International Journal of Engineering, Science and Mathematics
Vol. 7Issue 4, April 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

F-GS-OPEN , F-GS CLOSED AND F-GS CONTINUOUS MAPPINGS IN FINE TOPOLOGICAL SPACES.

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Abstract

In 1990 , S.P.Arya et al have introduced the concept of generalized semiclosed sets to characterize the s-normality. While in 1987 , P.Bhattacharyya et al have introduced the notion of semi generalized closed sets in topological spaces. Since then many works have been developed in the fields of generalized open and generalized closed sets .Powar P. L. and Rajak K.have introduced fine-topological space which is a special case of generalized topological space. Aim of this paper is we introduced and studies F-gs-Open Mappings , F-gs Closed Mappings and F-gs Continuos Mappings in fine topological spaces

Keywords:

F-gs closed set ,
F-gs-Open Mappings ,
F-gs Closed Mappings and
F-gs Continuous Mappings

AMS Mathematical Subject classification (2010): 54C08, 54C10

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1. Introduction

In 1987, Bhattacharyya and Lahiri[2]used semi open sets to define and investigate the notion of semi generalized closed sets . Later , in 1990 S.P.Arya et al [1] have introduced the concepts of generalized semi closed sets using semi closure to characterize the s-normality axiom . Now , we found the various papers in the field of generalized open sets and generalized closed sets. Powar P. L. [11]and Rajak K.have introduced fine-topological space which is a special case of generalized topological space. Aim of this paper is we introduced Fine gs closed sets in fine topological spaces

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and also we introduced and studies the concepts of F-gs-Open Mappings , F-gs Closed Mappings and F-gs Continuous Mappings in fine topological spaces

2. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

Definition:2.1 [10,11]

Let (X, τ) be a topological space we define, $\tau(A\alpha)=\tau_\alpha=\{G\alpha(\neq X):G\alpha\cap A\alpha \neq \varphi$, for $A\alpha\in\tau$ and $A\alpha \neq X, \varphi$ for some $\alpha \in J$, where J is the index set$\}$. Now, define $\tau_f = \{\varphi,X\}\cup\tau_\alpha\}$. The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine topological space X and generated by the topology τ on X.

The element of τf are called fine open sets in (X, τ, τ_f) and the complement of fine open set is called fine closed sets and it is denoted by τ_f^c

Example :2.2. [10,11]

Consider a topological space X = {p, q, r} with the topology

$\tau =\{X, \varphi ,\{p\}\} \cong\{X, \varphi ,A\alpha\}$ where $A_\alpha = \{p\}$. In view of Definition 2.1

we have, $\tau_\alpha=\tau(A_\alpha)= \tau \{p\} =\{\{p\}, \{p, q\},\{p, r\}\}$
then the fine collection is $\tau_f =\{ \varphi,X\}\cup\{\tau_\alpha\} =\{X, \varphi ,\{p\}, \{p, q\}, \{p, r\}\}$.
We quote some important properties of fine topological spaces.

Lemma: 2.3. [10,11]

Let (X, τ, τ_f) be a fine space then arbitrary union of fine open set in X is fine-open in X.

Lemma: 2.4. [10,11]

The intersection of two fine-open sets need not be a fine-open set as the following example shows.

Example:2.5 [10,11]

Let X = {p, q, r} be a topological space with the topology
$\tau = \{X, \varphi, \{p\},\{q\}, \{p, q\}\}$, $\tau_f = \{X, \varphi ,\{p\}, \{q\}, \{p, q\}, \{q, r\}, \{p, r\}\}$. It is easy to see that, the above collection τf is not a topology. Since, $\{p, r\}\cap\{q, r\}=\{r\}\notin\tau_f$. Hence, the collection of fine open sets in a fine space X does not form a topology on X, but it is a generalized topology on X.

Remark :2.6 [10,11]

In view of Definition 2.1 of generalized topological space and above Lemmas 2.3 and 2.4 it is apparent that (X, τ, τ_f) is a special case of generalized topological space. It may be noted specifically that the topological space plays a key role while defining the fine space as it is based on the topology of X but there is no topology in the back of generalized topological space.

