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PRIME LABELING OF CHAVATAL RELATED GRAPHS

Dr.C.Sekar*
Associate professer of Mathematics, Adithanar college, Tiruchendur, Tamilnadu, India
S.Chandrakala**
Assistant professor of Mathematics, T.D.M.N.S College, T. Kallikulam, Tamilnadu, India.

Abstract: A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers 1,2,…|V| such that for edge xy the labels assigned to x and y are relatively prime . A graph which admits prime labeling is called a prime graph. In this paper, we investigated prime labeling for Chavatal related graphs.

Keywords: Prime labeling, prime graph,Chavatal graph, corona, Duplication.

I.INTRODUCTION

In this paper, only finite simple undirected graphs are considered. The graph G has vertex set V =V (G) and edge set E = E (G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology we refer to Bondy and Murthy [1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [6].Two integers a and b are said to be relatively prime if their greatest common divisor is 1.Many researchers have studied prime graph. Fu.H[3] has proved that the path P_n on n vertices is a prime graph.Deretsky et al [2] have proved that the cycle C_n on n vertices is a prime graph .Around 1980 Roger Entringer conjectured that all trees have prime labeling which has not been settled so far.

In [7] S.K. Vaidhya and K.K. Kanmani have proved that the graphs obtained by identifying any two vertices duplicating arbitrary vertex and switching of any vertex in cycle C_n admit prime labeling. In [5] Meena and Vaithilingam have proved the Prime labeling for some helm related graphs.

II.PRELIMINARY DEFINITIONS

Definition 2.1

Let G = (V (G), E (G)) be a graph with P vertices .A bijection f:V (G)→ {1, 2…P} is called a prime labeling if for each edge e $=uv$, gcd$\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a prime graph.

Definition 2.2

Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that N$(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'.

Definition 2.3

The corona of two graphs $G_1$and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 2.4

$< G, K_{1,m} >, m \geq 1$ is the graph obtained by attaching $K_{1,m}$ to one vertex of the graph G.

III.MAIN RESULTS

Definition 3.1

A Chavatal graph is a triangle free 4 – regular graph with 12 vertices and 24 edges as shown in the figure 1.

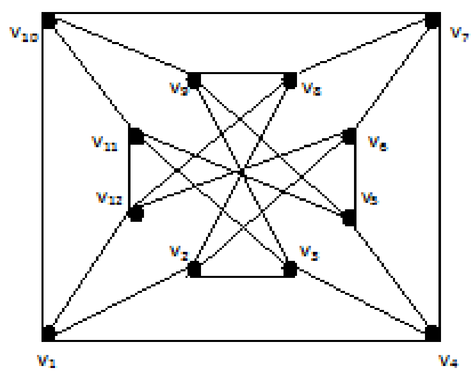

Figure:1 chavatal graph

Theorem 3.2

The Chavatal graph is not a prime graph.

Proof:

Let G be a Chavatal graph. The vertex set of G is $V(G) = \{v_1, v_2, \dots \dots \dots v_{12}\}$.

The edge set of G is $E(G) = \{v_i v_{i+1}/1 \leq i \leq 11\} \cup \{v_i v_{i+3} | i = 1,4,7\} \cup \{v_i v_{12} | i = 1,6,8\} \cup \{v_2 v_i\ ,\ i = 6,8\} \cup \{v_3 v_i |\ i = 9,11\} \cup \{v_5 v_i | i = 9,11\} \cup \{v_1 v_{10}\}$

Here G is a 4 regular graph and having 12 vertices ,24 edges .Therefore G must contain at the most two edges have even labels .Any prime labeling of G,it is not possible to assign even labels for two adjacent vertices. Hence chavatal graph is not a prime graph.

Theorem 3.3

Let G be a chavatal graph .Then $G\odot k_1$ is a prime graph.

Proof:

Let G be a Chavatal graph. Let $G^* = G\odot k_1$.

The vertex set of G^* is$V(G^*) = \{v_1, v_2, \dots \dots .., v_{12}, v_1{}', v_2{}', \dots .., v_{12}{}'\}$.

The edge set of G^* is

$E(G^*) = \{v_i v_{i+1}/1 \le i \le 11\} \cup \{v_i v_{i+3} | i = 1,4,7\} \cup \{v_i v_{12} | i = 1,6,8\} \cup \{v_2 v_i | i = 6,8\} \cup \{v_3 v_i | i = 9,11\} \cup \{v_5 v_i | i = 9,11\} \cup \{v_1 v_{10}\} \cup \{ v_i v_i{}'/1 \le i \le 12\}$.

