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Mathematical Modelling of Land Allocation Problem for Optimal Cropping Plan in Agricultural System Using Hybrid Goal Programming

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Keywords:

Agricultural planning;
Fuzzy Goal Programming;
Goal Programming;
Genetic Algorithm;
Hybrid Goal Programming.

Abstract

The history of human civilization shows that mankind prefers to settle on the places where water and plants are both plenty. Originally human settled near river basins to meet the two basic needs: food and water.

Biodiversity has enabled farming systems to evolve ever since agriculture was first developed some 10,000 years ago in regions across the world. Worldwide there is now a huge diversity of agricultural systems ranging, for example, from rice paddies of Asia, to dry land pastoral systems of Africa, and hill farms in the mountains of South America. However, the Earth's biodiversity is being lost at an alarming rate, putting in jeopardy the sustainability of ecosystem services and agriculture, and their ability to adapt to changing conditions. The conservation and sustainable use of biodiversity is essential for the future of agriculture and humanity.

This paper presents how fuzzy goal programming (FGP) method can be efficiently used for modeling and solving agricultural planning problems for achieving the aspiration levels of production of five seasonal crops cultivated in a planning period by allocating the arable land properly and utilizing the available productive resources efficiently in three different seasons such as the crop-cycles Pre-kharif, Kharif and Rabi successively throughout the planning year.

The land-use planning problem for production of the five principal crops such as Paddy, Wheat, Mustard, Potato, Pulses of the District Bardhaman of West Bengal (W.B.) in India is considered to illustrate the proposed FGP model.

Considering both the national and international scenario of agriculture, very scanty information is available regarding the application of Fuzzy Goal Programming approach to optimize the agricultural production inputs such as fertilizer to lower production cost and avoid environmental pollution. Thus the present study has planned to propose a fruitful solution of the problem for both short-and long-term farm profitability.

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1. Introduction

The history of human civilization has shown that mankind preferred to settle on the places where water and plants both are plenty, and originally human settled near river basin sides to meet the two basic needs food and water. Actually, water is the unique substance for evolution of life on the planet Earth.

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Among all the species, plants are the primitive species, and the major constituent of any kind of plant is water. Further, since irrigation water supply is a complicated issue which involves socio-economic and environmental impacts, various uncertainties are associated with demand patterns and availability of water. Again, it is worthy to mention that industrial emissions have now become a great threat to the environment of the planet Earth. It has now become a great challenge to obtain fresh water for the users. Particularly, agricultural sectors are facing water scarcity challenges along with serious threat from water pollution and climate change issues.

It is thought that domestication of plants went on as far as 7000 B.C. and plant-based food production system through forest gardening, the world's oldest known form of agriculture, was started as far back as 5200 BC. Actually, the development of agriculture made human civilization possible.

The history of agriculture shows that the significant improvements in the agricultural techniques and technology were taken place from the 12th to the 13th century.

In an agricultural planning situation, optimal production of seasonal crops highly depends on proper allocation of land and adequate supply of productive resources for cultivating the crops in different seasons of the planning period.

. During 1970s, linear programming (LP) models for farm planning problems were studied by the active researchers in the field. The extensive study of MP model to agricultural planning problems was surveyed by Nix [1] in 1979.

Since, the agricultural planning problems involve multiplicity of objectives, goal programming (GP) [2] as a prominent tool for multiobjective decision analysis has been widely used to farm management problems. The deep study in this area has been surveyed by Glen [3] in the past. The use of GP to farm planning has also been studied by Pal et al. [4] in the past.

However, the main weakness of GP formulation of real-life problems is that the different resource parameters involved with the problems need to be precisely defined. But, in most of the decision situations, they are found to be imprecise (fuzzy) in nature due to the expert's ambiguous understanding of their nature.

To overcome the above difficulties, fuzzy programming (FP) approach [5] as well as fuzzy goal programming (FGP) [6] approach to crops production planning has been studied [7] in the past.

Although, FGP has been successfully implemented to real-life problems, the difficulty of assigning imprecise aspiration levels to the goals sometimes arises in a highly sensitive inexact decision environment. To overcome the situation, interval- valued GP approach [8], where the parameters are considered in interval forms in decision problems, has been considered. But the use of such an approach to practical decision problems is at an early stage.

Now, it is to be observed that non-linearity in fractional form appears in most of the farm planning decision situations due to consideration of different ratios involved with the problems.

Due to the involvement of non-linearity in most of the cropping plan problems studied in the past, conventional linear approximation approaches [9] are used in which huge computational load and inherent approximation errors occur in the solution search process.

In such a situation, genetic algorithms (GAs) [10,11] based on natural selection and population genetics have appeared as robust computational tools for solving real-world optimization problems to overcome the computational load and decision error that inherently take place due to the use of conventional linearization techniques used in the traditional approaches.

The use of GAs to real-life problems in the framework of FGP has been studied by Pal et al. [12] in the recent past. But, exploration of the potential use of GAs to multiobjective decision making (MODM) problems is yet to be circulated in the literature.

Again, the use of an GA method to GP formulation having both the fuzzy and interval-valued goals is rare in literature.