Definition:2.7 [10,11]

A subset A of a Fine space (X, τ, τ_f) is called Fine semi-open if $A \subset Fcl(Fint(A))$.
The complement of Fine semi-open set is called Fine semi-closed.

The Fine semi-closure of a subset A of Fine space X, denoted by Fscl(A), is defined to be the intersection of all Fine semi-closed sets containing A in Fine space X.

Definition 2.8 [10]

Let A be a subset of a Fine space (X, τ, τ_f) is called generalized Fine closed (Fine g- closed) if F $clA\subseteq U$, whenever $A\subseteq U$ and U is Fine open. The complement of a Fine g- closed set is called the Fine g-open set.

Definition: 2.9 [13]

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called Fine g- closed if for each Fine closed set G of X , f(G) is Fine g-closed set.

Definition:2.10

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called a Fine pre semi closed if image of each Fine semi closed set is Fine semi closed .

Definition 2.11

A function $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called

(i) F-gs-continuous if $f^{-1}(V)$ is F-gs-closed in X for every closed subset V of Y;

(ii) F-gs-irresolute if $f^{-1}(V)$ is F-gs-closed in X for every F-gs-closed subset V of Y;

3. F-gs-CLOSED SETS AND F-gs-OPEN MAPPINGS

In this section we introduced Fine generalized semi closed set (F-gs-closed set) and F-gs-open mappings in Fine topological space

Definition:3.1

Let (X, τ, τ_f) be a Fine topological space . A subset A of a Fine space X is called Fine generalized semiclosed (F-gs-closed) if F-sclA$\subseteq$U whenever A$\subseteq$ U and U is Fine open.
The complement of a F-gs-closed set is called a F-gs-open set .

Definition 3.2

Let (X, τ, τ_f) and (Y, σ, σ_f) be Fine topological spaces. A function $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is Called F-gs-open map if for every Fine open set G in Fine space X, f (G) is a F-gs-open set in Fine space Y.

Theorem 3.3

Prove that a mapping $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-open if and only if for each $x \in X$, and $U \in \tau_f$ such that $x \in U$, there exists a F-gs-open set W$\subseteq$ Y containing f (x) such that W$\subseteq$ f(U) .

Proof.

Follows immediately from Definition 3.1

Theorem. 3.4

Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be F-gs - open. If W$\subseteq$ Y and G$\subseteq$ X is a Fine closed set containing $f^{-1}(W)$, then there exists a F-gs-closed H $\subseteq$ Y containing W such that $f^{-1}(H) \subseteq G$.

Proof.

Let H = Y-f (X- G) .from the definition of H,H is a F-gs-closed .By our assumption $f^{-1}(W) \subseteq H$, we have f (X - G) $\subseteq$ (Y -W) .Hence $f^{-1}(H) = X - f^{-1}[f(X-G)] \subseteq X - (X-G) = G$.

Theorem 3.5

Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be F-gs-open and let B$\subseteq$ Y. Then f^{-1}[F-gs-Cl(F-gs-Int(F-gs-Cl(B)))] $\subseteq$ F-Cl [f^{-1} (B)]

Proof.

Assume that B $\subseteq$ Y .we know that $f^{-1}(B) \subseteq$F -Cl [f^{-1} (B)] for any set.By Theorem 3.3, there exists a F-gs-closed set B$\subseteq$H$\subseteq$ Y, such that f^{-1} (H) $\subseteq$Cl [f^{-1} (B)] . Thus, f^{-1}[F-gs-Cl(F-gs-Int(F-gs-Cl(B)))] $\subseteq$ f^{-1}[F-gs-Cl(F-gs-Int(F-gs-Cl(H)))]$\subseteq f^{-1}$(H) $\subseteq$F-Cl[f^{-1}(B)] .

Theorem 3.6

Prove that a function $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-open if and only iff [F-Int (A)] $\subseteq$F-gs-Int[f (A)] , for all A$\subseteq$X.

Proof.

Necessity. Let A$\subseteq$ X. Let x$\in$ F-Int(A).Then there exists Ux$\in \tau_f$ such that x$\in$Ux$\subseteq \tau_f$ A.So f (x)$\in$f (Ux) $\subseteq$f (A) and by hypothesis,
f (Ux)$\in$F-gs-σ. Hence f (x) $\in$ F-gs-Int [f (A)] .Thus f [F-Int(A)] $\subseteq$F-gs-Int[f(A)] .

