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HIGH DENSITY DISTRIBUTION OF 3-PRIMEFACTORS NUMBERS TILL 1 TRILLION

Neeraj Anant Pande *

Abstract

Prime numbers have been recently generalized to '*k*-PrimeFactors Numbers', wherein instead of unique prime divisor, exactly *k* prime divisors are there to the concerned number. 3-PrimeFactors numbers are just a particular case with $k = 3$. Analogous to the analysis of 2-PrimeFactors numbers, in this work, 3-PrimeFactors numbers are considered for their maximum count in blocks of size of usual base powers, i.e., powers of 10, in range of 1 trillion. First and last blocks of different sizes along with number of such blocks containing maximum number of 3-PrimeFactors numbers are determined. Then block wise analysis is done by fixing block sizes and inspecting increasing ranges for each block size for occurrence of maximum number of 3-PrimeFactors numbers.

Keywords:

Prime number;
k-PrimeFactors number;
3-PrimeFactors number,
High density distribution

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1. Introduction

Number theorists have always been fascinated by prime numbers [1]. Owing to the lack of exact formulae, they [3] and their special types [4] have been analysed individually in huge ranges using choosy generator algorithms [2]. Until we get formulae for them approaches like this will continue for exploring their properties.

2. 3-PrimeFactors Numbers

Depending upon the number of prime divisors of a positive integer, the author has defined the following [6].

Definition (*k*-PrimeFactors Number) : For any integer $k \geq 0$, a positive integer having *k* number of prime factors, which need not be necessarily distinct, is called as *k*-PrimeFactors number.

Since usual prime numbers are infinite, their infinite multiplicative combinations, taking *k* of them at a time, are possible and *k*-PrimeFactors numbers are also so. This has an exception for $k = 0$, as there is only one 0-PrimeFactors positive integer, viz., 1.

In this work, we consider the case of $k = 3$.

Definition (3-PrimeFactors Number) : A positive integer having 3 prime factors, which need not be necessarily distinct, is called as 3-PrimeFactors number.

For lesser value 2 of *k*, 2-PrimeFactors numbers have been explored quite in detail [6], [7], [8], [9], [10], [11].

List of first few 3-PrimeFactors numbers is as follows :

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$$8, 12, 18, 20, 28, 30, \cdots$$

Every member of this sequence has precisely 3 prime divisors : $8 = 2^3$, $12 = 2^2 \times 3$, $18 = 2 \times 3^2$, $20 = 2^2 \times 5$, $28 = 2^2 \times 7$, $30 = 2 \times 3 \times 5$ and so on.

The following outcome required determination of all 3-PrimeFactors numbers till 1 trillion first by using usual primes. For their own determination, suitable better performing algorithms [2] were selected.

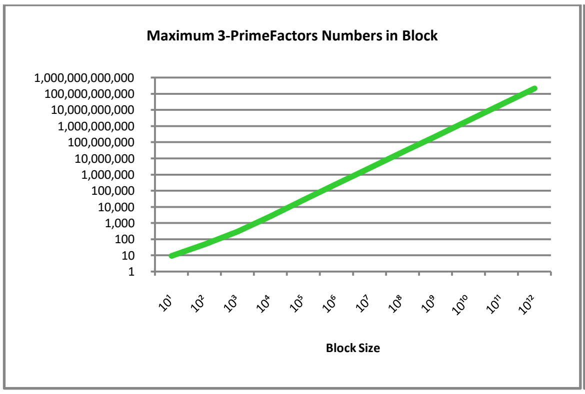
Similar analysis of 3-PrimeFactors numbers from other perspective of low density has been recently presented [12].

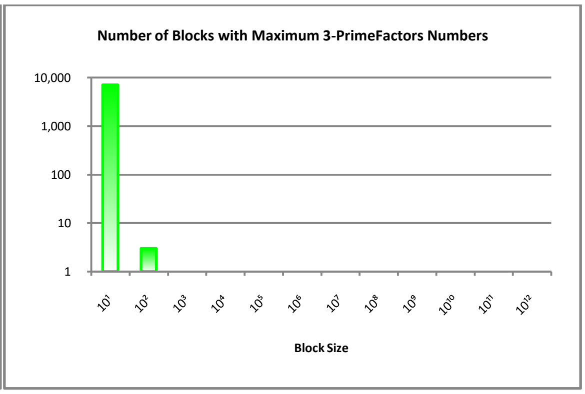
3. Maximum Number of 3-PrimeFactors Numbers in Blocks of Sizes 10^n