The potential use of the proposed approach is demonstrated by a case example of the Bardhaman District of West Bengal in India. The model solution is compared with the existing cropping plan. A solution scheme based on GA is introduced to reach a satisfactory decision on the basis of priorities of achieving the objectives of the problem in the decision making environment.

In this article, a GP formulation called hybrid GP (HGP) with incorporation of both the fuzzy and interval valued goals of a cropping plan system is introduced. In the solution process, an GA scheme as a goal satisficer rather than optimizer is adopted to reach a satisfactory decision for optimal production of crops. Now, the general model formulation of the problem is presented in the Section 2.

2. Problem Formulation

The general format of an MODM problem having fuzzy and interval-valued goals can be stated as:

Find X so as to:

satisfy $Z_k(X) = \begin{pmatrix} \gtrsim \\ \lesssim \end{pmatrix} g_k, \ k \in K_1$ (1)

$Z_k(X) : [a_k X + \alpha_k] = [t_k^L, t_k^U], \ k \in K_2$ (2)

subject to

$X \in S = \{X \in R^n \mid f(X) \begin{pmatrix} \geq \\ \leq \end{pmatrix} b, X \geq 0, b \in R^m\},$ (3)

where, X is the vector of decision variables, $Z_k(X), k \in K_1$ represents the k-th fuzzy goal with the aspiration level g_k, a_k is the vector of crisp coefficients and α_k is a constant, respectively. Also it is assumed that t_k^L and t_k^U are the lower- and upper-bounds of the target interval of the k-th objective $Z_k(X), k \in K_2$, where L and U stand for lower- and upper-bounds, respectively, $f(X)$ is a function (linear/nonlinear) representing the constraints set, b is the right-hand-side vector of resources of the system constraints. It is assumed that the feasible region S ($\neq \Phi$) is bounded, and $K_1 \cup K_2 = \{1, 2, ..., K\}$ with $K_1 \cap K_2 = \Phi$.

Now, in the model formulation of the problem, both types of goals are to be transformed into the standard goals.

Here, the fuzzy goals in (1) are first characterized by their membership functions to measure the degree of achievement of the goals. Then, they are transformed into fuzzy goals [7, 11] for achievement of the highest membership value (unity) to the extent possible.

Again, the interval valued goals in (2) are to be transformed into the conventional goals in GP for achievement of the goal values within the specified ranges by minimizing the regrets associated with them.

3. Construction of Membership Goals of Fuzzy Goals

The membership goal expression of the membership function $\mu_k(X)$ defined for the fuzzy goal $Z_k(X) \gtrsim g_k$ appears as:

$$\mu_k(X) : \frac{Z_k(X) - g_{lk}}{g_k - g_{lk}} + d_k^- - d_k^+ = 1, \ k \in K_1 \quad (4)$$

where, g_{lk} and $(g_k - g_{lk})$ represent the lower tolerance limit and tolerance range, respectively, for achievement of the associated k-th fuzzy goal. Also, $d_k^- \geq 0$ and $d_k^+ \geq 0$ are the under- and over- deviational variables, respectively, of the k-th membership goal $\mu_k(X)$.

Similarly, the membership goal expression for the fuzzy goal $Z_k(X) \lesssim g_k$ takes the form:

$$\mu_k(X) : \frac{g_{uk} - Z_k(X)}{g_{uk} - g_k} + d_k^- - d_k^+ = 1, \ k \in K_1 \quad (5)$$

where, g_{uk} and $(g_{uk} - g_k)$ represent the upper tolerance limit and tolerance range, respectively, for achievement of the associated k-th fuzzy goal.

4. Construction of Conventional Goal of Interval-Valued Goal

Using the concept of mid-point rule in interval programming (IvP) approach [8] and introducing under- and over-deviational variables, the two goal expressions from the expressions in (2) can be explicitly obtained as

$$\sum_{j=1}^{n} a_{kj} x_j + \alpha_k + d_{kL}^- - d_{kL}^+ = t_k^L,$$

$$\sum_{j=1}^{n} a_{kj} x_j + \alpha_k + d_{kU}^- - d_{kU}^+ = t_k^U, \ k \in K_2$$

(6)

where, $(d_{kL}^{-}, d_{kU}^{-}), (d_{kL}^{+}, d_{kU}^{+}) \geq 0$ represent lower- and upper-deviational variables associated with the respective goals.

Now, construction of the HGP model for goal achievement on the basis of the needs and desires of the decision maker (DM) in the decision making context is presented in the Section 5.

5. HGP Model Formulation

In the farm planning decision situation, since the DM's objective is to achieve the fuzzy goal values by achieving certain goal levels of interval-valued goals within the specified ranges, consideration of both types of goals in the GP model formulation of the problem with regard to attainment of their aspiration levels is called the hybridization of GP model.

Now, from the optimistic point of view of the DM, *minsum* GP approach to fuzzy goals as well as both *minsum* GP and *minmax* GP approaches [2] as a convex combination of them to the interval-valued goals are simultaneously taken into account to reach a satisfactory decision by minimizing the possible regrets on the basis of the weights of importance of achieving the goals in the decision situation.

