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TWO DIMENSIONAL NON NEWTONIAN BOUNDARY LAYER FLOW OVER A FLAT PLATE WITH POWER LAW FLUID WITH SUCTION/INJECTION THROUGH POROUS MEDIA

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Abstract

The present paper we give numerical solution of the Falkner-Skan equation for the study of two-dimensional permeable steady boundary-layer viscous flow over a flat plate in the presence of non-Newtonian power law fluid which is represented by a power law model. The outer free stream velocity is defined in the form of a power-law manner i.e., it varies as a power of a distance from the leading boundary-layer. Generalized similarity transformations are used to convert the the governing boundary layer equations in to a third order nonlinear differential equation which is famous Falkner- Skan equation for non-Newtonian fluid. This equation contains three flow parameters that is the Stream-wise pressure gradient (β), the porous parameter (Ω), and (m) is the power law relation parrameter. The governing equations (nonlinear partial differential equations) have been converted to an equivalent nonlinear ordinary differential equation along with boundary conditions by means of which is solved using the Keller-box method. The results are obtained for velocity profiles, viscosity profiles and skin friction for various values of physical parameters and are discussed in detail. It is also found that the drag force is reduced for dilatant fluids compared to pseudo-plastic fluids. The Physical significance of the flow parameters are also discussed in detail.

Keywords:

Boundary-layer equations;
Falkner-Skan equation;
Numerical solution;
porous media;
Non-Newtonian fluid;
power-law fluid;

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1. Introduction

The study of the boundary-layer flow of the Newtonian and non-Newtonian fluids providesvaluable insights into industrial and technological applications. The Newtonianfluids such as air or water serve as a benchmark for most of the fluid flow behaviour.However the behaviour of non-Newtonian fluids that are found many industrial applicationsis markedly different from those of the Newtonian fluids. Particulate slurries,coal in water, sewage sludge, inks and also multiphase mixtures i.e. oil-water emulsions,foams, gas-liquid dispersions are classified as non-Newtonian fluids. Generally,these fluids have the property of a variable viscosity. One class of material of considerableinterest is that the effective viscosity depends entirely on the rate of shearingon the total flow rate. For example the most commonly used models for the variableviscosity are Ostwald-de Waele, Carreau rheological fluid, Carreau-Yasuda, etc whichform a relationship between shear-stress and shear ratio. Because of such applicationsAcrivos et al (1960) and Schowalter (1960) have obtained equations for the boundarylayerflow of a non-Newtonian fluid particularly the numerical simulations of Acrivoset al (1960) show that thickness of the boundary-layer for the shear-thinning fluids israther large compared to the shear-thickening fluids. It is further shown by Wu andThomson (1996) that for modirate values of the Reynolds number, the boundary-layerequations for shear-thinning fluids provide accurate solutions. However, it is commonpractice that the boundary-layer forms when the Reynolds number is quite large.

For shear-thickening fluids, Andersson and Irgens (1998) have shown that the boundary-layerequations predict finite-width of the boundary-layer. To support these resultsFilipuss et al (2001) gave rigorous mathematical analysis that also predicts that samefinite-width of the boundary-layer. On the other hand, a self-similar solution of theboundary-layer equations results into a overshoot in the velocity profiles. In a smallregion in the boundary-layer, these velocity profiles exceeds the velocity of the mainstreamflow. Denier and Dabrowiski (2004) have even shown that these are doublesolutions for the boundary-layer equations when a self similar form is assumed. Theyfurther showed that mode 1 solution represents forward flow while mode 2 or mode3 solutions become increasingly oscillatory with alternatively forward and reverse regionsof flows. Results of Griffiths (2017) shown that the effects of shear-thinning areto stabilize the boundary-layer flow.

In this paper, we consider the solutions of the Navier-Stokes equations under thelimit of large-Reynolds number flow that exhibiting a power-law rheological model.Among the many possible non-Newtonian fluids we have chosen the Ostawald-deWaele fluid which has a sound theoretical basis represents the complex viscosity andalso it is often adopted to describe the rheological phenomena of the pseudo plastic fluids. [The boundary-layer equations admit the self-similar solutions since the mainstreamflow outside the boundary-layer is approximated in power of the distance alongthe wedge wall.]. To solve the transformedboundary-layer equation numerically, we use the Keller-box method which is second-order accurate (Keller 1971)is used for full non-linear problem. This enables us to precisely identify the behaviour of theboundary-layer flow of the Ostwald-de Waele fluid.