Complete range of all numbers less than 1000000000000 was investigated. For blocks size ten, hundred, thousand and so on till trillion itself, maximum number of 3-PrimeFactors numbers in blocks of all these sizes, first and last block of such high density of these numbers and number of occurrence of blocks of high density were looked for. For a block size $1\underbrace{0\cdots0}_{k\text{ times}}$, a block $x\underbrace{0\cdots0}_{k\text{ times}}$ means number range $x\underbrace{0\cdots0}_{k\text{ times}}$ to $x\underbrace{9\cdots9}_{k\text{ times}}$.

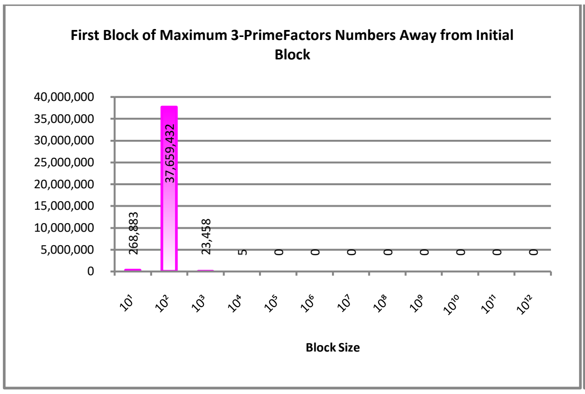
Sr. No.	Block -Size	Maximum 3-PrimeFactors Numbers in Block	First Block of Maximum 3-PrimeFactors Numbers	Last Block of Maximum 3-PrimeFactors Numbers	Number of Blocks with Maximum 3-PrimeFactors Numbers
1	10^1	9	2,688,830	999,892,219,310	7,110
2	10^2	46	3,765,943,200	619,858,239,900	3
3	10^3	289	23,458,000	23,458,000	1
4	10^4	2,598	50,000	50,000	1
5	10^5	25,556	0	0	1
6	10^6	250,853	0	0	1
7	10^7	2,444,359	0	0	1
8	10^8	23,727,305	0	0	1
9	10^9	229,924,367	0	0	1
10	10^{10}	2,227,121,996	0	0	1
11	10^{11}	21,578,747,909	0	0	1
12	10^{12}	209,214,982,911	0	0	1

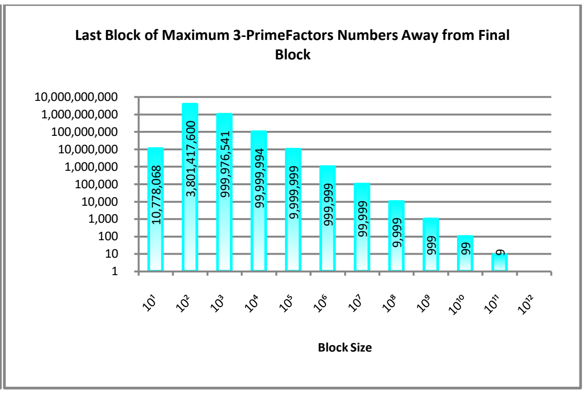
Similar to usual primes [3] and 2-PrimeFactors numbers [7], high density of 3-PrimeFactors Numbers comes in earlier blocks more often than in later ones. In fact, from block size 10^5 onwards, for each size, its very first and also the only block having highest density of 3-PrimeFactors numbers.





The first and last blocks of various sizes with highest density of 3-PrimeFactors numbers have following number from respective ends.





In section that follows, block-wise analysis is undertaken in increasing ranges.

4. Maximum Number of 3-PrimeFactors Numbers in Blocks of Particular Sizes in Different Ranges

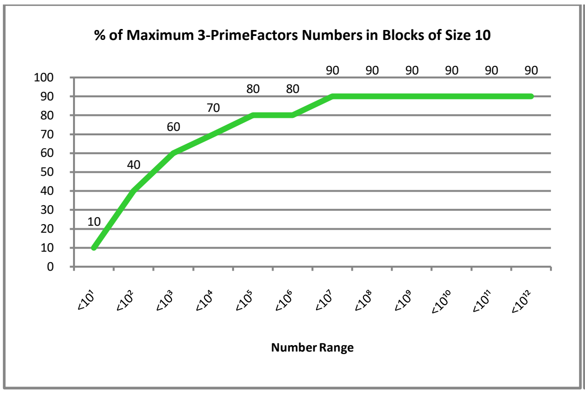
4.1 Maximum Number of 3-PrimeFactors Numbers in Blocks of Particular Sizes in Different Ranges