Sufficiency.

Let U∈τ_f. Then by hypothesis, f [F-Int (U)]F⊆-gs-Int[f (U)] .Since F-Int(U) =U as U is Fine open. Also F-gs-Int[f (U)] ⊆ f(U) . Hence f (U)=F-gs-Int[f(U)] .Thus f (U) is F-gs-open in Y. So f is F-gs-open.

Theorem 3.7

Prove that a function f : (X, τ, τ_f) → (Y, σ, σ_f) is F-gs-open if and only if F-Int [f^{-1} (B)] ⊆f^{-1}[F-gs-Int(B)] , for all B⊆Y.

Proof.

Necessity.

Let B⊆Y. Since F-Int [f^{-1}(B)] is Fine open in Fine space X and f is F-gs-open,f [FInt (f^{-1} (B))] is F-gs-open in Fine space Y. Also we have f [F-Int (f^{-1}(B))]⊆f [f^{-1}(B)]⊆B. Hence,f [F-Int (f^{-1}(B))] ⊆ F-gs-Int (B) . Therefore F-Int[f^{-1} (B)]⊆f^{-1}[F-gs-Int(B)] .

Sufficiency.

Let A ⊆ X. Then f(A)⊆Y. Hence by hypothesis, we obtain F-Int(A)⊆F-Int [f^{-1}(f (A))]⊆f^{-1}[F-gs-Int(f (A))].Thus f [F-Int(A)]⊆F-gs-Int[f (A)] , for all A⊆X. Hence,by Theorem 3.5, f is F-gs-open.

Theorem 3.8

Let f : (X, τ, τ_f) → (Y, σ, σ_f) be a mapping. Then a necessary and sufficient condition for f to be F-gs-open is that f^{-1}[F-gs-Cl(B)]⊆F-Cl[f^{-1}(B)] for every subset B of Fine space Y.

Proof.

Necessity.

Assume f is F-gs-open. Let B⊆Y. Let x∈ f^{-1}[F-gs-Cl(B)] .Then f(x)∈F-gs-Cl (B) . Let U∈τ such that x∈ U. Since f is F-gs -open, then f (U) is a F-gs-open set in Fine space Y. Therefore, B∩f (U)≠φ Then U ∩f^{-1} (B)≠φ. Hence x ∈FCl [f^{-1} (B)].We concludethat f^{-1}[F-gs-Cl (B)] ⊆ FCl[f^{-1}(B)] .

Sufficiency.

Let B⊆Y.Then (Y - B) ⊆ Y. By hypothesis, f^{-1}[F-gs-Cl (Y - B)] ⊆ F-Cl [f^{-1} (Y - B)] . This implies X –F-Cl [f^{-1} (Y - B)] ⊆ X – f^{-1}[F-gs-Cl (Y - B)] . Hence X- F-Cl [X-f^{-1} (B)] ⊆ f^{-1} [Y-F-gsCl(Y - B)] . By applying F-Int [f^{-1}(B)] ⊆ f^{-1} [F-gs -Int(B)] . Now form Theorem 3.6, it follows that f is F-gs-open.

4.F-gs-CLOSED MAPPING

In this section we introduce F-gs-closed functions and study certain properties and characterizations of this type of functions.

Definition 4.1

A mapping f : (X, τ, τ_f) → (Y, σ, σ_f) is called F-gs-closed if the image of each Fine closed set in Fine space X is a F-gs-closed set in Fine space Y .

Theorem 4.2

Prove that a mapping f : (X, τ, τ_f) → (Y, σ, σ_f) is F-gs-closed if and only F-gs-Cl [f (A)] ⊆f [F-Cl (A)] for each A ⊆ X.

Proof.

Necessity

Let f be F-gs-closed and let A⊆ X. Then f (A) ⊆ f [F-Cl (A)] and f [F-Cl(A)] is a F-gs-closed set in Y. Thus F-gs-Cl [f (A)] ⊆f [F-Cl (A)] .