The executable HGP model can be presented as:

Find X so as to:

Minimize $Z = (\sum_{k=1}^{K_1} w_k^{-} d_k^{-}) + \{\lambda \sum_{k=K_1+1}^{K} (w_{kL}^{-} d_{kL}^{-} + w_{kU}^{+} d_{kU}^{+}) + (1-\lambda)V\}$

and satisfy the goal expressions in (11.4)-(11.6),

subject to $d_{kL}^{-} + d_{kU}^{+} \leq V,\ k = K_1+1, K_1+2, ..., K$

(7)

where, $V = \max_{k \in K_2}(d_{kL}^{-} + d_{kU}^{+})$, Z represents the goal achievement function, w_k^{-} (> 0), $k = 1,2,\ldots, K_1$, represent the numerical weights associated with the respective under-deviational variables, where w_k^{-} (k=1,2,…, K_1) are determined as [12].

$$w_k^{-} = \begin{cases} \dfrac{1}{(g_k - g_{lk})}, & \text{for } \mu_k \text{ defined in (4)} \\ \dfrac{1}{(g_{uk} - g_k)}, & \text{for } \mu_k \text{ defined in (5)} \end{cases}$$

(8)

Again, $w_{kL}^{-}, w_{kU}^{+} (> 0)$ with $\sum_{k=K_1+1}^{K} (w_{kL}^{-} + w_{kU}^{+}) = 1$, represent the numerical weights associated with the respective deviational variables.

Now, in an MODM situation, it is to be observed that the objective goals often conflict each other for achieving their goal levels in the decision environment. Further, when nonlinearity in an objective or in a system constraint is involved, the computational complexity arises in the decision process. Here, the use of conventional approximation approach involves inherent error and increases computational load.

To overcome the above difficulty, an GA scheme is adopted here to make a proper cropping plan in the decision situation.

In the literature of the GAs, there are a number of schemes for generation of new populations with the use of the different operators: selection, crossover and mutation. Here, the binary coded representation of a candidate solution called chromosome is considered to perform genetic operations in the solution search process.

For the present problem, the fitness function is defined as:

eval (E_v) = $(Z)_v$ = $[(\sum_{k=1}^{K_1} w_k^{-} d_k^{-}) + \{\lambda \sum_{k=K_1+1}^{K} (w_{kL}^{-} d_{kL}^{-} + w_{kU}^{+} d_{kU}^{+}) + (1-\lambda)V\}]_v$, where, Z represents the objective function, and the subscript v refers to the fitness value of the selected v-th chromosome, v =1,2,…,pop_size. The least objective function values of the fittest chromosome can be obtained as:

$$E^* = \min\{eval(E_v) \mid v = 1, 2, ..., pop\_size\},$$

in searching of the best value of the objective.

Now, the HGP model formulation of the proposed problem is presented in the Section 6.

6. HGP Model of the Problem

The decision variables and different types of parameters are defined first in the context of developing the model of the problem.

Definition of Decision Variables and Parameters

- Decision variables:

  l_{cs} = Allocation of land for cultivating the crop c during the season s, c =1,2,…,C ; s =1,2,…,S.

- Parameters:

1) *Fuzzy goal levels*:

   TL_s = Total area of land (in hectares (ha)) currently in use for cultivating the crop c in the season s.

   AP_c = Annual production level (in qtls.) of the crop c.

   EM = Estimated total amount of money (in Rupees (Rs.)) required per annum for supply of the productive resources.

   EMV = Estimated total market value (in Rs.) of all the crops yield during the plan period.

2) *Crisp coefficients*:

   MH_{cs} = Average machine-hours (in hrs.) required for tillage per ha of land for cultivating the crop c during the season s.

   MD_{cs} = Man-days (in days) required per ha of land for cultivating the crop c during the season s.

   WS_{cs} = Amount of water consumed (in inch) per ha of land for cultivating the crop c during the season s.

   EP_{cs} = Estimated production of the crop c per ha of land cultivated during the season s.

   CP_{cs} = Average cost for the purchase of fertilizers, seeds and other different farm related materials per ha of land cultivated for the crop c during the season s.

   MP_{cs} = Market price (Rs. / qtl.) at the time of harvesting the crop c cultivated during the season s.

3) *Crisp target levels:*

   PR_{ij} = Ratio of annual production of the i-th and j-th crop, (i, j = 1, 2, ...,C ; $i \neq j$).

   pr_{ij} = Ratio of annual profits obtained from the i-th and the j-th crops, (i, j=1,2,...,C ; $i \neq j$).

4) *Interval-valued production resources:*

   $[MH_s^L, MH_s^U]$ = Target interval specified for total machine-hours (in hours (hrs)) required during the season s.

   $[MD_s^L, MD_s^U]$ = Target interval specified for total man-days (in days) required during the season s.

   $[WS_s^L, WS_s^U]$ = Target interval specified for total water supply (in inch) required during the season s.

Description of Goals and Constraints

1) Fuzzy goals :

For the defined variables and different types of parameters involved with the problem, the algebraic structures of the fuzzy goals appear as follows.

(i) Land utilization goal

The fuzzy goal expression for utilization of total cultivable land in different seasons appears as

$$\sum_{c=1}^{C} x_{cs} \lesssim TL_s, \quad s = 1,2,...,S$$

(ii) Production achievement goal

To meet the increasing demand of agricultural products in society, the fuzzy production achievement goal for each crop cultivated in different seasons appears as:

$$\sum_{s=1}^{S} EP_{cs}.x_{cs} \gtrsim AP_c, \quad c = 1,2,...,C$$

(iii) Cash requirement goal

An estimated amount of money (in Rs.) is essentially involved for the purpose of acquiring the productive resources.

