International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 4, April 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

SIGNED MEAN E-CORDIAL LABELING

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Abstract:

Let G be a simple (p,q) graph and let $f: E(G) \rightarrow \{-1, +2, -3, \dots\}$ be a mapping with the induced labeling $f^*: V(G) \rightarrow \{0,1\}$ defined by $f^*(V) = \sum f(u+v)/(u+v \in E(G)(mod\ 2)$ then f is called a Signed mean E- cordial labeling of a graph G if the number of vertices labeled with 0 and number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with alternative signed integers.

A graph G which admits Signed mean E-cordial labeling is called a Signed mean E-cordial graph.

Here ,we have proved that wheel graph , circulant graph, Petersen graph and Gear graph admit Signed mean E- cordial labeling.

Mathematical subject classification: 05C78.

Keywords:

Labeling, Mean labeling, Graceful labeling ,Cordial labeling, signed labeling, E-cordial labeling, Signed mean E- cordial labeling.

Introduction :

We begin with a graph $G = (V(G), E(G))$ with p vertices and q edges we mean G to be simple, finite ,connected and undirected graph .For any undefined notation and terminology ,we refer Gross and Yellen[8].

Definition 1.1:

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

In 1967,Rosa introduced the labeling on G ,called graceful labeling.

Definition 1.2:

A function $f: V(G) \rightarrow \{0,1 \ldots |E(G)|\}$ is called a graceful labeling of a graph G if f is injective and the induced function $f^*(e = uv) = | f(u) - f(v) |$ is bijective. A graph which admits graceful labeling is called a graceful graph.

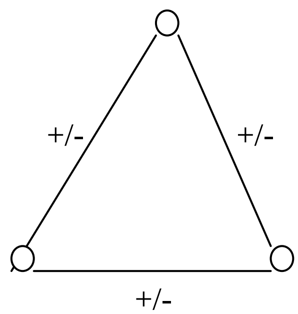
Definition 1.3:

A binary vertex labeling of a graph G with induced edge labeling $f^*: E(G) \rightarrow \{0,1\}$ defined by $f^*(e = uv) = | f(u) - f(v) |$ is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$. A graph G is a cordial graph if G admits a cordial labeling.

The concept of cordial labeling was introduced by Ebrahim Cahit(Turkey) as a Weaker version of graceful and harmonious labelings. He also investigated several results on this newly defined concept.

Definition 1.4:

In graph theory, Signed graph is a graph in which each edge or each vertex or both have a positive or negative sign.



The concept on Signed graph appeared first in 1953 by Frank Harary , at the center for Group Dynamics at the university of Michigan.

Definition 1.5:

Let G be a graph with vertex set V(G) and edge set E(G) and let $f: E(G) \rightarrow \{0,1\}$. Define a mapping f^* on V(G) by $f^*(V) = \sum f(uv)/(uv \,\epsilon\, E(G)(mod\ 2)$. The function f is called an E-cordial labeling of G if $|v_f(0) - v_f(1)\,| \leq 1$ and$|\ e_f(0) - e_f(1)\,| \leq 1$. A graph G is called E-cordial graph if it admits an E-cordial labeling.

In 1997, Yilmaz and Cahit [12] introduced E- cordial labeling as a weaker version of edge-graceful labeling and with the blend of cordial labeling.

Definition 1.6:

A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to $\{0,1,2, \dots .\, q\}$ such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$

if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd then the resulting edges are distinctly labeled.

Definition 1.7:

The Wheel graph W_n is defined to be the join $K_1 + C_n$. The vertex corresponding to K_1 is known as apex vertex and vertices corresponding to the cycle C_n are known as rim edges.

Definition 1.8:

The Gear graph G_n is obtained from the Wheel by subdividing each of its rim edges .

Main Results:

Theorem 2.1:

The Circulant graph $C_n(n = 6)$ admits Signed mean E- cordial labeling with generating set (1,2).

Proof:

Let $G = C_n(1,2)$ be the 4- regular graph with n=6.

Let $V(G) = \left\{\frac{V_i}{i} = 0,1,2..n - 1\right\}$

We define a mapping $f: E(G) \to \{-1, -2, -3, -4, -5, -6\}$

We have assigned the inner edges with positive integers.

Define the function $f(V_iV_{i+1})$ = -i-1 for i=1,2,3,4,5.

