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ON INTUITIONISTIC FUZZY MODAL OPERATORS IN MEDICAL DIAGNOSIS

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Abstract

In this paper we propose a new approach for Medical diagnosis with the symptoms of disease using IFS with modal operators. These operators apply to identify the disease of the patient with symptoms in the data. The membership and non-membership values are not always possible upto our satisfaction, but in deterministic(hesitation) part has more important role here, the fact that in decision making, particularly in case of medical diagnosis, there is a fair chance of the existence of a non-zero hesitation part at each moment of evaluation.

Keywords:

Intuitionistic Fuzzy set (IFS), Intuitionistic Fuzzy Relation (IFR), Intuitionistic Fuzzy Modal Operators [IFMO], Intuitionistic Medical Diagnosis(IMD).

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1. Introduction

The field of medicine is one of the best areas of application of fuzzy set theory. In the discrimination analysis, the symptoms are ranked according to the grade of discrimination of each disease by a particular symptom.

In real world, we frequently deal with vague or imprecise information. Information available is sometimes vague, sometimes inexact or sometimes insufficient. Out of several higher order fuzzy sets, intuitionistic fuzzy sets (IFS)[2,3] have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not possible to our satisfaction. Due to some reason, evaluation of non-membership values is not also always possible and consequently there remains a part in deterministic on which hesitation survives. Certainly Fuzzy sets theory is not appropriate to deal with such problem, rather IFS theory is more suitable. Out of several generalizations of fuzzy set theory for various objectives, the notion introduced by Atanassov[2] in defining intuitionistic fuzzy sets is interesting and useful. Fuzzy sets are intuitionistic fuzzy sets but the converse is not necessarily true [2]. In fact there are situations where IFS theory is more appropriate to deal with[5]. Besides, it has been cultured in [6] that vague sets[10] are nothing but IFS.

In the present paper we study Sanchez's method[11] for medical diagnosis using intuitionistic fuzzy modal operators [IFMO] in [12]. The method of intuitionistic medical diagnosis [IMD] involves intuitionistic fuzzy relations[IFR] as defined in [4].

2. Preliminaries

We give here some basic definitions, which are used in our next section.

2.1 Definition

Let a set E be fixed. An intuitionistic fuzzy set (IFS) A in E is an object having the form. $\check{A} = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / \ x \in X\}$ where the function $\mu_A: E \to [0,1]$ *and* $\gamma_A: E \to [0,1]$ define the degree of membership and degree of non-membership resepectively of the element $x \in E$ to the set A. which is a subset of E and for every $x \in E, 0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

The amount $\pi_A(x) = 1 - (\mu_A(x) + \gamma_A(x))$ is called the hesitation part which may cater to either membership value or non-membership value or both.

3. Methodology I

3.1 Definition

If A and B are two IFS of the set E, then

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$$
$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$$

3.2 Definition

An operator over an intuitionistic fuzzy set A (IFS A), given the fixed numbers α, β, $\alpha + \beta \in [0,1]$, as

$$S_{\alpha,\beta}(A) = \{\langle x, \alpha(\mu_A(x) + (1-\beta)\gamma_A(x), \beta(\gamma_A(x) + (1-\alpha)\mu_A(x) + \alpha)\rangle / x \in E\}$$

Where $\alpha + \beta \leq 1$

4. Medical Diagnosis

Suppose S is a set of symptoms, D is a set of Disease and P is a set of patient. Let U_1 be an intuitionistic fuzzy relations[IFR] $U_1(P \to S)$ and U_2 be an intuitionistic fuzzy relations[IFR] $U_2(S \to D)$.

Then

$$U_1 = A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$$
$$U_2 = A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$$
$$U_3 = (U_1 \circ U_2)$$
$$U_4 = S_{\alpha,\beta}(A) = \{\langle x, \alpha(\mu_A(x) + (1-\beta)\gamma_A(x), \beta(\gamma_A(x) + (1-\alpha)\mu_A(x) + \alpha)\rangle / x \in E\}$$

Here $\alpha, \beta, = 0.5$

$$U_5 = \mu_A(x) \wedge \gamma_A(x) = \min\{\mu_A(x), \gamma_A(x)\}$$

4.1 Algorithm

Step1: $U_1(P \to S)$ and $U_2(S \to D)$ are applied in Table 1 and Table 2 , we get the results is named Table 3 (ie, compute $U_3 = (U_1 \circ U_2)$)

Step2: The Table 3 values are applied in the formula U_4, and get the results is named Table 4.

Step3: The Table 4 values applied in U_5 and get the result is named Table 5.

Step4: Finally, we select the maximum value from(Table 5)each row, and then we conclude that the Patients $P_i (i = 1,2,3,4)$ is suffering from the Disease $D_j (j = 1,2,3,4,5)$

4.2 Case Study

Let there be four Patients $P = \{P_1, P_2, P_3, P_4\}$ and the set of symptoms
S = {Headache, Acidity, Burning Eyes, Back pain, Depression}.

