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AN EPQ MODEL WITH VARIABLE PRODUCTION RATE AND MARKDOWN POLICY FOR STOCK AND SALES PRICE SENSITIVE DEMAND WITH DETERIORATION

Monami Das Roy*

Abstract

This article deals with an Economic Production Quantity (EPQ) model for a single type of product. It is assumed that the demand of customer is not only influenced by the stock display but also the unit selling price. The production rate varies with the demand rate of the items. Deterioration of products takes place after remaining in stock for a long time. The rate of deterioration is assumed to be constant. Price markdown is taken into consideration when deterioration of product begins. Shortage occurs at the end of the cycle. The objective of this research work is to determine the optimal values of selling price, production lot size, production run time, markdown time and price markdown to maximize the average profit function. Numerical example with graphical illustration is provided to establish the model.

Keywords:

EPQ;
Variable production rate;
Price markdown policy;
Deterioration;
Shortage.

Author correspondence:

Monami Das Roy,
Department of Mathematics,
Haldia Government College,
Vidyasagar University,
PurbaMedinipur 721657, West Bengal, India.

1. Introduction (10pt)

Generally, all products deteriorate with time but the rate of deterioration is different for different products. Some products have high deterioration rate such as vegetables, fruits, milk, fish, etc. and some have comparatively low deterioration rate such as grocery products, cosmetics, cloths, etc. A number of production-inventory models have been investigated for deteriorating products. Among them, some are investigated by considering that the deterioration of items take place at the beginning of the production process and in some studies it is assumed to be started after some time span. A few of them are Teng and Chang [1], He *et al*. [2], Singh and Singh [3], Wee and Widyadana [4], Chen *et al*. [5] and Shah *et al*. [6]. The present study has considered that the product deterioration begins after remain in stock for a long time.

Many strategies may be used to handle the deteriorating products. Price markdown is one of them. It is a useful policy to reduce stock and increase profit as demand of item increases with price

*Department of Mathematics, Haldia Government College, Vidyasagar University, PurbaMedinipur 721657, West Bengal, India.

decrease. Nair and Closs [7] have examined the impact of coordinating supply chain policies and price markdowns on retail performance of short lifecycle product while a replenishment policy for deteriorating items is studied by Widyadana and Wee [8] where they have considered price sensitive demand and markdown policy. Srivastava and Gupta [9] have studied an EPQ model for time and price dependent demand. They have adopted markdown policy for both types of items: fresh and deteriorated. Multiple price markdowns over time is discussed by Chung, Talluri and Narasimhan [10].

Stock of goods and suitable price of an item are the two effective factors which may influence customers demand to some extent. Datta and Paul [11] have investigated an inventory model where they have considered stock and price sensitive demand pattern while an EPQ model for deteriorating items having stock and price sensitive demand rate is discussed by Teng and Chang [1]. Chang *et al*. [12] have introduced the concept of limited shelf space for an inventory model. They have considered stock- and price- dependent demand rate together with deterioration. An EPLS model for stock-price sensitive demand and deterioration is presented by Roy and Chaudhuri [13] where they have assumed variable production rate. Das Roy *et al*. [14] have discussed an economic production lot size model for defective items having stochastic demand where they have considered variable production rate and backlogging. Besides these, several researchers have developed their model either stock or price or both stock and price dependent demand pattern. Some of the significant works in this direction are Hou [15], Goyal and Chang [16], Roy *et al*. [17], Lee and Dye [18], Alfares [19] and Das Roy *et al*. [20].

In this study, an Economic Production Quantity (EPQ) model is analyzed for stock and price sensitive demand. The production rate of the items is directly proportional with the demand rate of the customers. Product deterioration starts after remaining in stock for a long time. The concept of markdown is implemented when deterioration starts. Due to continuous demand and deterioration, stock-out situation occurs at the end of the cycle. The aim of this research work is to maximize the average profit of the EPQ model and determine the optimal values of the selling price, production lot size, price markdown, production run time, markdown time and the maximum average profit.

The entire article is organized into six sections. Section 1 contains the introduction part while Section 2 states the notation and assumptions of the model. Model formulation and solution are given in Section 3 while the solution procedure is described in Section 4. Section 5 provides the numerical example and Section 6 presents the summary and conclusion of the research work.

2. Notation and assumptions

The notations and assumptions used to develop the model are as follows.