Rest of the paper is organized as follows. In section 2, we set-up the problem inquestion in which the Cauchy momentum equations for non-Newtonian fluid. Thesereduce to the boundary-layer equations with inclusion of a non-linear term in the equation which models the viscosity variations. The appropriate similarity transformationsare also presented. Section 3 devotes the full numerical solution of the problem. Detailsof numerical Keller-box method are also presented. Final section presents importantfindings of the problem. Here we discuss all significant results for shear-thinning andShear-thickening fluids in terms the velocity and viscosity shapes. Interestingly, thegoverning equation exhibits solutions for some parameter ranges.

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2 Formulation of the problem

We consider the two dimensional laminar boundary-layer flow of a viscous and incompressiblefluid over a flat plate through porous media with a non-Newtonian power-lawfluid. The positive x-coordinate is measured along the surface and the positive ycoordinateis measured normal to the x-axis in the outward direction towards the fluid.The fundamental equations for the flow of an incompressible fluid are the conservationof mass, linear momentum. We express these equations in the absence of body forces as follows

$$\nabla \cdot \vec{\vec{q}} = 0 \qquad (1)$$

$$\rho\left(\frac{\partial}{\partial t} + \vec{q}.\nabla)\vec{q}\right) = -\nabla p + \nabla \cdot \tau - kp \qquad \textbf{(2)}$$

where ρ is the fluid density, p is the pressure, k is the permeability of the porous medium and τ is the deviatroic stress tensorand is defined as

$$\tau = \mu(\dot{q}) \qquad (3)$$

where $\dot{q}$ is the second invariant of the strain-ratetensor. The shear rate $\dot{\mathbf{q}}$ is given by

$$\dot{q} = \frac{1}{2}(\dot{\mathbf{q}} : \dot{\mathbf{q}})^{\frac{1}{2}} \qquad (4)$$

with

$$\dot{\mathbf{q}} = (\nabla\vec{u} + \nabla\vec{u}^{T}) \qquad (5)$$

The constitutive viscosity relation μ forthe Ostwald-de Waele power-law model is given by

$$\mu = K(\dot{\mathbf{q}})^{m} \qquad (6)$$

where K is the material constant and the index m represents the degree of shear thickening or thinning. We note that the Newtonian viscosity relationship is recovered for $m = 1$. This parameter m is an important index which subdivides the fluids into pseudo-plastic fluids or shear- thickening when $m > 1$ and dilatants or shear-thinning for $m < 1$. Bird et al (1987) can be referred to the through account of the rheological data on m. The hydrodynamics of other values of m shall be discussed later. The velocity vector $\dot{q} = (u, v)$ where u and v are the velocity components in x and y directions respectively, and thus from (4), we have that

$$\dot{q} = \left[\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]^{\frac{1}{2}} \qquad (7)$$

using (5). We consider the problem of two-dimensional, incompressible and steady state laminar boundary-layer flow over a wedge which moves with velocity $U_{0w}(x)$ in a non-Newtonian power-law fluid. The positive x-coordinate is measured along the surface of the wedge with the apex as origin, and the positive y-coordinate is measured normal to the x-axis in the outward direction towards the fluid. Under these approximations, the governing equations for the steady two-dimensional laminar viscous flow of a non-Newtonian fluid. It is considered that the wedge moves with velocity $U_w(x)$ along or opposite to the mainstream flows $U(x)$. Using the standard boundary-layer approximations and for large R_e we have that $\left|\frac{\partial u}{\partial y}\right| >> \left|\frac{\partial u}{\partial x}\right|$ and $\left|\frac{\partial p}{\partial y}\right| << \left|\frac{\partial p}{\partial x}\right|$. Thus the system (1)-(2) can be written as

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (8)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^{m}-kp \qquad (9)$$