The fuzzy goal takes the form

$$\sum_{s=1}^{S}\sum_{c=1}^{C} CP_{cs}.x_{cs} \lesssim EM$$

(iv) Profit achievement goal

A minimum level of profit from the farm is highly expected by the farm manager.

The fuzzy profit goal appears as

$$\sum_{s=1}^{S}\sum_{c=1}^{C} (MP_{cs}.EP_{cs} - CP_{cs})x_{cs} \gtrsim EMV$$

2) Interval-valued productive resource goals :

The interval-valued productive resource goals appear as follows.

(i) Machine-hour goal

An estimated number of machine-hours (in hrs.) within an interval are required to till the land in the season *s*.

The interval-valued resource goal appear as

$$\sum_{c=1}^{C} MH_{cs}.x_{cs} = [MH_s^L, MH_s^U], s = 1,2,...,S$$

(ii) Manpower goal

To avoid the uncertainty of labourers and involvement of extra money for hiring them at the peak time, a certain number of labourers in an estimated interval should be employed during the period.

The interval-valued goal appear as

$$\sum_{c=1}^{C} MD_{cs}.x_{cs} = [MD_s^L, MD_s^U], s = 1,2,...,S$$

(iii) Water supply goal

Water is an essential input for yielding the crops. To meet the target levels of production of all the seasonal crops, water supply within a certain target interval must be provided during any season *s*.

The interval valued water supply goal appears as:

$$\sum_{c=1}^{C} WS_{cs}.x_{cs} = [WS_s^L, WS_s^U], s = 1,2,...,S$$

3) Crisp constraints :

(i) Production ratio constraint

It is to be mentioned that the few major crops serve almost the same purpose in terms of their consumption. So certain ratios between the total productions of two major crops sowed have to be maintained.

The ratio constraints appear as:

$$\sum_{s=1}^{S} EP_{is}.x_{is} / \sum_{s=1}^{S} EP_{js}.x_{js} = PR_{ij};\ i, j = 1,2,...,C, i \neq j,$$

(ii) Profit ratio constraint

From the socio-economic point of view, beyond the meeting of demand of food grains in society, the attention for cultivation of the profitable crops need be paid in the planning horizon. Here, certain ratios of crops production are to be maintained from the view point of making profit from the farm. The profit ratio constraint appear as:

$$\sum_{s=1}^{S} (MP_{is}.EP_{is} - CP_{cs}).x_{is} / \sum_{s=1}^{S} (M.P_{js}.EP_{js} - CP_{cs}).x_{js} = pr_{ij};\ i, j = 1,2,...,C, i \neq j,$$

7. Solutions and Recommendations

An Illustrative Example: A Case Study

The land-use planning problem for production of the principal crops of the District Bardhaman of West Bengal (W.B.) in India is considered to illustrate the proposed FGP model. Now, the three seasonal crop-cycles: Pre-kharif, Kharif and Rabi successively appear in W.B. during a planning year, and they designate the time periods for crop production during summer, rainy and winter seasons, respectively. The data were collected from different sources recorded District Statistical Hand Book, 2008 [13]; Economic Review; Basak, 2010) [14].

The decision variables and different types of model data are summarized in the Tables 1–4.

Table 1. Summary of decision variables and arable land for crop cultivation

Season(s)	**Prekharif**			**Kharif**	**Rabi**				
Crop (c)	Jute	Sugarcane	Aus	Aman	Boro	Wheat	Mustard	Potato	Pulses
Variable (x_{cs})	l_{11}	l_{21}	l_{31}	l_{42}	l_{53}	l_{63}	l_{73}	l_{83}	l_{93}

Table 2. Data description of the aspired goal levels and tolerance limits

Goal	Aspiration Level	Tolerance Limit	
		Lower	Upper
1. Water supply (in MCM)			
(i)Canal –water	1645.50		1840.00
(ii) Groundwater	1440.45		1620. 10
2. a) Man-hours (in ’000 hrs) :			
(i) Pre-Kharif season	2360.00	11103.30	----
(ii) Kharif season	49280.00	132866.70	----
(iii) Rabi season	01450.40	92313.00	----
3. Production (in ’000 metric ton) :			
(a) Jute	0.80	8.30	----

(b) Rice	965.00	468.80	----
(c) Wheat	2.80	7.00	----
(d) Mustard	2.45	0.20	----
(e) Potato	335.80	22.30	----
(f) Rabi pulse	.50	.20	----
4. Budget allocation (in Rupee (₹ `) Lakh)	28800.40	----	154550.30
5. Profit (in ₹ ` Lakh)	77615.80	159850.50	----