Sufficiency.

suppose that F-gs-Cl [f (A)] ⊆ f [F-Cl (A)] , for each A ⊆ X. Let A⊆ X be a Fine closed set. Then F-gs-Cl[f (A)]⊆f [F-Cl (A)] = f (A) . This shows that f (A) is a F-gs-closed set.Hence f is F-gs-closed.

Theorem 4.3

Let $f : (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ be F-gs-closed. If $V \subseteq Y$ and $E \subseteq X$ is an Fine open set containing $f^{-1}(V)$, then there exists a F-gs-open set $G \subseteq Y$ containing V such that $f^{-1}(G) \subseteq E$.

Proof.

Let G = Y-f (X - E) . Since $f^{-1}(V) \subseteq E$, we have $f(X - E) \subseteq Y - V$. Since f is F-gs-closed, then G is a F-gs-open set and $f^{-1}(G)=X-f^{-1}[f(X - E)] \subseteq X-(X - E) = E$.

Theorem 4.4

Suppose that $f : (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ is a F-gs-closed mapping. Then F-gs-Int[F-gs-Cl(f (A))] ⊆f[F-Cl(A)] for every subset A of Fine space X.

Proof.

Suppose f is a F-gs-closed mapping and A is an arbitrary subset of Fine space X. Then f [Cl(A)] is F-gs-closed in Fine space Y. Then F-gs-Int[F-gs-Cl(f(F-Cl (A)))] ⊆ f [F-Cl (A)] . But also F-gs-Int[F-gs-Cl(f(A))] ⊆F-gs-Int[F-gs-Cl(f(FCl(A)))] . Hence F-gs-Int[F-gs-Cl(f (A))] ⊆ f [F Cl (A)] .

Theorem 4.5

Let $f : (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ be a F-gs-closed function, and $B, C \subseteq Y$.

(i) If U is an Fine open neighborhood of $f^{-1}(B)$, then there exists a F-gs-open neighborhood V of B such that $f^{-1}(B) \subseteq f^{-1}(V) \subseteq U$.

(ii) If f is also onto, then if $f^{-1}(B)$ and $f^{-1}(C)$ have disjoint Fine open neighbourhoods , so have B and C.

Proof.

(i) Let V = Y-f (X - U) . Then $V^C = Y - V = f(U^C)$. Since f is F-gs-closed, so V is a F-gs-open set. Since $f^{-1}(B) \subseteq U$, we have $V^C = f(U^C) \subseteq f[f^{-1}(B^C)] \subseteq B^C$. Hence $B \subseteq V$, and thus V is a F-gs-open neighborhood of B. Further $U^C \subseteq f^{-1}[f(U^C)] = f^{-1}(V^C) = [f^{-1}(V)]^C$ .This proves that $f^{-1}(V) \subseteq U$.

(ii) If $f^{-1}(B)$ and $f^{-1}(C)$ have disjoint Fine open neighborhoods M and N, then by (i) ,we have F-gs-open neighborhoods U and V of B and C respectively such that $f^{-1}(B) \subseteq f^{-1}(U) \subseteq$ F-gs-Int(M) and $f^{-1}(C) \subseteq f^{-1}(V) \subseteq$ F-gs-Int(N) . Since M and N are disjoint, so are F-gs-Int(M) and F-gs-Int(N) , and hence so $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint as well. It follows that U and V are disjoint too as f is onto.

Theorem 4.6

Prove that a surjective mapping $f : (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ is F-gs-closed if and only if for each subset B of Fine space Y and each Fine open set U in Fine space X containing $f^{-1}(B)$, there exists a F-gs-open set V in Y containing B such that $f^{-1}(V) \subseteq U$.

Proof.

Necessity. This follows from (1) of Theorem 4.5.

Sufficiency.

Suppose G is an arbitrary Fine closed set in Fine space X. Let y be an arbitrary point in Y - f (G) . Then $f^{-1}(y) \subseteq X - f^{-1}[f(G)] \subseteq (X - G)$ and (X - G) is Fine open in Fine space X. Hence by hypothesis, there existsa F-gs-open set Vy containing y such that $f^{-1}(Vy) \subseteq (X - G)$. This implies that $y \in Vy \subseteq [Y - f(G)]$. Thus $Y - f(G) = \cup\{Vy : y \in Y - f(G)\}$. Hence Y - f (G) , being a union of F-gs-open sets, is F-gs-open. Thus its complement f (G) is F-gs-closed. This shows that f isF-gs-closed.