Similarely,we get

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{dp}{dy}+\frac{K}{\rho}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)^{m}-kp \qquad (10)$$

where

$$\dot{\mathbf{q}}=\frac{\partial u}{\partial y} \qquad (11)$$

To this end, we consider the two-dimensional incompressible flow of the non-Newtonian Ostwald-de Waele power-law fluid over a moving wedge which is moving wedge which is moving with velocity U_{0w} either along the mainstream flow with U_0 or opposite to it. The Cartesian co-ordinate system is adopted to the wedge wall the inviscid main stream velocity U_0 is assumed in the form of power of a distance that is

$$U_0(x)=U_{o\infty}x^{*n} \qquad (12)$$

where U_∞ is a non-negative constant and n is a constant related to the pressure gradient defined later in this section. Now, in order to derive boundary layer conditions, the physical quantities and variables specified in (1) and (2) are non-dimensionlized

$$x=\frac{x^*}{L},\ y=\frac{y^*}{\delta},\ u=\frac{u^*}{U},\ v=\frac{v^*}{U},\ p=\frac{p^*}{p_\infty} \qquad (13)$$

L,δ,U and P_∞ are certain reference values. These choices lead to define the Reynolds number for the Ostwald-de Waele power- law fluid as

$$\mathrm{Re}=\frac{\rho\delta^n U^{2-n}}{K\nu}$$

Where ν is the kinematic viscosity, for a large Re the flow divides in to near-field (boundary-layer region) and far field regions In the boundary-layer region of thickness of δ, a very large velocity gradient exists. The boundary layer equations can be derived based on the approximations concern the following measurements. Let $U_0(x)$ be the velocity of the mainstream flow along *x*-direction outside the boundary layer. The key idea involved in making the boundary layer approximation is that the viscosity effects are dominant in the adjacent to the surface. If δ is the thickness of the boundary layer, then $\delta << L$. Hence V is much smaller than U. Also other basic approximation is $\left|\frac{\partial u}{\partial y}\right| >> \left|\frac{\partial u}{\partial x}\right|$. Further, it is also assumed that

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$\left|\frac{\partial p}{\partial y}\right| << \left|\frac{\partial p}{\partial x}\right|$ in meaning that the pressure p in the boundary layer is a function of x only (to the approximation). With $\delta << L$ the term $\frac{\partial^2 u}{\partial x^2}$ can be neglected in comparison with $\frac{\partial^2 u}{\partial y^2}$. Velocity compared to the free stream velocity U_∞ with these assumptions, we have the number of component equations reduce to those in the flow directions. The number of viscous terms in the direction of flow can be reduced to only dominant term. This amounts viscous terms are measured in terms of the boundary-layer thickness. And the inertial terms of the characteristic length *L*. Thus along with these boundary- layer approximations. Equations (8), (9) and (10) for steady case may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (14)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^m-\frac{\nu}{k}(u-U) \qquad (15)$$

$$0=\frac{\partial p}{\partial y} \qquad (16)$$

where K is called the consistency coefficient and *m* is non-dimensional, and the dimension of K depends on the value of m. The two-parameter rheological equation (15) is known as the Ostwald-de-Waele model or more commonly, the power-law model. The parameter m is an important index to subdivide fluids into pseudo-plastic fluids $(m<1)$ and dilatant fluids $(m>1)$. The extreme cases of the power-law model are $(m=1)$ for Newtonian behaviour and $(m=0)$ for plastic or solid behaviour. To determine the pressure distribution, the velocity at the edge of the boundary layer is equal to the mainstream flow $U_0(x)$ and by Bernoullis theorem, the pressure would be constant in the inviscid flow influenced by the applied magnetic field. In order that equations (14) and (15) reduce to similarity form, we assume that the boundary conditions for these equations areof the following form

at $y=0: u=0, v=V_w(x)$

as $y\to\infty: u\to U_{0\infty}$ (17)

where $U_0 w(x)$ is the stretching surface velocity which obeys the power-law relation $U_0 w(x)=U_{0\infty}x^m$.. The conditions on the velocity at infinity mean that the velocity approaches the mainstream flow far-away from the wedge surface. Thus, the main boundary layer effects are restricted to the immediate neighbourhood of the surface. System (14) and (15) allows reducing both dependent and independent variables to one each by the following similarity transformations. This is further evidenced by the similar velocity profiles existing in the boundary layer for any x in the stream wise direction. The pressure change across the boundary layer is negligible (i.e., constant) and pressure can be treated as function of only flow direction.