Table 3

Data Description of the Aspiration Levels of Goals and Their Tolerance Limits

Goal	Aspiration Level	Tolerance Limit	
		Lower	Upper
1. Land utilization ('000 hectares) :			
(i) Pre-kharif season	458.20	----	510.50
(ii) Kharif season	458.20	----	510.50
(iii) Rabi season	458.20	----	510.50
2. Production ('000 metric ton) :			
(a) Jute	50.80	28.30	----
(b) Sugarcane	39.00	18.00	----
(c) Rice	1965.00	1468.80	----
(d) Wheat	12.80	7.00	----
(e) Mustard	42.45	30.20	----
(f) Potato	1335.80	922.30	----
(g) Rabi pulse	4.50	2.20	
Fertilizer requirement (in metric ton) :			
(a)Nitrogen	85.00	66.20	----
(b) Phosphate	65.00	54.30	----
(c) Potash	45.00	38.00	----

3. Cash expenditure (Rs. Lac.)	128800.40	-----	154550.30
4. Profit (Rs. Lac.)	177615.80	159850.50	----

Table 4. Data description of productive resource utilization, estimated cash and market value

Crops	MDS		W_{cs}	MH_{cs}	F_f			PA	EC	MV
	90				N	P	K			
Jute	90		5020	540	40	20	20	2555	17298.00	1600
Sugarcane	123		510	123	60	20	20	78666.60	30887.50	1300
Aus	60		8635	360	40	20	20	2954	14332.80	1200
Aman	60		12700	360	40	20	20	2720	12850.50	1380
Boro	60		17870	360	100	50	50	3335	23722.60	1300
Wheat	39		3820	234	100	50	50	2720	11120.50	1179.00
Mustard	30		2550	180	80	40	40	860	8400.50	2715.50
Potato	70		4570	420	150	75	75	24530	37315.00	355.00
Pulses	15		2550	90	20	50	20	540	4943.00	4475.00

Note: W_{cs} = Water requirement (in Cubic Meter (CM)/ha)), MH_{cs} = Man hours (hrs /ha), F_f = fertilizer (kg/ha): N=Nitrogen, P = Phosphate, K = Potash; PA = production achievement (kg/ha), EC = estimated cash (Rs./ ha), MV = market value (₹ / qtl).

Table 5. Data Description of the Interval-valued Production Resources, Production Ratio and Profit Ratio

Interval-valued Resources [LB, UB]		Season	
	Prekharif [ ]	Kharif [ ]	Rabi [ ]
a) Machine-hours (hrs.):	[60704, 72845]	[36422, 40469]	[114238, 117227]
b) Man-days (days) :	[13774, 13973]	[6702, 7176]	[14692, 15352]
c) Water supply (inch) :	[4037,4064]	[5026, 5979]	[95.86, 98.54]
d) Production ratio (Rice –Wheat)	…	7	
e) Profit ratio (Jute-Aus)	…	4	

Note: In [LB, UB]: LB= lower bound and UB= upper bound.

Now, using the data of Tables 1-5, the membership goals of the defined fuzzy goals and the conventional goals of the defined interval-valued goals can be constructed by using the expressions in (4), (5) and (6), respectively.

Here, it is to be noted that the three consecutive seasons are required for yielding the crop sugarcane and all the other crops are single-season based.

Now, the goals are described as follows.

Now, using the data Tables 1- 4, the membership functions of the defined fuzzy goals can be constructed by using the expressions in (4) and (5).

Farm Management Goals:

- **Land allocation goals**
- $\mu_1 : 9.80 - 0.019(x_{11} + x_{21} + x_{31}) + d_1^- - d_1^+ = 1$ (Pre-kharif)
- $\mu_2 : 9.80 - 0.019(x_{21} + x_{42}) + d_2^- - d_2^+ = 1$ (Kharif)
- $\mu_3 : 9.80 - 0.019(x_{21} + x_{53} + x_{63} + x_{73} + x_{83} + x_{93}) + d_3^- - d_3^+ = 1$ (Rabi)

- **Fertilizer requirement goals**

$$\mu_4 : 0.002\, l_{11} + 0.002 l_{21} + 0.002 l_{32} + 0.005 l_{43} + 0.0052 l_{53} + 0.004 l_{63} + 0.008 l_{73} + 0.001 l_{83} - 3.497 + d_6^- - d_6^+ = 1 \quad \text{(N)}$$

$$\mu_5 : 0.0017 l_{11} + 0.0017 l_{21} + 0.0017 l_{32} + 0.004 l_{43} + 0.004 l_{53} + 0.003 l_{63} + 0.006 l_{73} + 0.004 l_{83} - 4.567 + d_7^- - d_7^+ = 1 \quad \text{(P)}$$

$$\mu_6 : 0.007 l_{11} + 0.004 l_{21} + 0.004 l_{32} + 0.009 l_{43} + 0.009 l_{53} + 0.007 l_{63} + 0.014 l_{73} + 0.009 l_{83} - 7.269 + d_8^- - d_8^+ = 1 \quad \text{(K)}$$

- **Cash expenditure goal**

$$\mu_7 : 6.00 - (0.0067 l_{11} + 0.0055 l_{21} + 0.0049 l_{32} + 0.0092 l_{43} + 0.0043 l_{53} + 0.0033 l_{63} + 0.0144 l_{73} + 0.0019 l_{83}) + d_9^- - d_9^+ = 1$$

- **Production achievement goals**

$\mu_8 : 0.0059 l_{21} + 0.0054 l_{32} + 0.0067 l_{43} + d_{10}^- - d_{10}^+ = 1$ (Rice)