Then G* has 24 vertices and 36 edges.

Define a labeling $f: V(G^*) \to \{1,2, \dots \dots \dots \dots 24\}$ as follows.

$f(v_i) = 2i - 1, i = 1,2,3,4,6,7,9,10,11,12$;

$f(v_5) = 10$; $f(v_8) = 16$;

$f(v_i) = 2i, i = 1,2,3,4,6,7,9,10,11,12$;

$f(v_5') = 9$;

$f(v_8') = 15$.

Now,$g.c.d \{f(v_i), f(v_{i+1})\} = g.c.d\{2i - 1, 2i + 1\} = 1$ for $i = 1,2,3,4,6,7,9,10,11$

$g.c.d \{f(v_i), f(v_{i+3})\} = g.c.d\{2i - 1, 2i + 5\} = 1$ for $i = 1,4,7$

$g.c.d \{f(v_i), f(v_{12})\} = g.c.d\{2i - 1, 23\} = 1$ for $i = 1,6,8$

$g.c.d \{f(v_2), f(v_6)\} = g.c.d\{3,11\} = 1$

$g.c.d \{f(v_2), f(v_8)\} = g.c.d\{3,16\} = 1$

$g.c.d \{f(v_3), f(v_i)\} = g.c.d\{5, 2i - 1\} = 1$ for $i = 9,11$

$g.c.d \{f(v_5), f(v_i)\} = g.c.d\{10, 2i - 1\} = 1$ for $i = 9,11$

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Thus for any two adjacent vertices x & y are labeled and $g.c.d\{f(x), f(y)\} = 1 \forall xy \in E(G^*)$.

Hence f admits a prime labeling .Hence G*is a prime graph.

Illustration of theorem 3.3

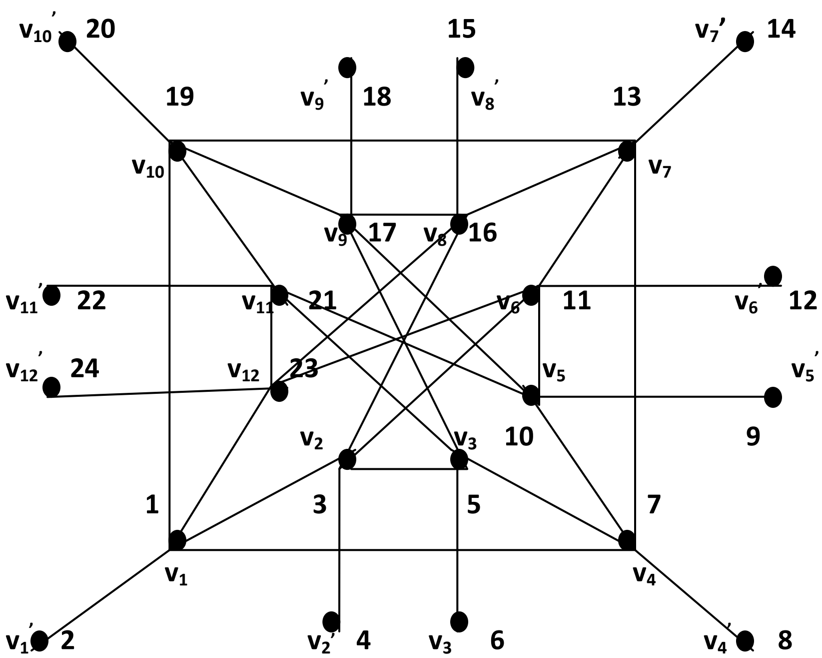

Figure :2 Prime labeling of $G \odot K_1$ where G is a Chavatal graph

Theorem 3.4

Let G be a chavatal graph .Then the graph $G \odot {K_2}^C$ is a prime graph.

Proof:

Let G be a chavatal graph .Let $G^* = G \odot {K_2}^C$.Let the vertex set of G* is $V(G^*) = \{v_1, v_2, \dots \dots \dots . v_{12}, v_1', v_2', \dots \dots \dots . v_{12}', u_1, u_2, \dots \dots \dots \dots . u_{12}\}$.The edge set of G* is

$E(G^*) = \{v_i v_{i+1} / 1 \le i \le 11\} \cup \{v_i v_{i+3} | i = 1,4,7\} \cup \{v_i v_{12} | i = 1,6,8\} \cup \{v_2 v_i , \ i =$ *6,8∪v3vi i=9,11∪v5vii=9,11∪{v1v10}∪{ vivi'| 1≤i≤12} {viui/1≤i≤12}.*

Then |V(G*)|=36 and |E(G*)|=48.Define the labeling $f: V(G^*) \to \{1,2, \dots \dots \dots \dots \dots .36\}$

by $f(v_i) = 2i - 1, i = 1,2,3,4,6,7,9,10,11,12$

$f(v_5) = 10$; $f(v_8) = 16$; $f(v_i') = 2i, i = 1,2,3,4,6,7,9,10,11,12$;

$f(v_5') = 9$; $f(u_8') = 15$; $f(u_{i+1}) = 24 + i, i = 1,2,3,6,7,8,9,10,11$;

$f(u_5) = 29$; $f(u_6) = 28$.