Since the pressure is uniform throughout the flow field from the Burnoullis equation, with $u=U_{0\infty}$ outside the boundary layer, we have

$$-\frac{1}{\rho}\frac{dp}{dx}=U_0\frac{dU_0}{dx} \qquad (18)$$

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$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_0\frac{dU_0}{dx}+\frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^m-\frac{\nu}{k}(u-U_0(x)) \qquad (19)$$

It is clearly observed that the system (14) and (15) with two unknown functions u and v are easily reduced to an equation with one unknown function by defining the stream function $\psi(x,y)$ as

$$u=\frac{\partial\psi}{\partial y} \text{ and } v=-\frac{\partial\psi}{\partial x} \qquad (20)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}+\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=U_0(x)\frac{dU_0(x)}{dx}+\frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial u}\right)^m-\frac{\nu}{k}(u-U_0(x) \qquad (21)$$

with boundary conditions

$$y=0:\quad \frac{\partial\psi}{\partial x}=0,\frac{\partial\psi}{\partial y}=V_w(x)^n \text{ and as } \quad \frac{y}{\partial}\to\infty:\frac{\partial\psi}{\partial y}=U_{0\infty}(x)^n \qquad (22)$$

The similar solutions of equation (21) can be obtained by using similarity transformation

$$\psi(x,y)=\sqrt{\frac{2\nu KU_\infty x^{1+m^*}}{\rho(n+1)}}\,f(\eta),\ \eta=\sqrt{\frac{(n+1)\rho U_\infty x^{-1+m^*}}{2K\nu}}\,y$$

(23)

where $m^*=\dfrac{(3n-1)(m-1)}{m+1}$

Substituting (23) in to (21) we get the following ordinary differential equation

$$\mu_0 f'''+\frac{2}{m+1}ff''+\frac{\beta}{1+(m-1)\beta}\left(1-f'^2\right)-\Omega(f'-1)=0$$

(24)

and a new set of boundary conditions,

$$f(0)=\alpha,\ f'(0)=0,\ \text{and } f'(\infty)=1 \qquad (25)$$

where $\mu_0(\eta)=m\left|f''(\eta)\right|^{m-1}$ where primes denote differentiation with respect to η and $\lambda=\dfrac{U_{0w}}{U_{0\infty}}$ is the ratio of the wall velocity to the free stream fluid velocity .And $\beta=\dfrac{2n}{n+1}$ is the pressure gradient variable parameter. The system (24)-(25) describes the two-dimensional permeable laminar boundary layer flow of a viscousfluid through porous media. Hence $\alpha=\sqrt{\dfrac{2x}{(m+1)\nu U(x)}}V_w(x)$ is the suction or injection parameter $\alpha>0$,represents suction and $\alpha<0$ is injection, whenever $\alpha=0$ is impermeable of the plate, $\beta>0$ is the favorable, and $\beta<0$ is

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adverse pressure gradient where as $\Omega = \dfrac{2\left(\dfrac{U_\infty}{\nu}\right)^{(m-2)}}{k(m+1)\,\mathrm{Re}^{(m-1)}}$ is the permeability. For $\beta = 0 = \Omega$, the above problem reducesto the Blasius flow that describes a two-dimensional flow over a flat plate with masstransfer and stretch of the plate, and is studied by several investigators with differentcases.The system (24)-(25) describes the flow of Ostwald-de-Waele fluid inthe two-dimension boundary-layer flow. Since any analytical solution is usually notpossible because of high non-linearity, we solve the system (24) and (25) numericallyusing the Keller-box method which is employed in most of boundary-layer simulations.

3. Numerical solution

We solve (24) with the boundary conditions (25) using Keller-box method for differentvalues of β, m and Ω. We briefly give about two-point Keller-box method for thesolution of (24). This scheme is very efficient and fast, and can be used to solve theboundary-layer problems. A detailed description about the method is given in Cebeciand Bradshaw (1977), Kudenatti et al (2013), and in the review paper given by Keller (1978).By using this method we are able to obtain approximations to the solutionsof the original differential equation at each grid point. This method is unconditionallyand has quadratic convergence even for non-uniform mesh points (Cebeci and Cousteix(1999)). To describe this method, the equation (24) with the boundary conditions (25)are rewritten in the form of system of first order ordinary differential equations and aregiven by

$$f' = U \qquad (26)$$

$$U' = V \qquad (27)$$

$$BV' + \frac{2}{m+1} fV + \frac{\beta}{1+(m-1)\beta}(1-U^2) - \Omega(U-1) = 0 \qquad (28)$$