$\mu_9 : 3.93 x_{21} - 5.9 + d_7^- - d_7^+ = 1$ (Sugarcane)

$\mu_{11} : 0..0115 l_{11} - 1.227 + d_{11}^- - d_{11}^+ = 1$ (Jute)

$\mu_{12} : 0.0456 l_{53} - 0.8955 + d_{12}^- - d_{12}^+ = 1$ (Wheat)

$\mu_{13} : 0.0745 l_{63} - 2.631 + d_{13}^- - d_{13}^+ = 1$ (Mustard)

$\mu_{14} : 0.0595 l_{73} - 2.2375 + d_{14}^- - d_{14}^+ = 1$ (Potato)

$\mu_{15} : 0.2304 l_{83} - 2.2375 + d_{15}^- - d_{15}^+ = 1$ (Pulses)

- **Profit achievement goal**

$$\mu_{16} : 0.0133 l_{11} + 0.0119 l_{21} + 0.0139 l_{32} + 0.0111 l_{43} + 0.0018 l_{53} + 0.0082 l_{63} + 0.028 l_{73} + 0.0105 l_{83} - 8.9999 + d_{16}^- - d_{16}^+ = 1$$

Standard Goals of Interval-Valued Goals

1) Machine-hour goals

$$204 l_{11} + 510 l_{21} + 425 l_{31} + d_{13L}^- - d_{13L}^+ = 60704,$$
$$204 l_{11} + 510 l_{21} + 425 l_{31} + d_{13U}^- - d_{13U}^+ = 72845,$$

(Pre-kharif)

$$204 l_{42} + d_{14L}^- - d_{14L}^+ = 36422,$$
$$204 l_{42} + d_{14U}^- - d_{14U}^+ = 40469,$$

(Kharif)

$$816 l_{53} + 204 l_{63} + 102 l_{73} + 340 l_{83} + 150 l_{93} + d_{15L}^- - d_{15L}^+ = 114238,$$
$$816 l_{53} + 204 l_{63} + 102 l_{73} + 340 l_{83} + 150 l_{93} + d_{15U}^- - d_{15U}^+ = 117227,$$

(Rabi)

2) Manpower goals

$$90\,l_{11} + 123l_{21} + 60l_{31} + d^-_{16L} - d^+_{16L} = 13774,$$
$$90\,l_{11} + 123l_{21} + 60l_{31} + d^-_{16U} - d^+_{16U} = 13973,$$

(Pre-kharif)

$$123\,l_{21} + 60l_{42} + d^-_{17L} - d^+_{17L} = 6702,$$
$$123l_{21} + 60l_{42} + d^-_{17U} - d^+_{17U} = 7176,$$

(Kharif)

$$123\,l_{21} + 60l_{53} + 39l_{63} + 30l_{73} + 70l_{83} + 15l_{93} + d^-_{18L} - d^+_{18L} = 14692,$$
$$123\,l_{21} + 60l_{53} + 39l_{63} + 30l_{73} + 70l_{83} + 15l_{93} + d^-_{18U} - d^+_{18U} = 15352,$$

(Rabi)

3) Water supply goal

$$20l_{11} + 60l_{21} + 34l_{31} + d^-_{19L} - d^+_{19L} = 4037,$$
$$20l_{11} + 60l_{21} + 34l_{31} + d^-_{19U} - d^+_{19U} = 4064,$$

(Pre-kharif)

$$60l_{21} + 50l_{42} + d^-_{20L} - d^+_{20L} = 5026,$$
$$60l_{21} + 50l_{42} + d^-_{20U} - d^+_{20U} = 5979,$$

(Kharif)

$$60\,l_{21} + 70l_{53} + 15l_{63} + 10l_{73} + 18l_{83} + 10l_{93} + d^-_{21L} - d^+_{21L} = 95.86,$$
$$60\,l_{21} + 70l_{53} + 15l_{63} + 10l_{73} + 18l_{83} + 10l_{93} + d^-_{21U} - d^+_{21U} = 98.54,$$

(Rabi)

3 Crisp Constraints

1) Production-ratio constraint

The ratio of the two crops, rice and wheat are considered here as the major agricultural products. The production ratio constraint appears as:

$$(2.187l_{31} + 2.285l_{42} + 3.336l_{53})/(1.882l_{63}) = 7,$$

2) Profit-ratio constraint

From the view point of making profit by exporting certain products the profit ratio of Jute and Aus-paddy in the pre-kharif season is taken into account here.

The profit-ratio constraint takes the form

$$(259.84\,l_{11})/(99.06\,l_{21} + 969.45l_{31}) = 4$$

Now, the HGP model of the problem appears as

Find $\{x_{cs} \mid c = 1,2,...,9;\ s = 1,2,3\}$ so as to

$$\text{Minimize } Z = (0.027d^-_1 + 0.027d^-_2 + 0.027d^-_3 + 0.015d^-_4 + 0.06d^-_5 + 0.088d^-_6 + 0.05d^-_7 + 0.14d^-_8$$
$$+ 0.03d^-_9 + 0.26d^-_{10} + 0.0012d^-_{11} + 0.0001d^-_{12}) + \lambda\{\sum_{k=13}^{21}(w^-_{KL}d^-_{KL} + w^+_{KU}d^+_{KU})\}$$
$$+ (1-\lambda)V,$$

and satisfy the goal expressions, subject to constraints and

$(d^-_{KL} + d^+_{KU}) \le V, k = 13,\ldots,21.$

Now, assigning the equal weights for minimizing the possible regrets to achieve the interval-valued goals within their specified intervals, taking $w^-_{KL} = w^+_{KU} = \dfrac{1}{18}$ and λ=0.5, the proposed GA scheme is employed to solve the problem in (11.18).

