International Journal of Engineering, Science and Mathematics
Vol. 7Issue 4, April2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

POISSON DISTRIBUTION IN REAL LIFE

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1. **Abstruct :** My present paper deals with poisson distribution in Real life. It also explain what the poisson distribution is. When it can be applied. It also throws light on history of poisson distribution. In this present paper I have applied this distribution to the menace of breast cancer in Chandigarh. This paper also explains how to poisson distribution can be used in exam and also how this distribution in useful in manufacturing problems and is also thrown light on how this is successfully employed in hospitals.

2. Introduction :

Poisson distribution was discovered by the French Mathematician and physicist Simeon Denis Poisson who published it in 1837. Poisson distribution is a limiting case of Binomial distribution under the following conditions.

(1) n, the number of trials is indefinitely large.
(2) The probability of success for each trial is constant and small.
(3) $np = \lambda$ (a constant), finite

2.1 **Key Words :** Probability mass function, poisson variate.

2.2 Definition:

A random variable x is said to follow a poisson distribution if it assumes only non negative values and its probability mass function is given by :

$$P\ (X = x) = \frac{e^{-\lambda}\lambda^{x}}{\lfloor x}; x = 0, 1, 2, 3, \ldots\ldots\ldots\ldots$$

= 0; otherwise

λ is called parameter of the poisson distribution and $\lambda > 0$

Now,

$$\text{Since } \sum_{x=0}^{\infty} P(X = x) = \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{\lfloor x}$$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{\lfloor 1} + \frac{\lambda}{\lfloor 2} + \frac{\lambda^{2}}{\lfloor 3} + \ldots\ldots\right]$$

$$= = e^{-\lambda} \times e^{\lambda} = e^{0} = 1$$

So, this probability mass function is allowed.

2.3 **Definition :** The corresponding distribution function is

$$F(x) = P[X \le x] = \sum_{r=0}^{x} p(r)$$

$$= \sum_{r=0}^{x} \frac{e^{-\lambda}\lambda^{r}}{\lfloor r}$$

$$= e^{-\lambda} \sum_{r=0}^{x} \frac{\lambda^{r}}{\lfloor r} ; x = 0, 1, 2, \ldots\ldots$$

2.4 **Mean of Poisson Distribution :** It is denoted by μ,'

And mean = E (x) = $\sum_{x=0}^{\infty} xp(X = x)$

$$= \sum_{x=0} x \frac{e^{-\lambda}\lambda^{x}}{\lfloor x}$$

$$= \sum_{x=1}^{\infty} x. \frac{e^{-\lambda}.\lambda^{x-1}.\lambda}{x\lfloor x-1}$$

$$= e^{-\lambda}\,\lambda\, e^{\lambda}$$

$$= \lambda. e^{-\lambda}. e^{\lambda}$$

$$= \lambda e^{-\lambda+\lambda}$$

$$= \lambda. e^{0}$$

$$= \lambda. 1$$

$$= \lambda$$

Hean Mean of Poisson distribution is λ.

2.4 **Variance :**

Now,

$$E(x^2) = \sum_{x=0}^{\infty} x^2 p(x = x)$$

$$= \sum_{x=0}^{\infty} x^2. \frac{e^{-\lambda}.\lambda^{x}}{\lfloor x}$$

$$= \sum \left(x^2 - x + x\right) \frac{e^{-\lambda}.\lambda^{x}}{\lfloor x}$$

$$= \sum \frac{\left\{x(x-1) + x\right\} e^{-\lambda}.\lambda^{x}}{\lfloor x}$$

$$= e^{-\lambda} \cdot \sum_{x=2}^{\infty} \frac{x(x-1)}{x(x-1)(x-2)} \lambda^x + e^{-\lambda} + \sum_{x=1}^{\infty} \frac{x\lambda^x}{x\lfloor x-1}$$

$$= e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^2 . \lambda^{x-2}}{\lfloor x-2} + e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda . \lambda^{x-1}}{\lfloor x-1}$$

$$= e^{-\lambda} . \lambda^2 \sum_{x=2}^{\infty} \lambda \frac{x-2}{\lfloor x-2} + e^{-\lambda} . \lambda \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{\lfloor x-1}$$

$$= e^{-\lambda} . \lambda^2 e^{\lambda} + e^{-\lambda} . \lambda . e^{\lambda}$$

$$= \lambda^2 + \lambda$$

$$= \lambda (\lambda + 1)$$

Now, variance of Poisson distribution is given by

$$= \mu_2 = E(x^2) - (E(x))^2$$

$$= \lambda (\lambda + 1) - \lambda^2 = \lambda$$

So, in case of poisson distribution mean and variance are equal.