where $B = \mu_0 = m|V|^{m-1}$ and boundary conditions equation (25) becomes

$$f(0) = \alpha,\ U(0) = 0 \ \text{ and } \ U(\infty) = 1 \qquad (29)$$

Using the backward finite difference operators for the system (26-28), we get

$$f_j^l - f_{j-1}^l - \frac{\eta_j}{2}\left(U_j^l - U_{j-1}^l\right) = 0$$

$$U_j^l - U_{j-1}^l - \frac{\eta_j}{2}\left(V_j^l - V_{j-1}^l\right) = 0$$

$$BV_j^l - BV_{j-1}^l - \frac{\eta_j}{2(m+1)}\left(f_j^l - f_{j-1}^l\right)\left(V_j^l - V_{j-1}^1\right) - \frac{\eta_j\beta}{4(1+(m-1)\beta)}\left(U_j^{l-1} + Uj - 1^{l-1}\right)^2 + \frac{2\eta_j\beta}{1+(m-1)\beta} -$$

$$\Omega\frac{\eta_j}{2}\left(U_j^l + U_{j-1}^l\right) + 2\Omega\eta_j = 0$$

The above system necessarily produces a nonlinear algebraic system of equations for each grid. We linearize the above system by introducing, $f_j^{(l+1)} = f_j^l + \delta f_j^l$, where δf_j^l has to be corrected at each step, we drop the product terms of like $\delta f_j, \delta V_j$ etc and also neglected square terms in δf_j, then, we get

and the boundary conditions are

$$\delta f_0 = \delta U_0 = 0,\ \delta U_N = 0$$

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$$f_0 = \alpha, U_0 = 0, U_N = 1 \tag{30}$$

$$\delta f_j - \delta f_{j-1} - \frac{\eta_j}{2}\left(\delta U_j + \delta U_{j-1}\right) = f_{j-1} - f_j + \frac{\eta_j}{2}\left(U_j - U_{j-1}\right) \tag{31}$$

$$\delta U_j - \delta U_{j-1} - \frac{\eta_j}{2}\left(\delta V_j + \delta V_{j-1}\right) = U_{j-1} - U_j + \frac{\eta_j}{2}\left(V_j - V_{j-1}\right) \tag{32}$$

$$c_1\delta V_j + c_2\delta V_{j-1} + c_3\delta f + c_4\delta f_{j-1} + c_5\delta U_j + c_6\delta U_{j-1} =$$

$$-\frac{4d\beta}{1+(m-1)\beta} + BV_{j-1} - BV_j - \frac{d}{m+1}\left(f_jV_j - f_{j-1}V_{j-1}\right) + \frac{d\beta}{2(1+(m-1)\beta)}\left(U_j + U_{j-1}\right)^2 + \Omega\frac{\eta_j}{2}\left(U_j + U_{j-1}\right) \tag{33}$$

Where $d = \frac{\eta_j}{2}$,

$$c_1 = B + \frac{d}{m+1}\left(f_j + f_{j-1}\right), c_2 = -B + \frac{d}{m+1}\left(f_j + f_{j-1}\right), c_4 = \frac{d}{m+1}\left(V_j + V_{j-1}\right)$$

$$c_5 = -\frac{d\beta}{1+(m-1)\beta}\left(U_j + U_{j-1}\right) - \Omega d\ ,\ c_6 = -\frac{d\beta}{1+(m-1)\beta}\left(U_j + U_{j-1}\right) - \Omega d\ ,$$

At $j = 1$,

from equations (31-33)we get,

$$\delta f_1 - \delta f_0 - d\delta U_1 - d\delta U_0 = f_0 - f_1 + dU_1 - dU_0$$

$$\delta U_1 - \delta U_0 - d\delta V_1 - d\delta V_0 = U_0 - U_1 + dU_1 - dU_0$$

$$c_1\delta V_1 + c_2\delta V_0 + c_3\delta f_1 + c_4\delta f_0 + c_5\delta U_1 + c_6\delta U_0 =$$

$$-\frac{4d\beta}{1+(m-1)\beta} + BV_0 - BV_1 - \frac{d}{m+1}\left(f_1V_1 - f_0V_0\right) + \frac{d\beta}{2(1+(m-1)\beta)}\left(U_1 + U_0\right)^2 + \Omega d\left(U_j + U_{j-1}\right) - 2d\Omega$$