2.1. Notation

A : Set up cost per cycle.
Q : Production lot size.
C_0 : Production cost per unit item per unit time.
C_H : Holding cost per unit item per unit time.
C_G : Deterioration cost per unit item per unit time.
C_S : Shortage cost per unit item per unit time.
θ : Deterioration rate, where $0 < \theta < 1$.
r : Markdown rate $0 < r < 1$.
p : Selling price per unit item before markdown.
$\tilde{p}$: Selling price per unit item after markdown.
t_1 : The production run-time.
t_2 : The time point at which deterioration begins or the markdown offering time.
t_3 : The time point at which the on-hand inventory reaches to zero.
T : The Length of the cycle.
$D(q, p)$: Demand of items at time $t \geq 0$.

$q(t)$: Inventory level at time t, $t \geq 0$.
$\chi(Q,p)$: The average profit for the model.

2.2. Assumptions

1. The EPLS model is studied for a single type of product.
2. Deterioration of items start after remains in stock for a long period of time and the rate of deterioration θ is a constant , where $0 < \theta < 1$.
3. To reduce the loss which may take place due to deterioration of products one time price markdown is considered in a cycle when deterioration of items begins.
4. Markdown rate is a known constant.
5. Selling price after markdown is $\tilde{p} = (1-r)p$.
6. Demand of items is influenced by stock and price of the product. The demand function is
$$\begin{aligned} D(q,p) &= \alpha + \beta q - \gamma p\,, \quad 0 \leq t \leq t_1 \\ &= \alpha + \beta q - \gamma p\,, \quad t_1 \leq t \leq t_2 \\ &= \alpha + \beta q - \delta \tilde{p}\,, \quad t_2 \leq t \leq t_3 \\ &= (\alpha - \delta \tilde{p}), \quad t_3 \leq t \leq T. \end{aligned}$$
$\alpha > 0$; $\beta > 0$; $\gamma > 0$; $\delta > 0$whereα is the initial demand, β denotes the rate at which the demand rate changes with stocks, γ and δ denotes the rate at which the demand rate changes with price.
7. The production rate varies with the demand rate i.e. production rate is $KD(q,p)$, where $K > 1$.
8. Shortage occurs at the end of the cycle.

3. Model Formulation and Solution

The production process starts at time $t = 0$ and ends at time $t = t_1$. The production lot size is Q. After satisfying the continuous consumption during the production run time $[0, t_1]$ the on-hand stock at time $t = t_1$ is S_1. Again, after meeting the demand in the time span $[t_1, t_2]$ the on-hand inventory at time $t = t_2$ becomesS_2. Suppose the deterioration of items starts at time $t = t_2$. Price markdown is introduced when deterioration of product begins. The on-hand stock reaches to zero at time $t = t_3$. Shortage takes place in the time interval$[t_3, T]$. The inventory model is shown in Fig. 1.

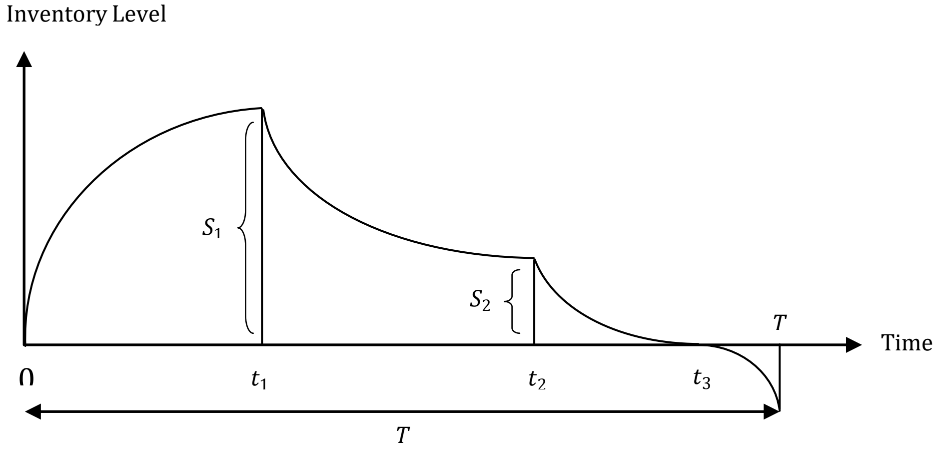

Fig. 1: Inventory level versus time.