The objective function of the model appears as the fitness function in the solution search process.

The following genetic parameter values are introduced in the search process:

- Probability of crossover p_c =0.8
- Probability of mutation p_m = 0.08
- Population size =100
- Chromosome length = 30

The GA based program is designed in Programming Language C. The execution is done in an Intel Pentium IV PC with 2.66 GHz CPS and 1 GB RAM. The optimal solution is reached after 200 generations. The achieved result of the cropping plan is presented in the following Table.

Now, employing the GA scheme, the achievement function Z appears the fitness function in the process of solving the problem. The number of generations = 300 is initially taken into account to conduct the experiment.

In the genetic search process, the following parameter values are introduced.

• probability of crossover Pc = 0.8
• probability of mutation Pm = 0.08
• population size = 100
• chromosome length = 150.

The GA-based programme is designed in Programming Language C++. The execution is done in an Intel Pentium IV with 2.66 GHz clock-pulse and 1 GB RAM. The optimal solution is reached at 200 generations.
The model solutions for goal achievement are presented in the Table 6 and Table 7.

Table 6. Land Allocation and Production of Crops under the Proposed Model

Crop (c)	**Jute**	**Sugarcane**	**Rice**	**Wheat**	**Mustard**	**Potato**	**Pulses**
Land allocation	123.37	1.9	334.72	55.05	87.15	5.5	49.95
Production	332.27	149.46	896.38	117.34	78.56	147.50	41.55

The total profit obtained under the proposed cropping plan is **Rs. 110299.47 Lac.**

The land allocation and production structure of the existing cropping plan (2005- 2006) of the District is presented in the Table 11.6.

Table7. Land Allocation and Production of Crops recorded in the year 2005-2006

Crop (c)	**Jute**	**Sugarcane**	**Rice**	**Wheat**	**Mustard**	**Potato**	**Pulses**
Land allocation	120.2	1.50	265.4	47.10	79.20	5.50	46.40
Production	325.15	118.0	732.4	100.4	71.40	147.50	38.60

The total profit obtained under the existing cropping plan is **Rs. 95803.0117 Lac.**

A comparison of the model solution with the results in Table 11.6 shows that the solution under the proposed approach is better and a satisfactory decision for the optimal cropping plan is obtained here in the decision making environment.

The diagrammatic representation of land allocation and crops production under the existing system as well as with the use of the proposed GA approach are given in the Figures1 and 2, respectively.