Let the set of Disease be D = {Stress, Ulcer, Vision problem, Spinal problems, Blood pressure}

Table1: IFR $U_1(P \to S)$

	Headache	Acidity	Burning Eyes	Back Pain	Depression
P_1	(0.9, 0.1)	(0.7, 0.2)	(0.2, 0.8)	(0.7, 0.2)	(0.2, 0.7)
P_2	(0.0, 0.7)	(0.4, 0.5)	(0.6, 0.2)	(0.2, 0.7)	(0.1, 0.2)
P_3	(0.7, 0.1)	(0.7, 0.1)	(0.0, 0.5)	(0.1, 0.7)	(0.0, 0.6)
P_4	(0.5, 0.1)	(0.4, 0.3)	(0.4, 0.5)	(0.8, 0.2)	(0.3, 0.4)

Table2: IFR $U_2(S \to D)$

	Stress	Ulcer	Vision Problem	Spinal Problems	Blood Pressure
Headache	(0.3, 0.0)	(0.0, 0.6)	(0.2, 0.2)	(0.2, 0.8)	(0.2, 0.8)
Acidity	(0.3, 0.5)	(0.2, 0.6)	(0.5, 0.2)	(0.1, 0.5)	(0.0, 0.7)
Burning Eyes	(0.2, 0.8)	(0.0, 0.8)	(0.1, 0.7)	(0.7, 0.0)	(0.2, 0.8)
Back Pain	(0.7, 0.3)	(0.5, 0.0)	(0.2, 0.6)	(0.1, 0.7)	(0.1, 0.8)
Depression	(0.2, 0.6)	(0.1, 0.8)	(0.2, 0.8)	(0.2, 0.7)	(0.8, 0.1)

Table3: Using Step 1(ie, compute $U_3 = (U_1 \circ U_2)$

	Stress	Ulcer	Vision Problem	Spinal Problems	Blood Pressure
P_1	(0.7, 0.1)	(0.5, 0.2)	(0.5, 0.2)	(0.2, 0.5)	(0.2, 0.7)
P_2	(0.3, 0.5)	(0.2, 0.6)	(0.4, 0.5)	(0.6, 0.2)	(0.2, 0.2)
P_3	(0.3, 0.1)	(0.2, 0.6)	(0.5, 0.2)	(0.2, 0.5)	(0.2, 0.6)
P_4	(0.7, 0.1)	(0.5, 0.2)	(0.4, 0.2)	(0.4, 0.5)	(0.3, 0.4)

Table4: Using Step 2

	Stress	Ulcer	Vision Problem	Spinal Problems	Blood Pressure
P_1	(0.37, 0.47)	(0.30, 0.47)	(0.30, 0.47)	(0.22, 0.55)	(0.27, 0.65)
P_2	(0.27, 0.57)	(0.25, 0.60)	(0.32, 0.60)	(0.35, 0.50)	(0.15, 0.40)
P_3	(0.17, 0.37)	(0.25, 0.60)	(0.30, 0.47)	(0.22, 0.55)	(0.25, 0.60)

P_4	(0.37, 0.47)	(0.30, 0.47)	(0.25, 0.45)	(0.25, 0.32)	(0.25, 0.52)

Table5: Using Step 3 and Step 4

	Stress	Ulcer	Vision Problem	Spinal Problems	Blood Pressure
P_1	**0.37**	0.30	0.30	0.22	0.27
P_2	0.27	0.25	0.32	**0.35**	0.15
P_3	0.17	0.25	**0.30**	0.22	0.25
P_4	**0.37**	0.30	0.25	0.25	0.25

From the results of the disease from Table 5, we see that the maximum value of $P_1$and P_4 is 0.37 and therefore both of them suffer from Stress. The maximum value of P_2 is 0.35. This concludes that P_2 faces Spinal Problem. Whereas the maximum value of P_3 is 0.30 and therefore P_3 faces Vision Problem.

5. Methodology II

5.1 Definition

If A and B are two IFS of the set E, then

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$$
$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$$

5.2 Definition

An operator over an intuitionistic fuzzy set A (IFS A), given the fixed numbers α, β, $\alpha + \beta \in [0,1]$, as

$$T_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\alpha)\gamma_A(x) + \alpha), \alpha(\gamma_A(x) + (1-\beta)\mu_A(x))\rangle / x \in E\}$$

Where $\alpha + \beta \leq 1$

6. Medical Diagnosis

Suppose S is a set of symptoms, D is a set of Disease and P is a set of patient. Let V_1 be an intuitionistic fuzzy relations [IFR] $V_1(P \to S)$ and V_2 be an intuitionistic fuzzy relations[IFR] $V_2(S \to D)$.