2.5 Moment Generating Function of the Poisson Distribution :

It in denoted by

$$M_x (t) = \sum_{x=0}^{\infty} e^{tx} . \frac{e^{-\lambda} . \lambda^x}{\lfloor x}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda} (\lambda . e^t)^x}{\lfloor x}$$

$$= e^{-\lambda} \left[1 + \lambda e^t + \frac{\lambda^2 e^{2t}}{\lfloor 2} + \frac{\lambda^3 e^{3t}}{\lfloor 3} ... \right]$$

$$= e^{-\lambda} . e^{\lambda e^t}$$

$$= e^{\lambda (e^t - 1)}$$

2.6 Characteristic Function of Poisson Distribution :

It is denoted by Q_x (t) and is given by

$$Q_x (t) = \sum_{x=0}^{\infty} e^{itx} p(x = x)$$

$$= \sum_{x=0}^{\infty} e^{itx} \frac{e^{-\lambda} . \lambda^x}{\lfloor x}$$

$$= \sum_{x=0}^{\infty} e^{-\lambda} \frac{(\lambda . e^{it})^x}{\lfloor x}$$

$$= e^{-\lambda} \left[1 + \frac{\lambda e^{it}}{\lfloor 1} + \frac{\lambda^2 e^{2it}}{\lfloor 2} + \frac{\lambda^3 e^{3it}}{\lfloor 3} + ... \right]$$

$$= e^{-\lambda} . e^{\lambda\left(e^{it}-1\right)}$$
$$= e^{-\lambda} (e^{it} - 1)$$

2.7 Additive Property of Poisson Variate :

If $X_1, X_2, X_3 \ldots X_n$ independent poisson variables with parameters $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$ is a poisson variable with parameter

$\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_n$

It can be proved as follows :

Product of moment generating function of Xi variables are given by

$$\prod M_{x_i}(t) = M_{x1}(t) \times M_{x2}(t) \times \ldots \times M_{x_n}(t)$$
$$= e^{\lambda 1\left(e^t-1\right)} e^{\lambda 2\left(e^t-1\right)} \ldots e^{\lambda n\left(e^t-1\right)}$$
$$= e^{(\lambda_1+\lambda_2+\ldots+\lambda_n)(e^t-1)}$$

Which is moment generating function of poisson variable with parameter

$\lambda_1 + \lambda_2 + \ldots + \lambda_n = \lambda$

i.e. $M_x(t) = e^{\lambda\left(e^t-1\right)}$

3. Application in Real Life :

Problem 3.1 :

Applied to cancer patients in Chandigarh. A latest report by Tata Memorial Hospital, Mumbai, in collaboration with Post Graduate Institute of Medical Education and Research (PGIMER), Chandigarh and Department of Health and Family Welfare revealed that Female breast cancer patients in Chandigarh are 37.5 per lakh. If a random sample of 1000 female are takent, then we wish to determine the probability that there are atmost four women who are suffering from breast cancer.

Let X denote the number of women suffering from breast cancer.

then $p = \dfrac{37.5}{100000}$

and np $= \dfrac{37.5}{100000} \times 1000 = \dfrac{375}{1000}$

Now, λ = np = 0.375

We can apply poisson distribution

So, P [X = r] $= \dfrac{e^{-\lambda}\lambda^r}{\lfloor r}$ r = 0, 1, 2, 3, ……….

$$= \frac{e^{-0.375}\lambda^r}{\lfloor r}$$

So, required probability is given by,

P [X≤4] = P [X = 0] + P [X = 1] + P [X = 2] + P [X = 3] + P [X = 4]

$$= e^{-0.375}\frac{\lambda^0}{\lfloor 0} + e^{-0.375}\frac{\lambda^1}{\lfloor 1} + e^{-0.375}\frac{\lambda^2}{\lfloor 2} + e^{-0.375}\frac{\lambda^3}{\lfloor 3} + e^{-0.375}\frac{\lambda^4}{\lfloor 4}$$

$$= e^{-0.375}\left[1 + 0.375 + \frac{(0.375)^2}{\lfloor 2} + \frac{(0.375)^3}{\lfloor 3} + \frac{(0.375)^4}{\lfloor 4}\right]$$

$$= 0.0687289 \times \left[1.375 + \frac{0.140625}{2} + \frac{0.052734375}{6} + \frac{0.0197753906}{24}\right]$$

$$= 0.0687289 \times 1.45492558$$

$$= 0.09999$$

Problem 3.2 : In Hospital

In a maternity ward of a reputed private hospital in one way 10 pregnant ladies were operated to deliver baby. The probability that one such operation will result in death of new born baby is 0.1125. Now we want to calculate the probability that at least nine operation will result in success and new born baby will survive.