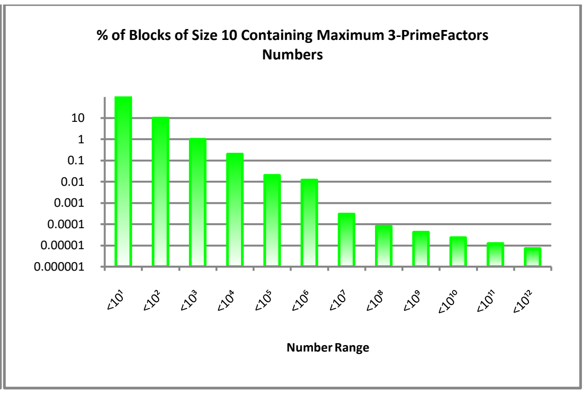
We interchange the roles of block size and range that they had in earlier analysis. In preceding section, for fixed range till 1 trillion, blocks of increasing sizes were analysed. Now fixing block size, we consider increasing ranges for occurrence of peak density of 3-PrimeFactors numbers in them.

The starter block size is 10, with block 0 standing for number range 0 to 9, block 10 standing for range 10 to 19 and higher once in similar pattern.

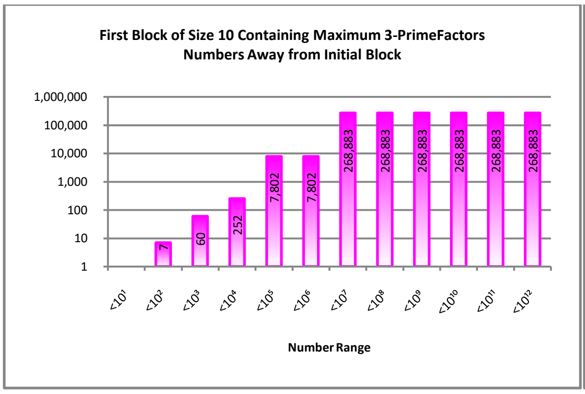
Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10-Size Block*	*First 10-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^1$	1	0	0	1
2	$<10^2$	4	70	70	1
3	$<10^3$	6	600	600	1
4	$<10^4$	7	2,520	4,920	2
5	$<10^5$	8	78,020	82,970	2
6	$<10^6$	8	78,020	771,020	12
7	$<10^7$	9	2,688,830	7,760,590	3
8	$<10^8$	9	2,688,830	97,424,710	8
9	$<10^9$	9	2,688,830	998,273,630	42
10	$<10^{10}$	9	2,688,830	9,988,639,190	232
11	$<10^{11}$	9	2,688,830	99,838,541,090	1,227
12	$<10^{12}$	9	2,688,830	999,892,219,310	7,110

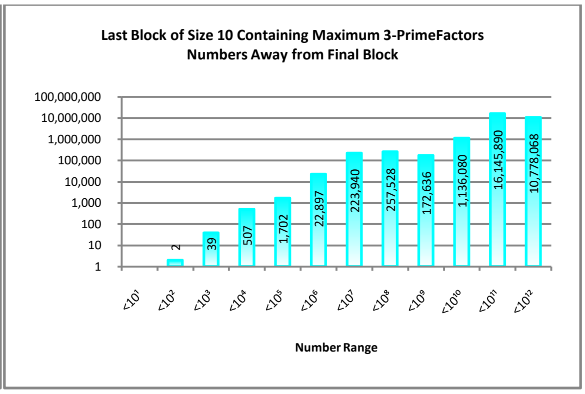
After the range 10^6, highest density of 3-PrimeFactors numbers in blocks of size 10 becomes 90%. Although the number of blocks containing maximum 3-PrimeFactors numbers increase in increasing ranges, their percentage with respect to inspection range goes down.





The block distances of first and last blocks of highest density in respective ranges from respective ends get following values.