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$$f(V_iV_{i+2}) = \begin{cases} i+7 & \text{for } i=0,5 \\ 3i+7 & \text{for } i=1 \\ i+6 & \text{for } i=2 \\ i+8 & \text{for } i=3 \\ i+5 & \text{for } i=4 \end{cases}$$

then apply $f^*(V) = \sum f(u+v)/(u+v \epsilon E(G)(mod\ 2)$, then we get the induced function $f^*: V(G) \to \{0,1\}$.

Thus using the above labeling pattern , We found that C_n admits Signed mean E-cordial labeling.

Illustration 1.1:

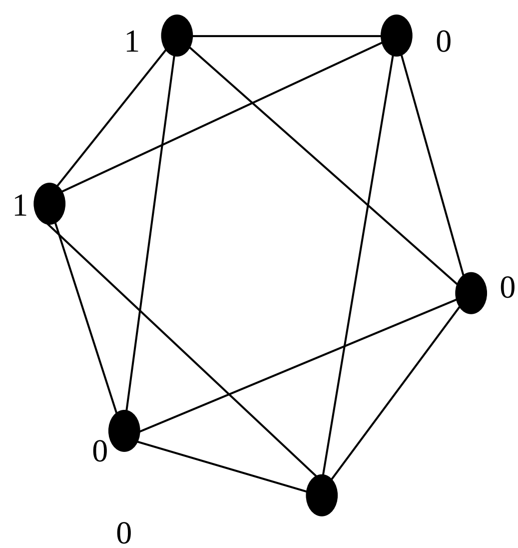

Figure 1: Signed mean E- cordial labeling of Circulant graph C_6 .

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Theorem 2.2:

The Gear graph G_n admits Signed mean E- cordial labeling.

Proof:

Let W_n be a Wheel graph with apex vertex v and rim vertices $v_1 v_2 \ldots v_n$.

To obtain the Gear graph G_n , sub divide each of the rim edges of the Wheel graph by the vertices $u_1 u_2 \ldots u_n$.

We define a mapping $f: E(G) \rightarrow \{-1, +2, -3, +4 \ldots . +12\}$

We have assigned the inner edges with positive integers .

then apply $f^*(V) = \sum f(u+v)/(u+v \, \epsilon \, E(G)(mod\ 2)$ then we get the induced function $f^*: V(G) \rightarrow \{0,1\}$.

Thus using the above labeling pattern , We found that G_n admits Signed mean E-cordial labeling.

Illustration 1.2:

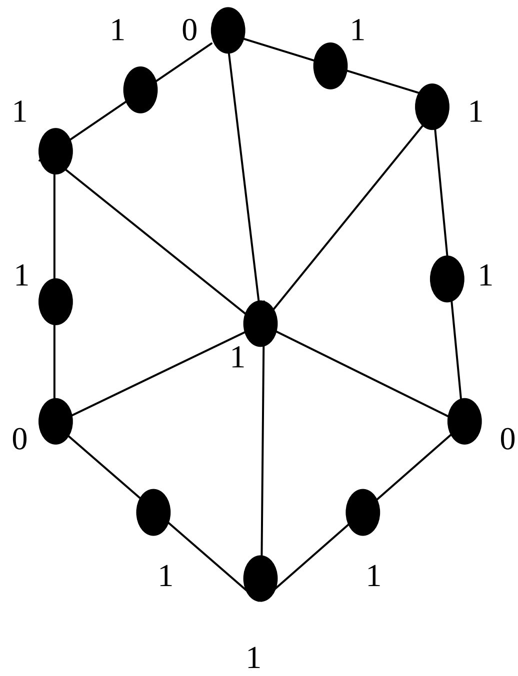

Figure 2: Signed mean E- cordial labeling of the Gear graph G_7.

Theorem 2.3:

Petersen graph admits Signed mean E- cordial labeling.

Proof:

Petersen graph is a 3-regular graph with 10 vertices and 15 edges.

Let $u_0, u_1 \dots u_{14}$ be the edges and

Let $v_0, v_1 \dots v_9$ be the vertices of Petersen graph.

Let $e_{1,} e_2, e_3, e_4, e_5$ be the inner edges.