Let p is the probability that one such operation will result in death of new born baby.

Then, p = 0.01125

n = 10

So, λ = np = 10 × 0.01125

= 0.1125

Let x denote the number of operation that result in failure and so death of new born baby. Probability that at least nine operation will result in success and so the new baby will survive is given by probability that there will be at most one failure.

So,

P (X≤1) = P (X = 0) + p (X = 1)

Now, $P(x = r) = \frac{e^{-0.1125}(0.1125)^r}{\lfloor r}\; r = 0, 1, \ldots$

So, $P(X \le 1) = e^{-0.1125}\frac{(0.1125)^0}{\lfloor 0} + e^{-0.1125}\frac{(0.1125)^1}{\lfloor 1}$

$= e^{-0.1125}[1 + 0.1125]$

= 0.893599 × 1.1125

= 0.99413

So, required probability is 0.99413.

Problem 3.3 : In Exam

Two hundred students of a reputed public school appeared in 10 + 2 CBSE Board Exam in year 2017. Probability that a student will fail in exam is 0.002. Let we find the probability that these are at most two students who fail.

We apply poisson distribution.

Here, p = 0.002

n = 200

So, np $= 200 \times 0.002$

$= 0.4$

So, $\lambda = 0.4$

Let X is number of student who fails

So,

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{\lfloor r}$$

$$= \frac{e^{-0.4}(0.4)^r}{\lfloor r}, \ r = 0, 1, 2, \ldots\ldots$$

So, required probability is given by

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= e^{-0.4}\frac{(0.4)^0}{\lfloor 0} + e^{-0.4}\frac{(0.4)^1}{\lfloor 1} + e^{-0.4}\frac{(0.4)^2}{\lfloor 2}$$

$= e^{-0.4} \times [1 + 0.4 + 0.08]$

$= 0.670320045 \times 1.48$

$= 0.9921$

So, required probability is 0.9921

Problem 3.4 : Manufacturing Problem

A cycle manufacturers knows that 5 out of his 1000 cycles are defective. He sells 100 cycles to retailer assuming him that not more that's two cycles are defective. Then we want to calculate the probability that his statements does that turn out to be reality that is lot has more than two defective cycles.

Let x denotes the number of defective cycles.

Let p is the probability that a cycle is defective

then $p = \frac{5}{1000} = \frac{1}{200}$

n = 100

$$np = \frac{1}{200} \times 100$$

$$= \frac{1}{2} = 0.5$$

Now np is finite. So, we can apply poisson distribution.

Here, $\lambda = np = 0.5$

Let $P(X = r) = e^{-0.5} \frac{(0.5)^r}{\lfloor r}$; $r = 0, 1, 2, \ldots\ldots\ldots$

$$p(X \le 2| = p(X = 0) + p(X = 1) + p(X = 2)$$

$$= e^{-0.5} + e^{-0.5} \frac{(0.5)^1}{\lfloor 1} + e^{-0.5} \frac{(0.5)^2}{\lfloor 2}$$

$$= e^{-0.5} \left[1 + 0.5 + \frac{0.25}{2} \right]$$

$$= e^{-0.5} [1.625]$$

$$= e^{-0.5} \times 1.625$$

$$= 0.6065 \times 1.625$$

$$= 0.9856$$

Required probability is

$$p(X > 2) = 1 - 0.9856 = 0.0144$$

1. **Conclusion :** Poisson distribution occurs when there are events which do not act as outcomes of a definite number of trials, but which occur at random points of time and space. It can also be applied when we have to find number of deaths from a disease such as heart attack, number of suicides reported in a particular city, number of air accidents in some unit of time, number of printing mistakes at each page of the book, number of radioactive particles.

   In such situations, we are often interested in whether the events occur randomly in time or space.

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