Using the above boundary conditions (30) in (31) - (33) we get a block tridiagonal matrix wherein each element is again 3×3 matrix, in the form

$$AD = R \tag{35}$$

is essentially a linear system

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ -d & 0 & -d \\ c_2 & c_3 & c_1 \end{bmatrix}, A_j = \begin{bmatrix} -d & 1 & 0 \\ -1 & 0 & -d \\ c_6 & c_3 & c_1 \end{bmatrix}$$

$$B_j = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -d \\ 0 & c_4 & c_2 \end{bmatrix}, C_{j-1} = \begin{bmatrix} -d & 0 & 0 \\ 1 & 0 & 0 \\ c_5 & 0 & 0 \end{bmatrix}$$

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$$D=\left[\delta_1,\quad \delta_2,\ldots\quad \delta_N\right]^T,\delta_j=\left[f,\quad U,\quad V\right]^T,\ R\left[R_1,\quad R_2,\ldots\quad R_N\right]^T,R_j=\left[R_{11},\quad R_{12},\quad R_{13}\right]^T,$$

$$\delta_1=\begin{bmatrix}\delta V_0\\ \delta f_1\\ \delta V_1\end{bmatrix},\delta_2=\begin{bmatrix}\delta U_1\\ \delta f_2\\ \delta V_2\end{bmatrix},\ldots\delta_N=\begin{bmatrix}\delta U_{N-1}\\ \delta f_N\\ \delta V_N\end{bmatrix}$$

where $j=2,3,4....$ The tridiagonal structure (35)can be solved using LU decompositionmethod. The velocity equation for similar for each pressure gradient and permeable parameters the Keller-box code also given other required derivedquantities such as the velocity profiles. The numerical solution of equation (24) fordifferent parameters, β,Ω , and m has been obtained. Results for the skin frictioncoefficient, velocity profiles and numerical solutions are reported. The drag force is reduced for dilatant fluids compared to pseudo-plastic fluids.

4 Results and Discussion

The similarity solutions of the permeable Falkner-Skan equation for non-Newtonianfluid are obtained for all physical parameters. This equation describes flow of a viscousfluid through porous media. The flow is governed by the nonlinear differentialequation of order three and is solved by different approaches. The validity and efficiencyof the solution method are tested for various parametric values of β,Ω and m are compared with the numerical solution of the permeability Falkner-Skan equation.We also investigated the nature of the distribution of velocity in the boundary layerregion at which the effects of permeability taken into account. Numerical values forthese parameters are taken which have been extensively used in the previous theoreticalstudies. In particular, we have taken the range of values for which the solutions are predictedand boundary layer flows are realized. Further, the direct numerical solutions ofthe permeability Falkner-Skan equation are obtained via finite difference based Kellerboxmethod. This is a standard method for solving nonlinear boundary value problemon a closed interval, in which the Falkner-Skan is converted into an equivalent systemof first order equations. The outer boundary condition is taken at very large value of η that is $\eta_{\max}>>1$The standard central difference schemes are used for the first orderequations, and resulting nonlinear algebraic equations are linearized and solved. OurKeller-box code adapts a variable discretization step size to ensure the desired accuracyin a double precision which was set to 10^{-8} in all our computations. This is because aprecise value of $f''(0)$ would be required to compare solution with the numerical ones.

The values for β and m are so chosen to be in the range of parameters that havebeen used in the previous studies (Bird et al (1987)). Also, fullnumerical Keller-box results using$\Delta\eta=0.01$ are compared with those obtained withstill- smaller $\Delta\eta=0.001$ values and the velocity shapes between two are graphicallymatchable; thus, most of the simulations are performed with $\Delta\eta=0.01$. Further, thevalue of N (the number of grid points) depends on the value $\Delta\eta$ of η and the largevalues of integration-domain lengths (see figures for details). We discuss most of ourresults pertaining to $m\neq1$ which is regarded as a generic problem as $m=1$ case hasbeen fully discussed. The case $m=1$ also serves as the bench-mark of present study.

In order to get the physical insight, numericalcomputations are carried out for various sets of physical parameters on skin friction coefficient, pressure gradient variable parameter β , suction or injection parameterα and permeability parameterΩ to obtain the effects of those parameters ondimensionless velocity. The obtained computational results (variations in velocity andviscosity profiles) are presented graphically in from figures 1 to 8

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Figures 1 and 4 depicts that the variation of velocity profiles $f'(\eta)$ as a function of η fordifferent values of permeability parameter. There have been simulated using theKeller-box numerical method that is described in section 3. This code starts to predictpermeability effects on the boundary-layer flow. It is noticed that thickness of theboundary-layer thickness increases for increasing permeability. It is very clear that form the boundar-layer shear-thickening when $m>1$ (i.e.in dilatant fluids) and when $m<1$the boundary-layer shear- thinning (i.e. in pseuo-plastic fluids) for fixed values of α, β and m. The two extreme cases of the power-law model are $m=1$ for Newtonian behaviour and $m=0$ for plastic or solid behaviour.

