International Journal of Engineering, Science and Mathematics
Vol.7, Special Issue 4(3), April 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Relativistic Accretion and Wind Flows around Rotating Black Holes

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Abstract

In this communication, we study the relativistic hydrodynamics of accretion and wind flows around rotating black holes and investigate the possibilities of shocks in the flows. We find that shocks are possible in the presence of the two saddle type sonic points in the flow. This happens for the choice of constant ultra-relativistic adiabatic index $\gamma = 4/3$ or close to it. However, employing the relativistic equation of state(EOS) [7] in which the adiabatic index γ varies smoothly from 5/3 to 4/3 the non relativistic to relativistic regime with temperature we notice that number of the saddle type sonic point reduces to one [1][2] indicating that the possibilities of formation of shocks in the flow becomes rare [1][8].

Keywords:

Accretion discs;
Equation of State;
Relativity;
Hydrodynamics;
Shock Waves..

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1. Introduction

In the case of accretion, matter remains cool (non relativistic(NR)) at the outer boundary and becomes very hot (extreme relativistic) at the inner edge, (the situation is reverse in the wind flows). Thus, the variation of the adiabatic index γ needs to be taken into account while solving the hydrodynamic equations. However in the conventional study of accretion/wind flows people used constant ultra-relativistic (UR) value of $\gamma = 4/3$. This particular choice of γ, although takes into account some ofthe UR-effects, misses the essential feature namely, the variation of γ with temperature. A proper study using the relativistic EOS has not made so far and in the popular studies of numerical relativistic hydrodynamics people prefer to use various alternative models of EOS.

In this work our purpose is to find out the accretion and wind solutions. For that we consider hydrodynamics equations in a fully relativistic framework, (e.g., relativistic Euler equation, continuity equation and EOS with variable adiabatic index). The pressure and density of the flow is expressed in terms of sound speed/temperature of the flow. The velocity gradient is derived in a similar way as in the previous study [3]; [4]; [9], The only difference is that now we modified by incorporating this extra piece of information. Once

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velocity gradiant is obtained, we then numerically integrate the velocity gradient to find the accretion solutions *v(r)* for the black hole, neutron star and the wind solutions [8]. Conventionally, these solutions are ploted in terms of the Mach no.(ratio of velocity *v* to sound speed (*Cs*) in order to predict the location of shock in the flow [4];[9]. In the present analysis, we follow the same formalism to investigate the possibilities of shock in. the flow and find that the solutions have one or two critical points in general and in some cases more (three) [8].

2. Solution Topologies and the nature of the EOS at different radii of the disk

In the present study, γ is not constant but varies from 5/3 to 4/3 (from outer to inner boundary). The variation of γ can easily be obtained once we find the solution topologies. We obtain solution topologies by integrating velocity gradient $(dv/dr=N/D)$ through the the critical point condition $N=D=0$ as to make dv/dr to be finite and then plot the Mach no. (=velocity/sound speed) as a function of radius r for different inner and outer boundary values [1][8]. Considering all the different types of solutions (black hole, neutron star and winds), we investigate the variation of γ with the disk radius r. The results are plotted in the Fig.1. The solutions corresponding to the accretion and wind flows is shown in the sub-panel of the figure. Here the solid curves represent the variation of γ for the accretion flow while the dotted curves represent the variations for the wind solutions. Note that in both the figures upto the critical point (crossing points), γ does not vary much and remain same for both the flows. The variation becomes prominent only after crossing the inner critical point. We also see that in the wind solution the variation of γ is faster than the black hole accretion solution. However, in case of black hole accretion, there is a sharp jump of γ values close to the horizon. Note that the change in adiabatic index with radius indicates the nature of the EOS at the different location of the disk and its variation at different region. Therefore, we conclude that except for the region very nearby the compact object, γ does not change significantly from its NR value and thus, the EOS remains non relativistic in nature.

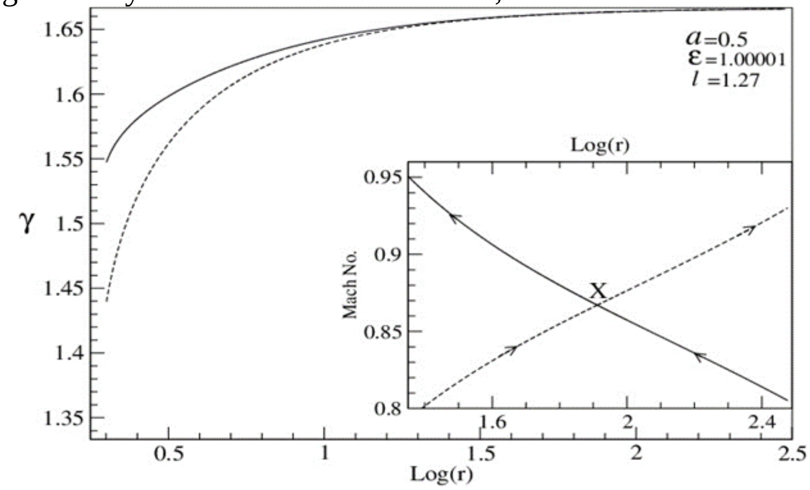

Figure 1. The variation of the adiabatic index γ as a function of radius r is shown in the figure for accretion(solid) and wind(dotted) flow topologies plotted in the sub-panel. Flow parameters are also given in the right corner of the panel.