The governing differential equations of the inventory system at time t are

$$\frac{dq(t)}{dt} = (K-1)(\alpha + \beta q - \gamma p), \quad 0 \le t \le t_1, \quad q(0) = 0 \quad \text{... (1)}$$

$$\frac{dq(t)}{dt} = -(\alpha + \beta q - \gamma p), \quad t_1 \le t \le t_2, \quad q(t_1) = S_1 \quad \text{... (2)}$$

$$\frac{dq(t)}{dt} + \theta q(t) = -(\alpha + \beta q - \delta\tilde{p}), \; t_2 \le t \le t_3, \quad q(t_2) = S_2 \quad \text{... (3)}$$

and
$$\frac{dq(t)}{dt} = -(\alpha - \delta\tilde{p}), \quad t_3 \le t \le T, \quad q(t_3) = 0. \quad \text{... (4)}$$

With the help of the above boundary conditions the solutions of equations (1) to (4) become

$$q(t) = \frac{(\alpha - \gamma p)}{\beta}\left(e^{\beta(K-1)t} - 1\right), \qquad 0 \le t \le t_1 \quad \text{.... (5)}$$

$$= S_1 e^{\beta(t_1 - t)} - \frac{(\alpha - \gamma p)}{\beta}\left(1 - e^{\beta(t_1 - t)}\right), \quad t_1 \le t \le t_2 \quad \text{... (6)}$$

$$= S_2 e^{(\theta+\beta)(t_2 - t)} - \frac{(\alpha - \delta\tilde{p})}{\theta+\beta}\left(1 - e^{(\theta+\beta)(t_2 - t)}\right), \; t_2 \le t \le t_3 \quad \text{... (7)}$$

$$= (\alpha - \delta\tilde{p})(t_3 - t), \qquad t_3 \le t \le T. \quad \text{... (8)}$$

Theorem 1: If the on-hand stock at time $t = t_1$is S_1 then the production run time will be

$$\text{or,} \quad t_1 = \frac{1}{\beta(K-1)} \ln\left[1 + \frac{\beta S_1}{(\alpha - \gamma p)}\right]. \quad \text{... (9)}$$

Proof: Substituting $t = t_1$ and $q(t_1) = S_1$ in equation (5), which gives

$$q(t_1) = \frac{(\alpha - \gamma p)}{\beta}\left(e^{\beta(K-1)t_1} - 1\right)$$

$$\text{or,} \quad S_1 = \frac{(\alpha - \gamma p)}{\beta}\left(e^{\beta(K-1)t_1} - 1\right)$$

$$\text{or,} \quad t_1 = \frac{1}{\beta(K-1)} \ln\left[1 + \frac{\beta S_1}{(\alpha - \gamma p)}\right].$$

Hence the proof. ■

Theorem 2: If Theorem 1 holds good, then the markdown offering time will be

$$t_2 = \frac{1}{\beta}\left\{\frac{1}{(K-1)} \ln\left[\frac{\beta S_1 + \alpha - \gamma p}{\alpha - \gamma p}\right] - \ln\left[\frac{\beta S_2 + \alpha - \gamma p}{\beta S_1 + \alpha - \gamma p}\right]\right\}. \quad \text{... (10)}$$

Proof: Using the condition $q(t_2) = S_2$ equation (6) becomes

$$S_2 = S_1 e^{\beta(t_1-t_2)} - \frac{(\alpha - \gamma p)}{\beta}\left(1 - e^{\beta(t_1-t_2)}\right),$$

$$or,\ \ \beta S_2 + \alpha - \gamma p = (\beta S_1 + \alpha - \gamma p) e^{\beta(t_1-t_2)}$$

$$or,\quad e^{\beta(t_1-t_2)} = \frac{\beta S_2 + \alpha - \gamma p}{\beta S_1 + \alpha - \gamma p}$$

$$or,\quad t_1 - t_2 = \frac{1}{\beta}\ln\left[\frac{\beta S_2 + \alpha - \gamma p}{\beta S_1 + \alpha - \gamma p}\right].$$

With the help of Theorem 1, the markdown offering time becomes

$$t_2 = \frac{1}{\beta}\left\{\frac{1}{(K-1)}\ln\left[\frac{\beta S_1 + \alpha - \gamma p}{\alpha - \gamma p}\right] - \ln\left[\frac{\beta S_2 + \alpha - \gamma p}{\beta S_1 + \alpha - \gamma p}\right]\right\}.$$

Hence the proof. ■

Theorem 3: If shortages take place then the time point when shortage begins is

$$t_3 = \frac{1}{\beta}\left\{\frac{1}{(K-1)}\ln\left[\frac{\beta S_1 + \alpha - \gamma p}{\alpha - \gamma p}\right] - \ln\left[\frac{\beta S_2 + \alpha - \gamma p}{\beta S_1 + \alpha - \gamma p}\right]\right\} - \frac{1}{(\theta+\beta)}\ln\left[\frac{\alpha - \delta\tilde{p}}{S_2(\theta+\beta) + \alpha - \delta\tilde{p}}\right].$$
... (11)