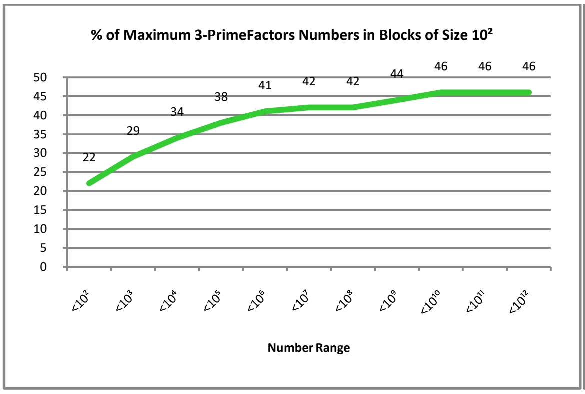


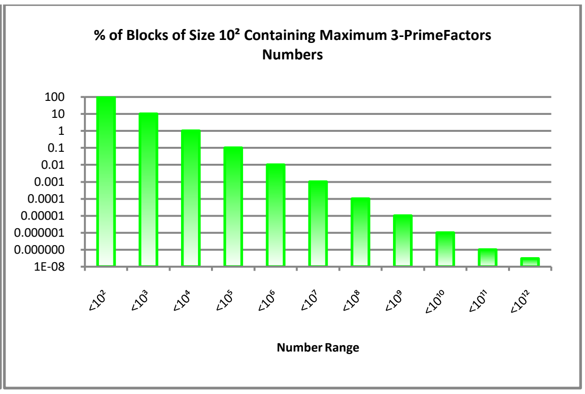
4.2 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^2

Second number is of block size 10^2, i.e., 100. For this, block 0 will represent number range 0 to 99, block 100 will represent range 100 to 199 and similarly for higher values.

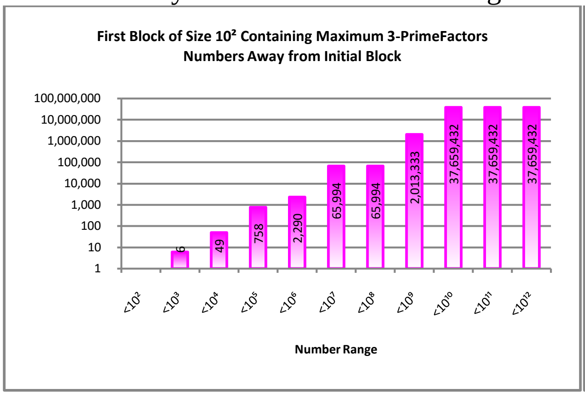
Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^2-Size Block*	*First 10^2-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^2-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^2-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^2$	22	0	0	1
2	$<10^3$	29	600	600	1
3	$<10^4$	34	4,900	4,900	1
4	$<10^5$	38	75,800	75,800	1
5	$<10^6$	41	229,000	229,000	1
6	$<10^7$	42	6,599,400	6,599,400	1
7	$<10^8$	42	6,599,400	6,599,400	1
8	$<10^9$	44	201,333,300	201,333,300	1
9	$<10^{10}$	46	3,765,943,200	3,765,943,200	1
10	$<10^{11}$	46	3,765,943,200	3,765,943,200	1
11	$<10^{12}$	46	3,765,943,200	619,858,239,900	3

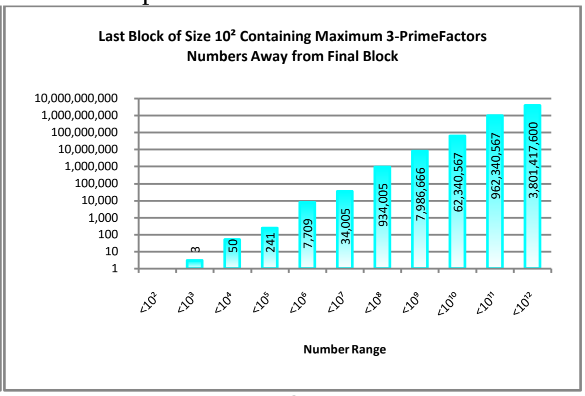
There is a steady stepwise increase in maximum number of 3-PrimeFactors numbers in blocks of size 100 with increasing ranges. Except for the last range, there is unique block of highest density.





Peak density blocks maintain following distances from respective ends.