3. Necessary conditions for the formation of shocks in the flows.

To form a shock, one requires the presence of two saddle('X')-type (inner and outer) critical points [3][9]. The subsonic matter crossing the outer sonic point becomes super sonic and may jump to subsonic branch through a shock transition if the flow satisfies Rankine - Hugoniot shock conditions (balance of pressure, mass flux and specific energy at the interface of the shock). These conditions hold in between two 'X'-type sonic points for a large range of parameters specific energy(ε) and angular momentum(*l*) values and therefore, a stable or oscillatory shock may form [3] [4]. The possibility of occurrence such a shock in the flow is also confirmed numerically [5] [6]. However, employing relativistic EOS, we re-investigate this issue in the entire range of flow parameters ε, *l* and spin parameter(*a*) of compact object. In a similar manner of [4][9], using the sonic point conditions, we express ε as a function of the critical radius *rc*. To find the number of critical points in the flow, we plot in Fig.2 for the different pairs of (*a*, *l*)=(0, 1.3)(top), (0, 1.4)(bottom), (0.5, 1.3)(top), (0.5, 1.33)(bottom), (0.9, 1.299)(top), (0.9, 1.301)(bottom), (0.999, 1.265)(top), (0.999, 1.27)(bottom) values. The Kerr parameter's (*a*) are printed on the individual curve. For a given (a, l)-curve the number of intersections with a constant ε line (see in figure ε=1.00001 line) give the number of critical points in the flow. As we can now see, with the increase of *l*, in most of the cases curves have two critical points. The situation does not change even if *a* increases up to 0.9. Therefore, in the entire range of ε no trace of three critical points is observed. Rather only mono-type/single 'X'-type critical point is frequent. Unfortunately, this is unusual. In [3][4] [9] when $\gamma = 4/3$ used, shocks was favourable because a large range of parameters pertained two 'X'-type critical point.

4. Parameters Space and Possibilities of Shocks

In our study, we find three critical points among them two are 'X'-type are rare. We see three critical points just start to occur beyond the *a*=0.9 values, and become more prominent when *a* close to 1 (see the curve-(0.999, 1.27) in Fig.2). However, the value of ε is small and close to one e.g. at ε = 1.00001, curve-(0.999, 1.27) has three critical points: I-inner('X'), M-middle(circle), and O-outer('X'). In-spite of rare possibility, we identify the parameters space (ε, *l*) at a = 0.999, for three critical points. We see in the sub-panel of Fig.2 that the available range of parameters is small and tiny. This implies a very small window of parameters (ε, *l* & *a*) can have three critical points in the flow, when temperature dependent adiabatic index is considered. Therefore, possibilities of shocks in the flow which require the presence of two saddle type ('X') critical points becomes very unlikely in case of relativistic EOS of accretion and wind flows around black holes.

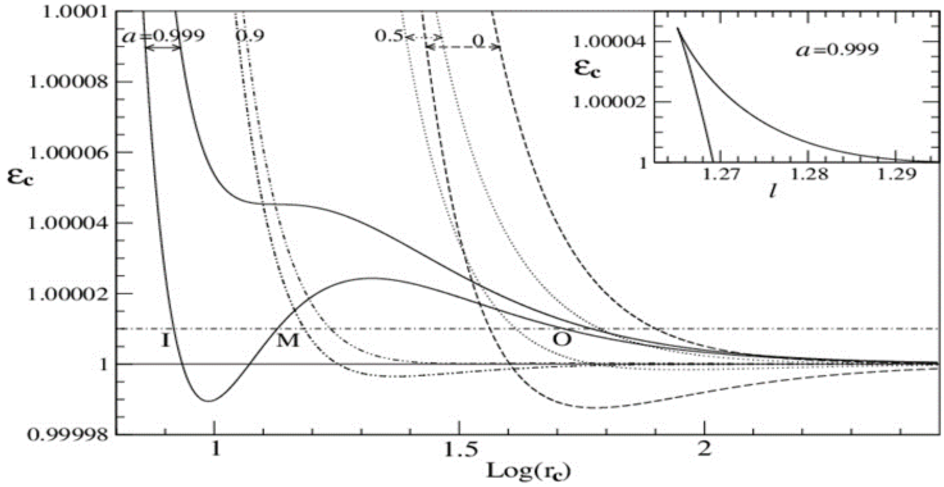

Figure 2. *The variation with is shown for (a, l)=(0, 1.3)(top), (0, 1.4)(bottom), (0.5, 1.3)(top), (0.5, 1.33)(bottom), (0.9, 1.299)(top), (0.9, 1.301)(bottom), (0.999, 1.265)(top), (0.999, 1.27)(bottom with three sonic points). In the sub-panel a parameter space (ε, l) is also drawn when two saddle type ('X') critical points exists.*

5. Conclusion

In this study, we review hydrodynamic solutions of accretion and wind solutions to investigate the possibilities of shocks in the presence of relativistic EOS [7]. In the previous study [3], the flow has mostly two saddle('X')-type sonic points and may contain stable shocks in between two ‘X’-type sonic points when the ultra-relativistic γ(=4/3) is used. However, in the present study with the correct choice of EOS in which γ varies continuously from 5/3 to 4/3, we find that the EOS of the matter mostly remains non-relativistic in nature i.e., γ lies close to value 5/3. Because of its nonrelativistic nature, flow has mostly one saddle type critical points which unfortunately is not favourable for the formation of shock in the flow. These results also agree with our previous similar study [8].

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