Proof: By using the boundary condition $q(t_3) = 0$, equation (7) gives

$$S_2 e^{(\theta+\beta)(t_2-t_3)} - \frac{(\alpha - \delta\tilde{p})}{\theta+\beta}\left(1 - e^{(\theta+\beta)(t_2-t_3)}\right) = 0$$

$$\text{or,}\quad \left\{S_2 + \frac{(\alpha - \delta\tilde{p})}{\theta+\beta}\right\} e^{(\theta+\beta)(t_2-t_3)} = \frac{(\alpha - \delta\tilde{p})}{\theta+\beta}$$

$$\text{or,}\quad e^{(\theta+\beta)(t_2-t_3)} = \frac{\alpha - \delta\tilde{p}}{S_2(\theta+\beta) + \alpha - \delta\tilde{p}}$$

$$\text{or,}\quad t_2 - t_3 = \frac{1}{(\theta+\beta)}\ln\left[\frac{\alpha - \delta\tilde{p}}{S_2(\theta+\beta) + \alpha - \delta\tilde{p}}\right].$$

Using Equation (10), the shortage time point becomes

$$t_3 = \frac{1}{\beta}\left\{\frac{1}{(K-1)}\ln\left[\frac{\beta S_1 + \alpha - \gamma p}{\alpha - \gamma p}\right] - \ln\left[\frac{\beta S_2 + \alpha - \gamma p}{\beta S_1 + \alpha - \gamma p}\right]\right\} - \frac{1}{(\theta+\beta)}\ln\left[\frac{\alpha - \delta\tilde{p}}{S_2(\theta+\beta) + \alpha - \delta\tilde{p}}\right].$$

Hence the proof. ■

The production lot size is Q. It is assumed that after satisfying the demand during the production period$[0, t_1]$, $\mu\,\%$ of the production lot size (Q) remains in stock (S_1) at time $t = t_1$. After satisfying the demand of the customers in the interval $[t_1, t_2]$, $\eta\,\%$ of the previous stock i.e., S_1 remain in stock i.e. S_2 which are used to meet the demand during the period $[t_2, t_3]$. These implies the following relations

$$S_1 = \mu Q, \qquad 0 < \mu < 1$$
... (12)

and $$S_2 = \eta S_1 = \eta\mu Q, \qquad 0 < \eta < 1.$$
... (13)

Now the various costs of the system in a cycle are as follows.

Set up cost $= A$.
Production cost $= C_0 Q$.
Holding cost $= C_H\left[\int_0^{t_1} q(t)\,dt + \int_{t_1}^{t_2} q(t)\,dt + \int_{t_2}^{t_3} q(t)\,dt\right]$

$$= C_H\Big[\frac{S_1}{\beta(K-1)} + \frac{(S_1 - S_2)}{\beta} + \frac{S_2}{(\theta+\beta)} + \frac{(\alpha-\gamma\tilde{p})}{(\theta+\beta)^2}\ln\left[\frac{\alpha-\delta\tilde{p}}{S_2(\theta+\beta)+\alpha-\delta\tilde{p}}\right] - \frac{(\alpha-\gamma p)}{\beta^2}\left\{\frac{1}{(K-1)}\ln\left[\frac{\beta S_1+\alpha-\gamma p}{\alpha-\gamma p}\right] - \ln\left[\frac{\beta S_2+\alpha-\gamma p}{\beta S_1+\alpha-\gamma p}\right]\right\}\Big].$$

Deterioration cost $= C_G\theta\int_{t_2}^{t_3} q(t)\,dt$

$$= C_G\theta\left\{\frac{S_2}{(\theta+\beta)} + \frac{(\alpha-\delta\tilde{p})}{(\theta+\beta)^2}\ln\left[\frac{\alpha-\delta\tilde{p}}{S_2(\theta+\beta)+\alpha-\gamma\tilde{p}}\right]\right\}.$$

Shortage cost $= C_S\int_{t_3}^{T}(-q(t))\,dt$

$$= \frac{C_S(\alpha-\gamma\tilde{p})}{2}\left[T + \frac{1}{(\theta+\beta)}\ln\left[\frac{\alpha-\delta\tilde{p}}{S_2(\theta+\beta)+\alpha-\delta\tilde{p}}\right] - \frac{1}{\beta}\left\{\frac{1}{(K-1)}\ln\left[\frac{\beta S_1+\alpha-\gamma p}{\alpha-\gamma p}\right] - \ln\left[\frac{\beta S_2+\alpha-\gamma p}{\beta S_1+\alpha-\gamma p}\right]\right\}\right]^2.$$

Total revenue = Revenue from items sold at price p + Revenue from items sold at price $\tilde{p}$
$= p(Q - S_2) + \tilde{p}S_2 = (1 - r\eta\mu)pQ$.