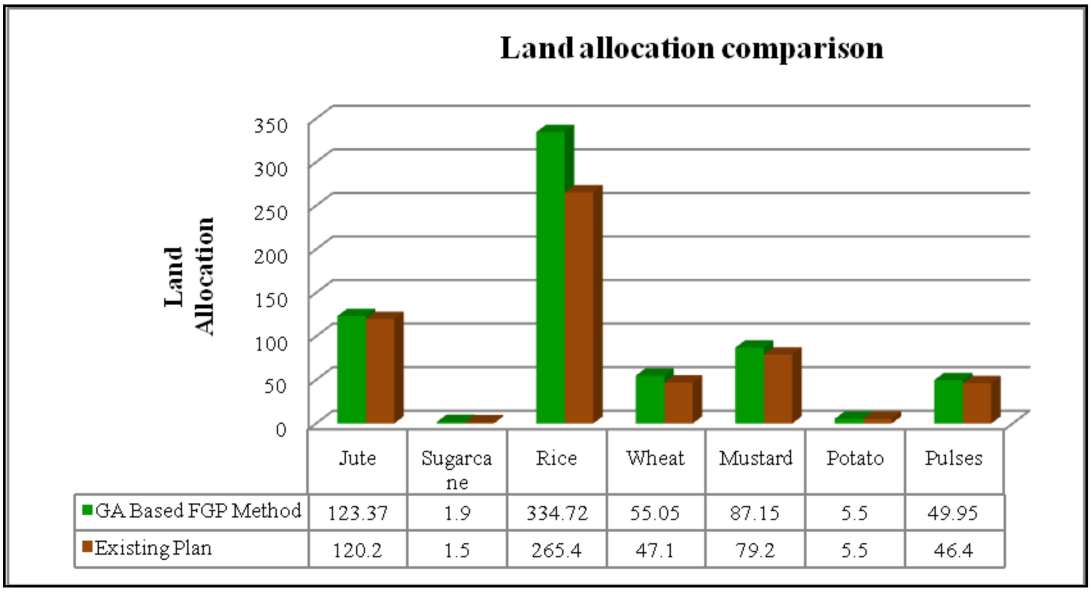

Figure 1: Land allocation comparison

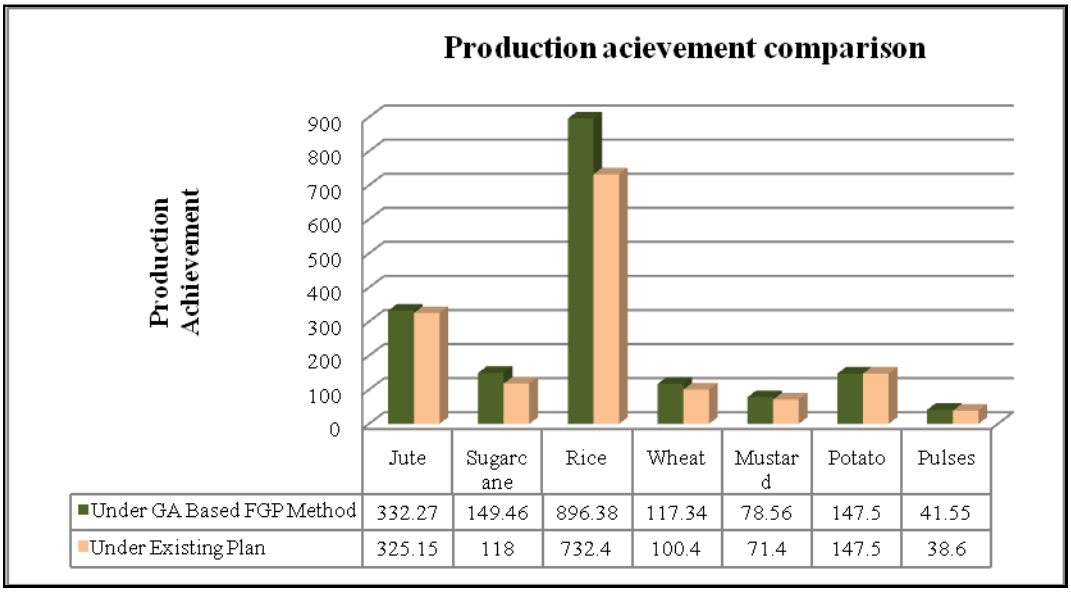

Figure 2: Crop Production comparison

The following Figure 3 is used to represent diagrammatically the profit achievements under the existing plan and proposed GA approach.

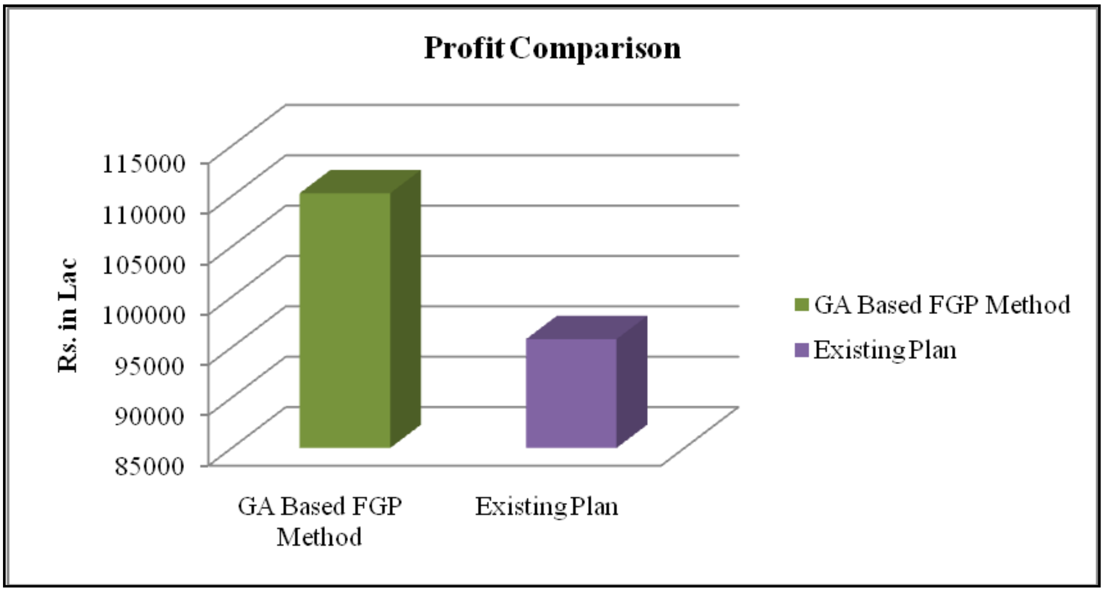

Figure 3: Pictorial Representation of profit comparison

The result reflects that the proposed GA approach provides a more satisfactory solution in the decision making environment.

7 Conclusion

The land-use planning approach outlined in this article provides a good basis for analyzing the DM's perception on the use of both FGP and interval-valued GP in the framework of the production planning model in farm management. Here, it is worth mentioning that achievement of all the aspired goal levels may not always be possible due to the limitation of productive resources, but the best possible decision can be obtained here under the proposed model.

Again, in different agricultural planning horizons, different regional based environmental constraints generally occur which can easily be incorporated under the framework of the proposed planning model.

Also in future studies, the proposed approach can be extended to solve agricultural production planning problems with probabilistic data in an uncertain decision environment.

However, the solution method presented in this article may open up many new vistas into the way of making proper decision in the current complex multiobjective planning problems in farm management.

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