectation, maintaining current process capability will be an essential task.

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