Then
$$V_1 = A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$$
$$V_2 = A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$$
$$V_3 = (V_1 \circ V_2)$$
$$V_4 = T_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\alpha)\gamma_A(x) + \alpha), \alpha(\gamma_A(x) + (1-\beta)\mu_A(x))\rangle / x \in E\}$$

Here $\alpha, \beta, = 0.5$

$$V_5 = \mu_A(x) \vee \gamma_A(x) = \max\{\mu_A(x), \gamma_A(x)\}$$

6.1Algorithm

Step1: $V_1(P \to S)$ and $V_2(S \to D)$ are applied in Table 1 and Table 2 , we get the results is named Table 3 (ie, compute $V_3 = (V_1 \circ V_2)$)

Step2: The Table 3 values are applied in the formula V_4, and get the results is named Table 4.

Step3: The Table 4 values applied in V_5 and get the result is named Table 5.

Step4: Finally, we select the maximum value from(Table 5)each row, and then we conclude that the Patients $P_i (i = 1,2,3,4)$ is suffering from the Disease $D_j (j = 1,2,3,4,5)$

6.2. Case Study

Let there be four Patients $P = \{P_1, P_2, P_{3,} P_4\}$ and the set of symptoms
S = {Headache, Acidity, Burning Eyes, Back pain, Depression}.

Let the set of Disease be D = {Stress, Ulcer, Vision problem, Spinal problems, Blood pressure}

Table1: IFR $V_1(P \rightarrow S)$

	Headache	Acidity	Burning Eyes	Back Pain	Depression
P_1	(0.9, 0.1)	(0.7, 0.2)	(0.2, 0.8)	(0.7, 0.2)	(0.2, 0.7)
P_2	(0.0, 0.7)	(0.4, 0.5)	(0.6, 0.2)	(0.2, 0.7)	(0.1, 0.2)
P_3	(0.7, 0.1)	(0.7, 0.1)	(0.0, 0.5)	(0.1, 0.7)	(0.0, 0.6)
P_4	(0.5, 0.1)	(0.4, 0.3)	(0.4, 0.5)	(0.8, 0.2)	(0.3, 0.4)

Table2: IFR $V_2(S \rightarrow D)$

	Stress	Ulcer	Vision Problem	Spinal Problems	Blood Pressure
Headache	(0.3, 0.0)	(0.0, 0.6)	(0.2, 0.2)	(0.2, 0.8)	(0.2, 0.8)
Acidity	(0.3, 0.5)	(0.2, 0.6)	(0.5, 0.2)	(0.1, 0.5)	(0.0, 0.7)
Burning Eyes	(0.2, 0.8)	(0.0, 0.8)	(0.1, 0.7)	(0.7, 0.0)	(0.2, 0.8)
Back Pain	(0.7, 0.3)	(0.5, 0.0)	(0.2, 0.6)	(0.1, 0.7)	(0.1, 0.8)
Depression	(0.2, 0.6)	(0.1, 0.8)	(0.2, 0.8)	(0.2, 0.7)	(0.8, 0.1)

Table3: Using Step 1(ie, compute $V_3 = (V_1 \circ V_2)$)

	Stress	Ulcer	Vision Problem	Spinal Problems	Blood Pressure
P_1	(0.7, 0.1)	(0.5, 0.2)	(0.5, 0.2)	(0.2, 0.5)	(0.2, 0.7)
P_2	(0.3, 0.5)	(0.2, 0.6)	(0.4, 0.5)	(0.6, 0.2)	(0.2, 0.2)
P_3	(0.3, 0.1)	(0.2, 0.6)	(0.5, 0.2)	(0.2, 0.5)	(0.2, 0.6)
P_4	(0.7, 0.1)	(0.5, 0.2)	(0.4, 0.2)	(0.4, 0.5)	(0.3, 0.4)

Table4: Using Step 2

	Stress	Ulcer	Vision Problem	Spinal Problems	Blood Pressure
P_1	(0.62, 0.22)	(0.55, 0.22)	(0.55, 0.22)	(0.47, 0.30)	(0.52, 0.40)
P_2	(0.52, 0.32)	(0.50, 0.35)	(0.57, 0.35)	(0.60, 0.25)	(0.40, 0.15)
P_3	(0.42, 0.12)	(0.50, 0.35)	(0.55, 0.22)	(0.47, 0.30)	(0.50, 0.35)
P_4	(0.62, 0.22)	(0.55, 0.22)	(0.55, 0.20)	(0.57, 0.35)	(0.50, 0.27)

Table5: Using Step 3 and Step 4

	Stress	Ulcer	Vision	Spinal	Blood

			Problem	Problems	Pressure
P_1	**0.62**	0.55	0.55	0.47	0.52
P_2	0.52	0.50	0.57	**0.60**	0.40
P_3	0.42	0.50	**0.55**	0.47	0.50
P_4	**0.62**	0.55	0.55	0.57	0.50

From the results of the disease from Table 5, we see that the maximum value of P_1 and P_4 is 0.62 and therefore both of them suffer from Stress. The maximum value of P_2 is 0.60. This concludes that P_2 faces Spinal Problem. Whereas the maximum value of P_3 is 0.55 and therefore P_3 faces Vision Problem.

7. Conclusion

Though the tabulating the final results are straight forward, i.e P_1 and P_4 suffer from Stress, P_2 faces Spinal problem, P_3 faces Vision problem.

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