Theorem 4.7

Let $f : (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ be a bijection. Then the following are equivalent:

(i) f is F-gs-closed.

(ii) f is F-gs-open.

(iii) f^{-1} is F-gs-continuous.

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Proof.

(i) = (ii): Let U ∈ τ. Then X -U is Fine closed in Fine space X. By (i),f (X-U) is F-gs-closed in Fine space Y.But f (X-U)=f(X)-f(U)=Y-f(U).Thus f (U) is F-gs -open in Fine space Y. This shows that f is F-gs-open.

(ii) = (iii): Let U ⊆X be an Fine open set. Since f is F-gs-open. So f (U) = $(f^{-1})^{-1}$ (U) is F-gs- open in Fine space Y .Hence $f^{-1}$is F-gs-continuous.

(iii) = (i): Let A be an arbitrary Fine closed set in Fine space X. Then X-A is Fine open in Fine space X. Since f^{-1} is F-gs- continuous, $(f^{-1})^{-1}$(X -A) is F-gs-open in Y. But $(f^{-1})^{-1}$(X - A) = f (X - A) =Y - f (A) .Thus f (A) is F-gs- closed in Fine space Y. This shows that f is F-gs-closed.

5.F-gs -CONTINUOUS MAPS

Here we introduce and study Fine generalized semi-continous (F-gs-continuous) maps and Fine generalized semi-irresolute (F-gs irresolute) maps using the notion of Fine generalized semi-open sets

Definition 5.1.

A map f : $(X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called F-gs-continuous if f^{-1}(V) is F-gs-open in Fine space X for every Fine open set V in Fine space Y.

Example 5.2

Let X = Y = {a, b, c}, τ = { φ, X, {b}} ,τ_f={ φ, X, {b}, {a, b},{b,c} }and σ = { φ , Y, {c}}. σ_f= { φ , Y, {c},{a, c},{b,c,}}

Define a map f : $(X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ by f(a) = a,f(b) = c, f(c) = b. Then f is F-gs-continuous

Definition 5.3

A map f : $(X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called F-gs-irresolute if f^{-1} (V) is F-gs-open in Fine space X for every F-gs-open set V in Fine space Y.

Theorem 5.4

Every F-sg-open set in a Fine topological space (X, τ, τ_f) is F-gs-open but not conversely.

Proof

Let A be any F-sg-open set in Fine space X. Let G be any Fine closed set in Fine space X such that G⊂A. Since every Fine closed set is Fine semiclosed, G is a Fine semi-closed set in Fine space X such that G⊂A. Since A is F-sg-closed, G⊂F- sint(A). Therefore, A is F-gs-open. The converse need not be true as seen from the following example.

Theorem 5.5.

Let f : $(X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be F-sg-continuous. Then f is F-gs-continuous.

Proof

Let V be Fine open in Fine space Y. Then f^{-1}(V) is F-sg-open in Fine space X since f is F-sg-continuous. But every F-sg-open set is F-gs-open. Therefore f^{-1}(V) is F-gs-open. Hence f is F-gs-continuous.

Theorem 5.6

Let f : $(X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be Fine semi-continuous. Then f is F-gs-continuous.

Proof

We have already observed that Fine semi-continuity implies F-sg-continuity. By Theorem 1.5, F-sg-continuity implies F-gs continuity. Therefore f is F-gs-continuous.

Theorem 5.7

A map f : $(X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-continuous if and only if f^{-1}(V) is F-gs-closed in Fine space X for every Fine closed set V in Fine space Y.

Proof

Assume that f is F-gs-continuous. Let V be any Fine closed set in Fine space Y. Then V^C is Fine open in Fine space Y. Since f is F-gs-continuous, $f^{-1}(V^C)$ is F-gs-open in X. But $f^{-1}(V^C) = X-f^{-1}(V)$ and so $f^{-1}(V)$ is F-gs-closed in Fine space X. Conversely, assume that $f^{-1}(V)$ is F-gs-closed in Fine space X for every Fine closed set V in Fine space Y. Let V be any Fine open set in Fine space Y. Then V^C is Fine closed in Fine space Y. By assumption, $f^{-1}(V^C)$ is F-gs-closed in Fine space X. But $f^{-1}(V^C) = X-f^{-1}(V)$ and so $f^{-1}(V)$ is F-gs-open in Fine space X. Therefore,f is F-gs-continuous.