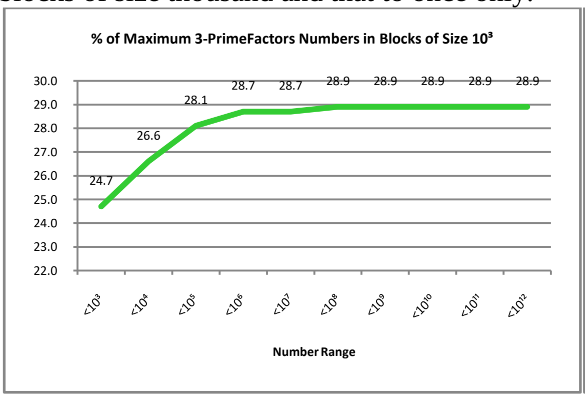


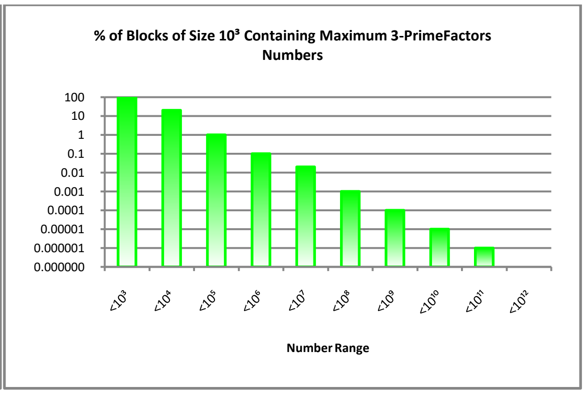
4.3 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^3

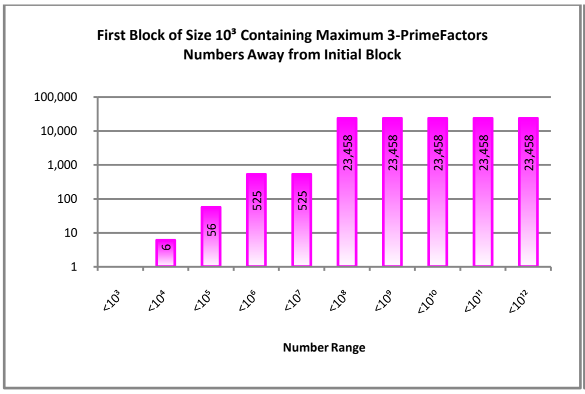
We now consider block size is 10^3, i.e., 1000, where block 0 indicates number range 0 to 999, block 1000 indicates range 1000 to 1999 and so on.

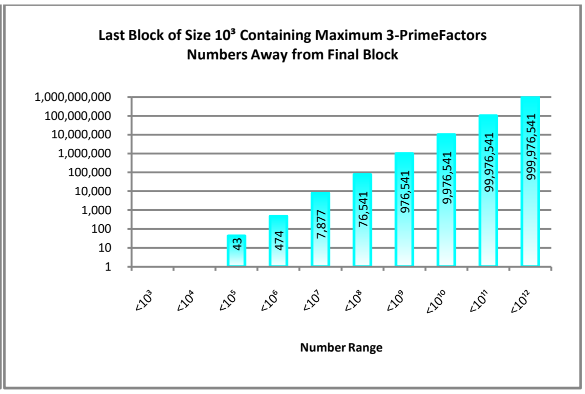
Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^3-Size Block*	*First 10^3-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^3-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^3-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^3$	247	0	0	1
2	$<10^4$	266	6,000	9,000	1
3	$<10^5$	281	56,000	56,000	1
4	$<10^6$	287	525,000	525,000	1
5	$<10^7$	287	525,000	2,122,000	2
6	$<10^8$, $<10^9$, … , $<10^{12}$	289	23,458,000	23,458,000	1

From range 10^8 and onwards till 10^{12}, maximum 28.9% 3-PrimeFactors numbers are found in blocks of size thousand and that to once only.