Figure 2 exhibits the nature of velocity profiles $f'(\eta)$ as a function of η. It is very clear thatfor fixed Ω, β and α the boundary-layer decreases as increase m. The effect of non-Newtonian parameter m for fixed values of Ω, β and α on velocity fields are depicted. When suction parameter $\alpha=1.0$ velocity increases exponentially as decreasein m increases monotonically as decrease in m for $\alpha=-1.0$.

It is worthwhile to note from thefigure 4that, varation of viscosity profiles $\mu_o(\eta)$ with η the suction parameter $\alpha>0$ and injection parameter $\alpha<0$ visocity decreases as increases m in both case when $\beta=0.46$ and $\beta=1.0$ for fixed values of Ω and β.

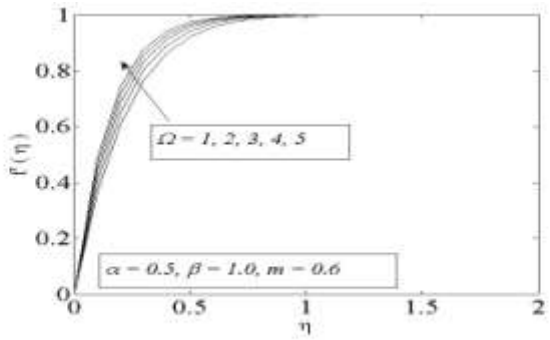

Figure 1(a): Variation of Velocity profiles $f'(\eta)$ with η for various various values of Ω , $\alpha=0.5,\ \beta=1,$ and $m=0.6$

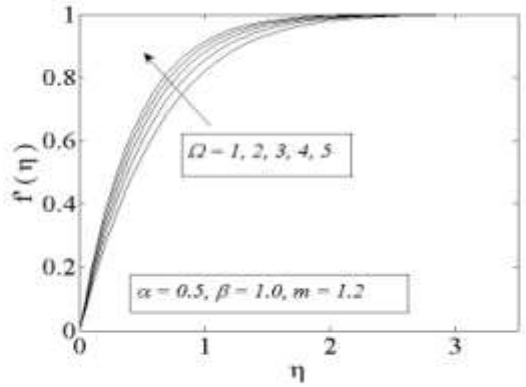

Figure 1(b): Variation of Velocity profiles $f'(\eta)$ with η for various various values of Ω , $\alpha=0.5,\ \beta=1,$ and $m=1.2$

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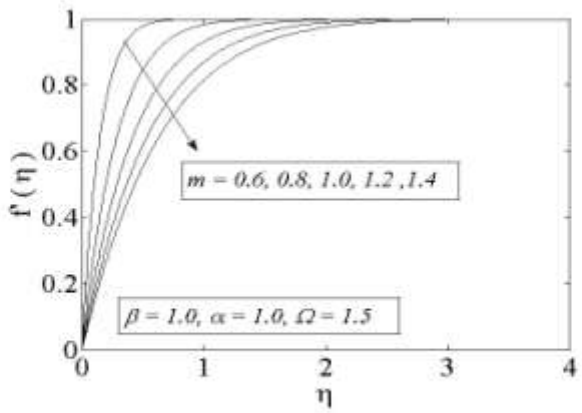

Figure 2(a): Variation of Velocity profiles $f'(\eta)$ with η for various various values of m , $\alpha = 1, \beta = 1,$ and $\Omega = 1.5$

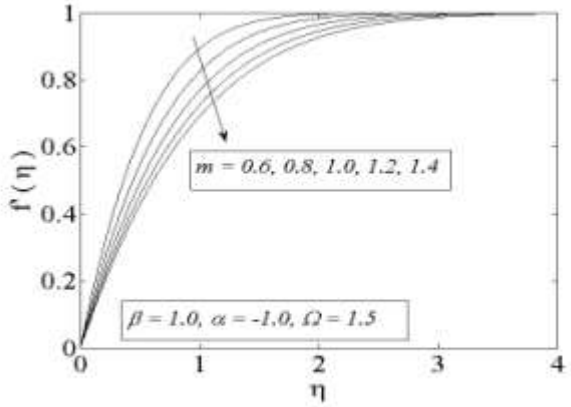

Figure 2(b): Variation of Velocity profiles $f'(\eta)$ with η for various various values of m , $\alpha = -1, \beta = 1,$ and $\Omega = 1.5$