The average profit is

$$\chi(Q,p) = \frac{1}{T}[\text{TotalRevenue} - \text{Set up cost} - \text{Production cost} - \text{Holding cost} - \text{Deterioration cost} - \text{Shortage cost}]$$

$$= \frac{1}{T}\Big[(1-r\eta\mu)pQ - A - C_0Q - C_H\Big[\frac{S_1}{\beta(K-1)} + \frac{(S_1-S_2)}{\beta} + \frac{S_2}{(\theta+\beta)} - \frac{(\alpha-\gamma p)}{\beta^2}\left\{\frac{1}{(K-1)}\ln\left[\frac{\beta S_1+\alpha-\gamma p}{\alpha-\gamma p}\right] - \ln\left[\frac{\beta S_2+\alpha-\gamma p}{\beta S_1+\alpha-\gamma p}\right]\right\} + \frac{(\alpha-\delta\tilde{p})}{(\theta+\beta)^2}\ln\left[\frac{\alpha-\delta\tilde{p}}{S_2(\theta+\beta)+\alpha-\delta\tilde{p}}\right] - C_G\theta\left\{\frac{S_2}{(\theta+\beta)} + \frac{(\alpha-\delta\tilde{p})}{(\theta+\beta)^2}\ln\left[\frac{\alpha-\delta\tilde{p}}{S_2(\theta+\beta)+\alpha-\delta\tilde{p}}\right]\right\} - \frac{C_S(\alpha-\delta\tilde{p})}{2}\left[T + \frac{1}{(\theta+\beta)}\ln\left[\frac{\alpha-\delta\tilde{p}}{S_2(\theta+\beta)+\alpha-\delta\tilde{p}}\right] - \frac{1}{\beta}\left\{\frac{1}{(K-1)}\ln\left[\frac{\beta S_1+\alpha-\gamma p}{\alpha-\gamma p}\right] - \ln\left[\frac{\beta S_2+\alpha-\gamma p}{\beta S_1+\alpha-\gamma p}\right]\right\}\right]^2\Big].$$

... (14)

Using relations (12) and (13) in equation (14), it becomes as follows.

$$\chi(Q,p)=\frac{1}{T}\Big[F_1pQ-A-F_2Q-F_6(\alpha-F_3p)\ln\left[\frac{\alpha-F_3p}{F_5Q+\alpha-F_3p}\right]$$
$$+\frac{C_H(\alpha-\gamma p)}{\beta^2}\left\{\frac{1}{(K-1)}\ln\left[\frac{F_4Q+\alpha-\gamma p}{\alpha-\gamma p}\right]-\ln\left[\frac{\eta F_4Q+\alpha-\gamma p}{F_4Q+\alpha-\gamma p}\right]\right\}$$
$$-\frac{C_S(\alpha-F_3p)}{2}\left[T+\frac{1}{(\theta+\beta)}\ln\left[\frac{\alpha-F_3p}{F_5Q+\alpha-F_3p}\right]-\frac{1}{\beta}\left\{\frac{1}{(K-1)}\ln\left[\frac{F_4Q+\alpha-\gamma p}{\alpha-\gamma p}\right]-\ln\left[\frac{\eta F_4Q+\alpha-\gamma p}{F_4Q+\alpha-\gamma p}\right]\right\}\right]^2\Big],$$
... (15)

where $F_1=(1-r\eta\mu), F_2=C_0+C_H\mu\left\{\frac{1}{\beta(K-1)}+\frac{1-\eta}{\beta}+\frac{\eta}{\theta+\beta}\right\}+\frac{C_G\theta\eta\mu}{\theta+\beta},\quad F_3=\delta(1-r)$

$F_4=\mu\beta,\quad F_5=\eta\mu(\theta+\beta),\quad F_6=\frac{C_H+\theta C_G}{(\theta+\beta)^2}.$

4. Solution Procedure

Here, in the present study the average profit function contains non-linear logarithmic expressions. "MATHEMATICA 8.0" software is used to obtain the optimal solution and examine the following optimization criteria.