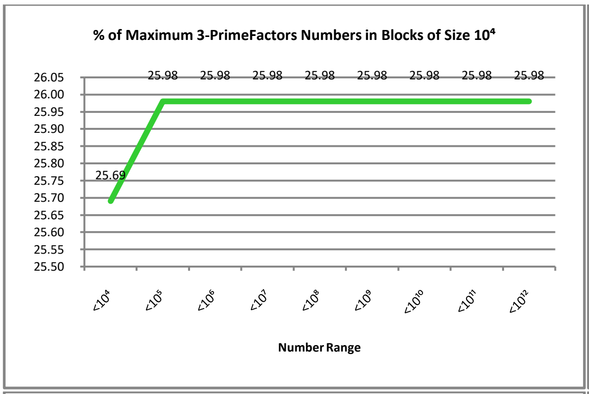


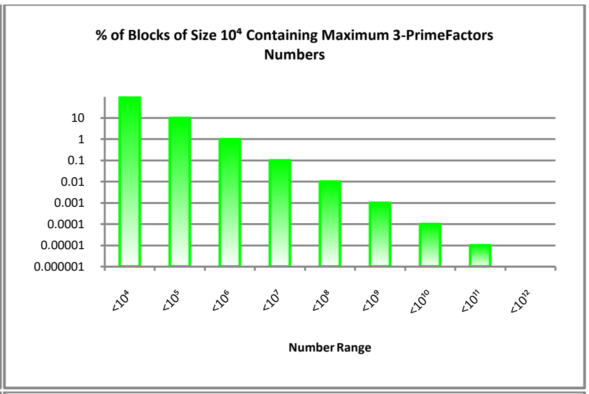
4.4 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^4

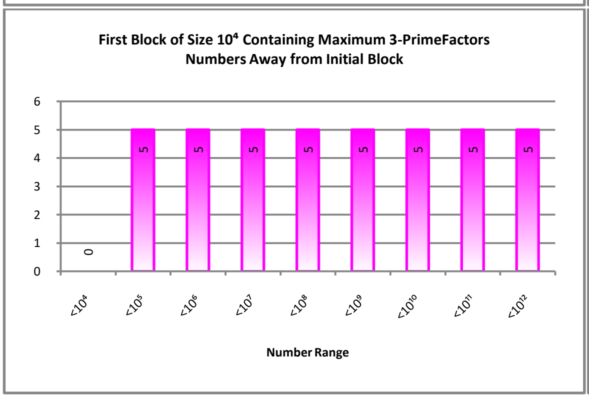
Next is block-size 10^4, i.e., 10000; Block 0 giving number range 0 to 9999, block 10000 giving range 10000 to 19999 and so on.

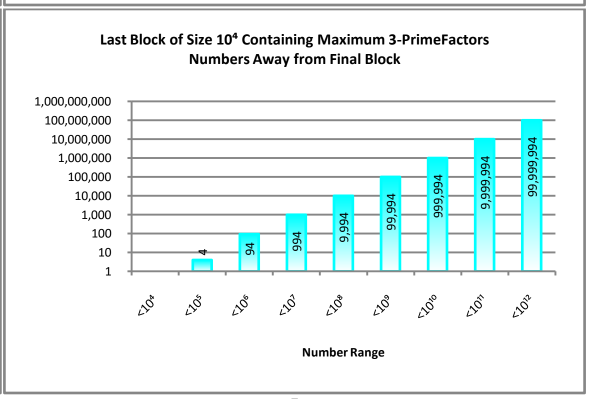
Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^4-Size Block*	*First 10^4-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^4-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^4-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^4$	2,569	0	0	1
2	$<10^5$, $<10^6$, … , $<10^{12}$	2,598	50,000	50,000	1

The maximality percentage is just above a quarter and in each range of study there is only one block with highest density of 3-PrimeFactors numbers in it.









4.5 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^5

Now let's take up block size 10^5, i.e., 100000, block 0 being number range 0 to 99999, block 100000 being range 100000 to 199999 and so on.

Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^5-Size Block*	*First 10^5-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^5-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^5-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^5$, $<10^6$, … , $<10^{12}$	25,556	0	0	1

The in-block 3-PrimeFactors numbers high density value for size 10^5 till trillion continues to maintain value above one fourth.

4.6 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^6

Blocks of size 10^6, i.e., 1000000 are under consideration, where block 0 means number range 0 to 999999, block 1000000 means range 1000000 to 1999999 and so on.

Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^6-Size Block*	*First 10^6-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^6-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^6-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^6$, $<10^7$, … , $<10^{12}$	250,853	0	0	1

The maximality percentage is still above 25% although very marginally.

4.7 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^7

Higher block size is 10^7, i.e., 10000000, block 0 referring to number range 0 to 9999999, block 10000000 referring to range 10000000 to 19999999 and so on.

Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^7-Size Block*	*First 10^7-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^7-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^7-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^7$, $<10^8$, … , $<10^{12}$	2,444,359	0	0	1

For the first time, the percentage of maximum 3-PrimeFactors numbers in blocks has gone below 25.