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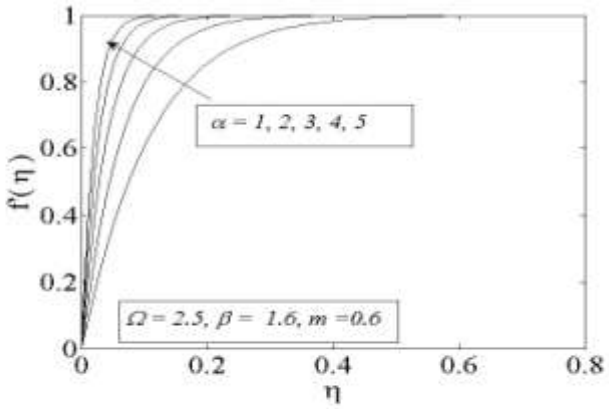

Figure 3(a): Variation of Velocity profiles $f'(\eta)$ with η for various various values of α , $m = 0.6, \beta = 1.6,$ and $\Omega = 2.5$

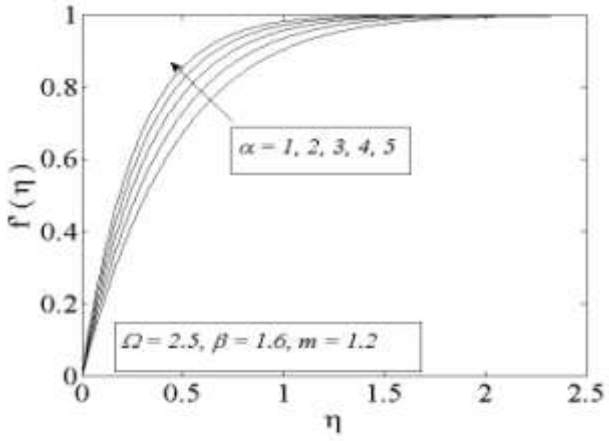

Figure 3(b): Variation of Velocity profiles $f'(\eta)$ with η for various various values of α , $m = 1.2, \beta = 1.6,$ and $\Omega = 2.5$

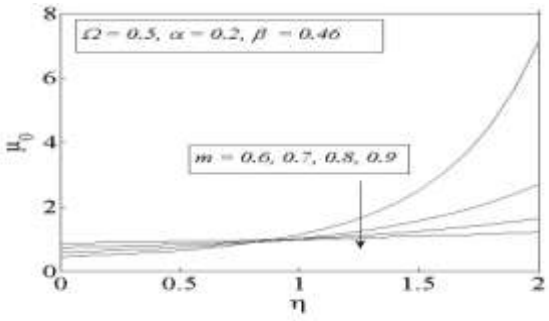

Figure 4(a): Variation of Velocity profiles μ_0 with η for various various values of m , $\alpha = 0.2$, $\beta = 0.46,$ and $\Omega = 0.5$

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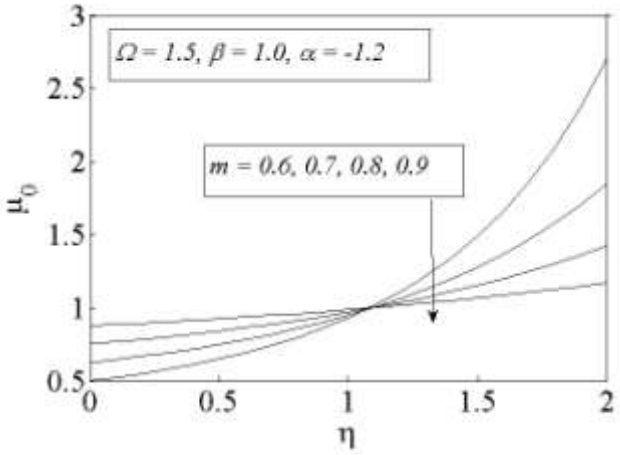

Figure 4(b): Variation of Velocity profiles μ_0 with η for various various values of m , $\alpha = -1.2$, $\beta = 1$ and $\Omega = 1.5$

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