$\frac{\partial^2\chi}{\partial Q^2}<0,\ \frac{\partial^2\chi}{\partial p^2}<0$ and the determinant of Hessian matrix i.e. $|H|=\left(\frac{\partial^2\chi}{\partial Q^2}\right)\left(\frac{\partial^2\chi}{\partial p^2}\right)-\left(\frac{\partial^2\chi}{\partial Q\partial p}\right)^2>0.$

The graph of the numerical example also shows the concavity nature of the average profit function.

5. Numerical Example

Example 1. Let us consider an EPQ model whose parameters values are:

$A=\$120$,	$C_0=\$20$,	$C_H=\$5.8$,	$C_G=\$3$,	$C_S=\$6.4$,
$r=.2$,	$\theta=0.3$,	$K=6$,	$\mu=0.7$,	$\eta=0.4$,
$\alpha=200$,	$\beta=0.01$,	$\gamma=2.35$,	$\delta=2.26$,	$T=12$months.

Using the above parameter values in equation (15) the optimal value of selling price i.e. $p^*=\$66.8824\cong\$66.88$, production lot size $Q^*=758.877\cong758.88$ units and the average profit is $\chi^*=\$1078.64$ (see Fig. 2).

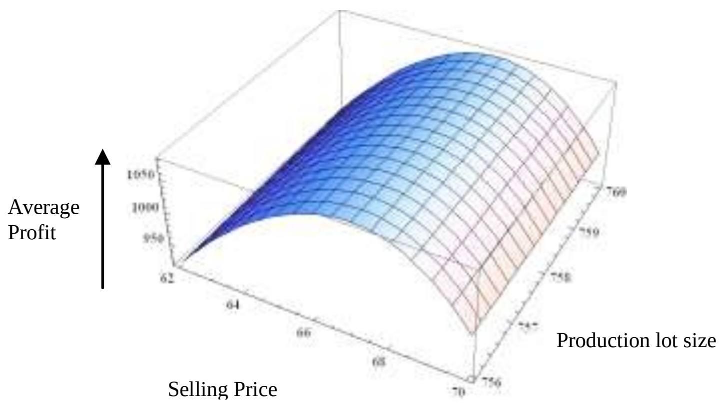

Fig. 2: Graphical representation of average profit (χ^*) versus production lot size (Q^*) and selling price (p^*).

Since at the optimal solutions $p^* = \$66.8824, Q^* = 758.877$,

$$\frac{\partial^2\chi}{\partial Q^2} = -0.0120289 < 0, \qquad \frac{\partial^2\chi}{\partial p^2} = -17.4074 < 0$$

and $H = \frac{\partial^2\chi}{\partial Q^2}.\frac{\partial^2\chi}{\partial p^2} - (\frac{\partial^2\chi}{\partial Q\partial p})^2 = 0.0663405 > 0$. Therefore, the Hessian matrix H is negative definite.

Hence, the required optimal solution is $p^* = \$66.8824 \cong \$66.88, Q^* = 758.877 \cong 758.88$ units and$\chi^* = \$1078.64$. Putting the value of Q^* in equation (12) and (13), the optimal value of S_1 and S_2 are ${S_1}^* = 531.214 \cong 531.21$ units and ${S_2}^* = 212.486 \cong 212.49$ units. Again, substituting the values of ${S_1}^*$ and ${S_2}^*$ in equation (9), (10) and (11) the optimal values of t_1, t_2 and t_3 are obtained as ${t_1}^* = 2.33857 \cong 2.34$ months, ${t_2}^* = 9.18901 \cong 9.19$ months and ${t_3}^* = 11.1437 \cong 11.14$ months respectively. Also, $\tilde{p} = \$53.5059 \cong \$53.51$. Therefore the price markdown is \$13.37 and the markdown percentage is 24.98%.

5. Summary and Conclusion

In this paper, an EPQ model is studied for stock and price sensitive demand. Usually, production rate is assumed to be constant but in this study it is considered as directly proportional with demand. Obviously, if items remain in stock for a long time then deterioration takes place. Therefore, deterioration and shortages are the other two realistic assumptions. Single markdown policy is introduced to reduce the loss that may cause due to deterioration of items. The model is established with suitable numerical and graphical results. All the assumptions taken together in this study are very much realistic. It is hoped that this study will help a production manager to take right decision regarding pricing and manufacturing policies.

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