We define the labeling function for the inner edges of the Petersen graph as follows:

$f: E(G) \rightarrow \{-1, -2, -3, -4, -5\}$

Let the remaining outer edges be the positive integers.

Let $f: E(G) \rightarrow \{6,7,8 \ldots 15\}$

then apply $f^*(V) = \sum f(u+v)/(u+v \, \epsilon \, E(G)(mod\ 2)$ then we get the induced function $f^*: V(G) \rightarrow \{0,1\}$.

Thus using the above labeling pattern , We found that Petersen graph admits Signed mean E- cordial labeling.

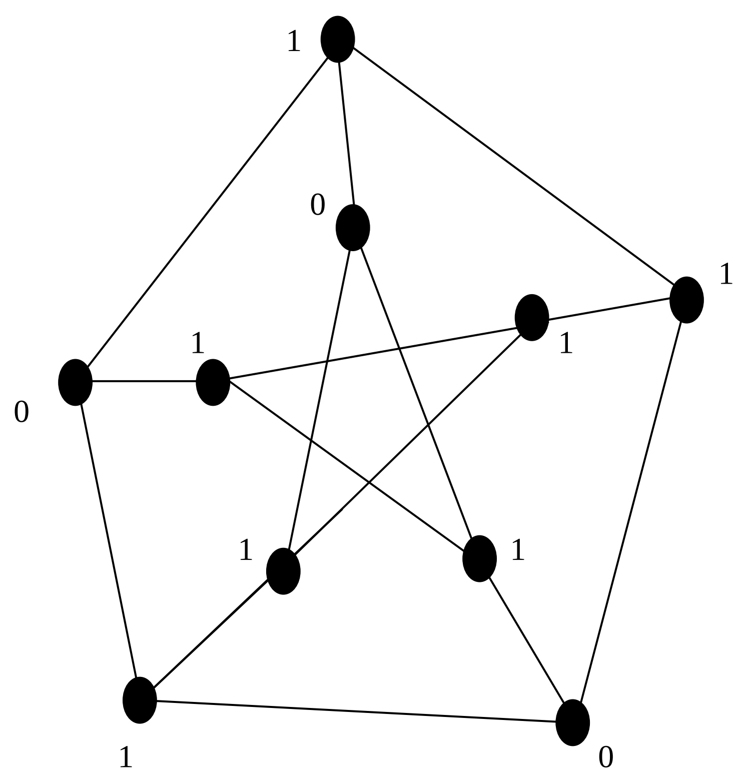

Figure 3: Signed mean E- cordial labeling G(10,15).

Theorem 2.4:

Let W_n be a wheel graph of order n=6 then W_n admits Signed mean E – cordial labeling.

Proof:

Let $W_n = (v_1 v_2 \dots v_n)$ be a Wheel of order n=6.

We define a mapping $f: E(G) \to \{-1, -2, -3, -4, -5\}$

We have assigned the inner edges with positive integers.

then apply $f^*(V) = \sum f(u+v)/(u+v \epsilon E(G)(mod\ 2)$ then we get the induced function $f^*: V(G) \to \{0,1\}$.

Thus using the above labeling pattern , We found that W_6 admits Signed mean E-cordial labeling.

Illustration 1.4:

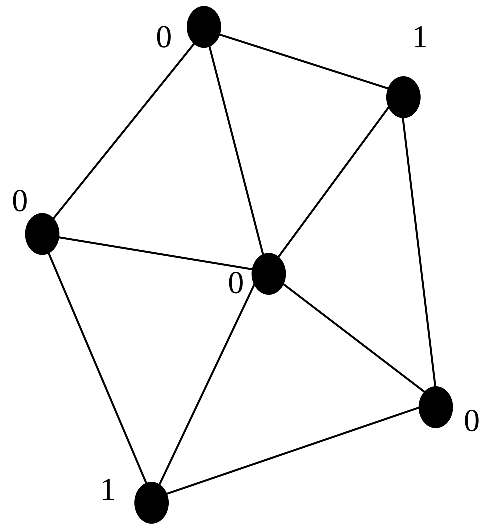

Figure 4: Signed mean E- cordial labeling of W_6.

Conclusion :

In this paper, We have obtained Signed mean E-cordial labeling for the Circulant graph C_n, Petersen graph, Wheel graph and the Gear graph of finite order. We further, motivated to verify the above labeling process, for some more special classes of graphs.

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