4.8 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^8

We go ahead with block size is of 10^8, i.e., 100000000. Here block 0 corresponds to number range 0 to 99999999, block 100000000 corresponds to range 100000000 to 199999999 and so on.

Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^8-Size Block*	*First 10^8-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^8-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^8-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^8$, $<10^9$, …, $<10^{12}$	23,727,305	0	0	1

The reduction in the percentage of maximum number of 3-PrimeFactors numbers in blocks of larger sizes is on.

4.9 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^9

Further we take up block size 10^9, i.e., 1000000000, block 0 denoting number range 0 to 999999999, block 1000000000 denoting range 1000000000 to 1999999999 and so on.

Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^9-Size Block*	*First 10^9-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^9-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^9-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^9$, $<10^{10}$, …, $<10^{12}$	229,924,367	0	0	1

The percentage decrease continues.

4.10 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^{10}

Now its turn of block size 10^{10}, i.e., 10000000000, where block 0 signifies number range 0 to 9999999999, block 10000000000 signifies range 10000000000 to 19999999999 and so on.

Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^{10}-Size Block*	*First 10^{10}-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^{10}-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^{10}-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^{10}$, $<10^{11}$, $<10^{12}$	2,227,121,996	0	0	1

Drop in percentage, uniqueness of highest density block and its being very first one, all have become quite consistent.

4.11 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^{11}

Block size 10^{11}, i.e., 100000000000, is considered next, where block 0 happens to be number range 0 to 99999999999, block 100000000000 happens to be range 100000000000 to 199999999999 and so on.

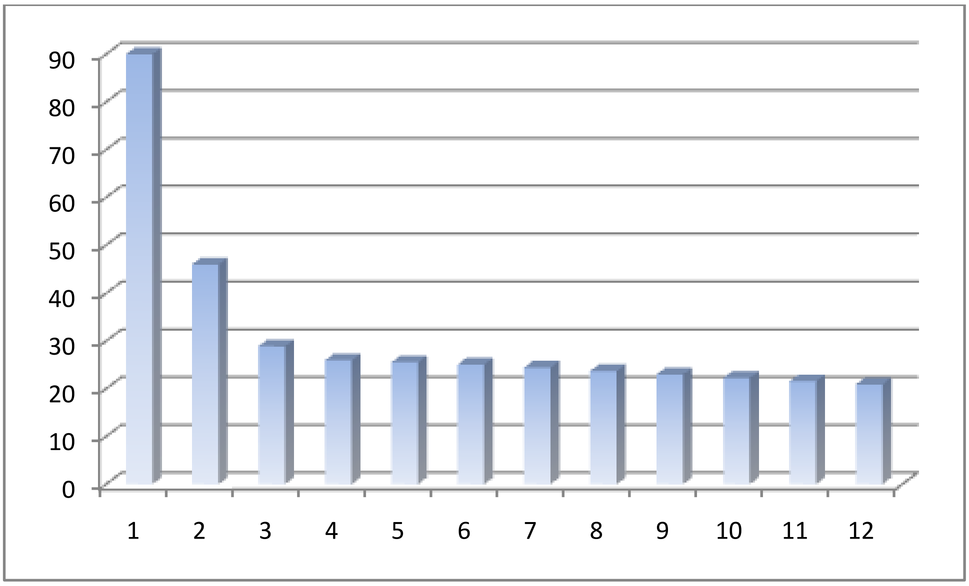
Sr. No.	*Range*	*Maximum 3-PrimeFactors Numbers in 10^{11}-Size Block*	*First 10^{11}-Size Block of Maximum 3-PrimeFactors Numbers*	*Last 10^{11}-Size Block of Maximum 3-PrimeFactors Numbers*	*Number of 10^{11}-Size Blocks with Maximum 3-PrimeFactors Numbers*
1	$<10^{11}$, $<10^{12}$	21,578,747,909	0	0	1

Similar trend carries on.

4.12 Maximum Number of 3-PrimeFactors Numbers in Blocks of Size 10^{12}

As there is only one block of size 10^{12} till that limit itself, the 209214982911 number of 3-PrimeFactors numbers present in it are maximum as well as minimum.

The percentages of in-block highest densities of 3-PrimeFactors numbers for blocks of different sizes in range of 1 trillion go down like below.



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