Theorem 5.8

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-irresolute if and only if $f^{-1}(V)$ is F-gs-closed in Fine space X for every F-gs-closed set V in Fine space Y.

Proof:

Assume that f is F-gs-irresolute. Let V be any F-gsclosed set in Fine space Y. Then V^C is F-gs-open in Fine space Y. Since f is F-gsirresolute, $f^{-1}(V^C)$ is F-gs-open in Fine space X. But $f^{-1}(V_0) = X-f^{-1}(V)$ and so $f^{-1}(V)$ is F-gs-closed in Fine space X

Conversely assume that $f^{-1}(V)$ is F-gs-closed in Fine space X for every F-gs-closed set V in Fine space Y. Let V be any F-gs-open set in Fine space Y. Then V^C is F-gs-closed in Fine space Y. By assumption, $f^{-1}(V^C)$ is F-gs closed in Fine space X. But $f^{-1}(V^C) = X-f^{-1}(V)$ and so $f^{-1}(V)$ is F-gs open in Fine space X. Therefore, f is F-gs-irresolute.

Theorem 5.9

Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be irresolute and Fine pre-semi closed. Then f is F-gs-irresolute.

Proof:

Let A be F-gs-closed in Fine space Y. Let O be an Fine open set in Fine space X such that $f^{-1}(A) \subset O$. Since f is Fine irresolute, $f(Fscl(f^{-1}(A)) \cap f(O^C) \subset f(Fscl(f^{-1}(A))) \cap f(O^C) \subset Fscl(f(f^{-1}(A))) \cap A^C \subset F\text{-}sc1(A)-A$. Since f is Fine pre-semi-closed and the set $F\text{-}scl(f^{-1}(A)) \cap O^C$ is Fine semi closed, F-scl(A)-A contains the Fine semi-closed set $f(F\text{-}scl(f^{-1}(A)) \cap O^C$,$f(Fscl(f^{-1}(A)) \cap O^C) = \varphi$.Therefore $Fscl(f^{-1}(A)) \cap O^C = \varphi$ and so, $Fscl(f^{-1}(A)) \subset O$. Hence $f^{-1}(A)$ is F-gs-closed in Fine space X and thus f is F-gs-irresolute.

Theorem 5.10.

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-irresolute and $g : (Y, \sigma, \sigma f) \rightarrow (Z, \rho, \rho_f)$ is F-gs-continuous then the composition $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$ is F-gs-continuous.

Proof

Let V be any Fine open set in Fine space Z. Since g is F-gs continuous, $g^{-1}(V)$ is F-gs-open in Fine space Y. Since f is F-gs-irresolute, $f^{-1}(g^{-1}(V))$ is F-gs-open in Fine space X. But $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$. Therefore, g o f is F-gs-continuous.

Theorem 5.11

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ and $g : (Y, \sigma, \sigma_f) \rightarrow (Z, \rho, \rho_f)$ are both F-gs-irresolute maps then the composition map $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$ is F-gs-irresolute.

Proof

Let V be any F-gs-open set in Fine space Z. Since g is F-gs-irresolute, $g^{-1}(V)$ is F-gs-open in Fine space Y. Since f is F-gs-irresolute, f (g (V)) is F-gs-open in Fine space X. But $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$. Therefore, g o f is F-gs-irresolute.

CONCLUSIONS

Many different forms of open functions and closed functions have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, this paper we introduce F-gs-Open Mappings , F-gs Closed Mappings and F-gs Continuos Mapping in Fine topological and investigate some of the basic properties . This shall be extended in the future Research with some applications

ACKNOWLEDGMENT

I wish to acknowledge friends of our institution and others those who extended their help to make this paper as successful one. I acknowledge the Editor in chief and other friends of this publication for providing the timing help to publish this paper.

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