Clearly any two adjacent vertices x and y are labeled and g $. c. d\{fx), f(y)\} = 1 \; \forall \; xy \; \in E(G^*)$.

Hence f admits a prime labeling and hence G* is a prime graph .

Illustration of Theorem 3.4

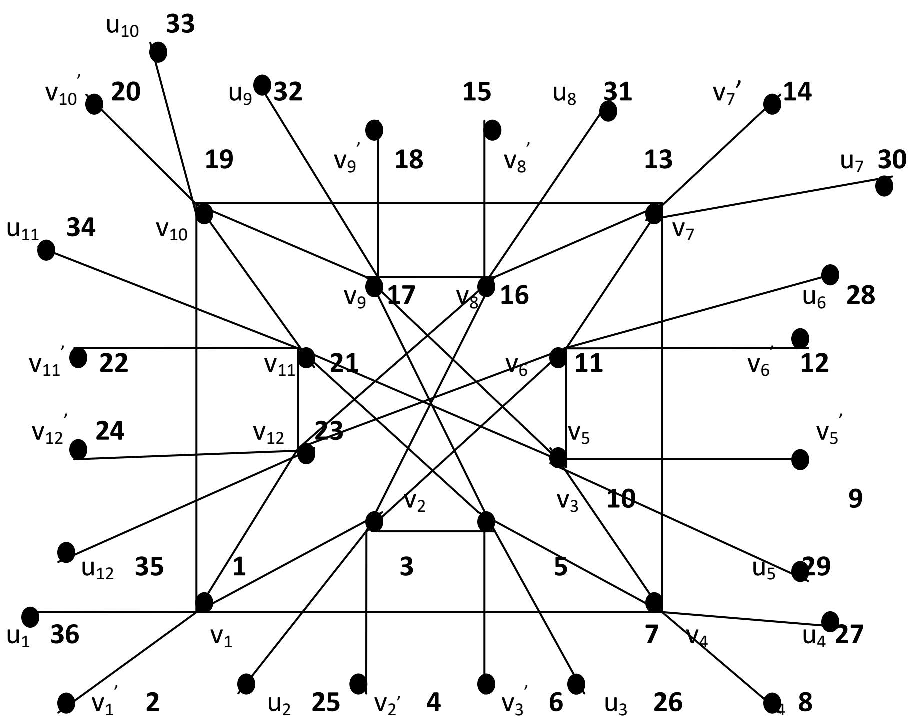

Figure :3 Prime labeling of G⊙$K_2{}^C$ where G is a chavatal graph.

Theorem 3.5

Let G be charatal graph .The graph obtained by duplicating the vertex of $G \odot K_2{}^C$ is a prime graph .

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Proof:

Let G* be the graph obtained by duplicating the vertex of $G\odot {K_2}^C$ where G is a chavatal graph. Let w be the new vertex by duplicating a vertex in $G\odot {K_2}^C$.

Let $(G^*) = \{v_1, v_2, \ldots \ldots \ldots \ldots . v_{12}, v_1', v_2', \ldots \ldots v_{12}', u_{1,} u_2, \ldots \ldots \ldots . u_{12}, w\}$ .Then $(G^*) = 37$.

Define a labeling $f: V(G^*) \to \{1,2, \ldots \ldots .37\}$ as follows

$f(v_i) = 2i - 1, i = 1,2,3,4,6,7,9,10,11,12$

$f(v_5) = 10$; $f(v_8) = 16$; $f(v_i') = 2i, i = 1,2,3,4,6,7,9,10,11,12$;

$f(v_5') = 9$; $f(u_8') = 15$; $f(u_{i+1}) = 24 + i, i = 1,2,3,6,7,8,9,10,11$;

$f(u_5) = 29$; $f(u_6) = 28$; $f(w) = 37$.

Then clearly any two adjacent vertices x and y are labeled and $g.c.d\{f(x), f(y)\} = 1 \forall xy \in E(G^*)$

Thus f admits a prime labeling and hence G* is a prime graph.

Illustration of theorem 3.5

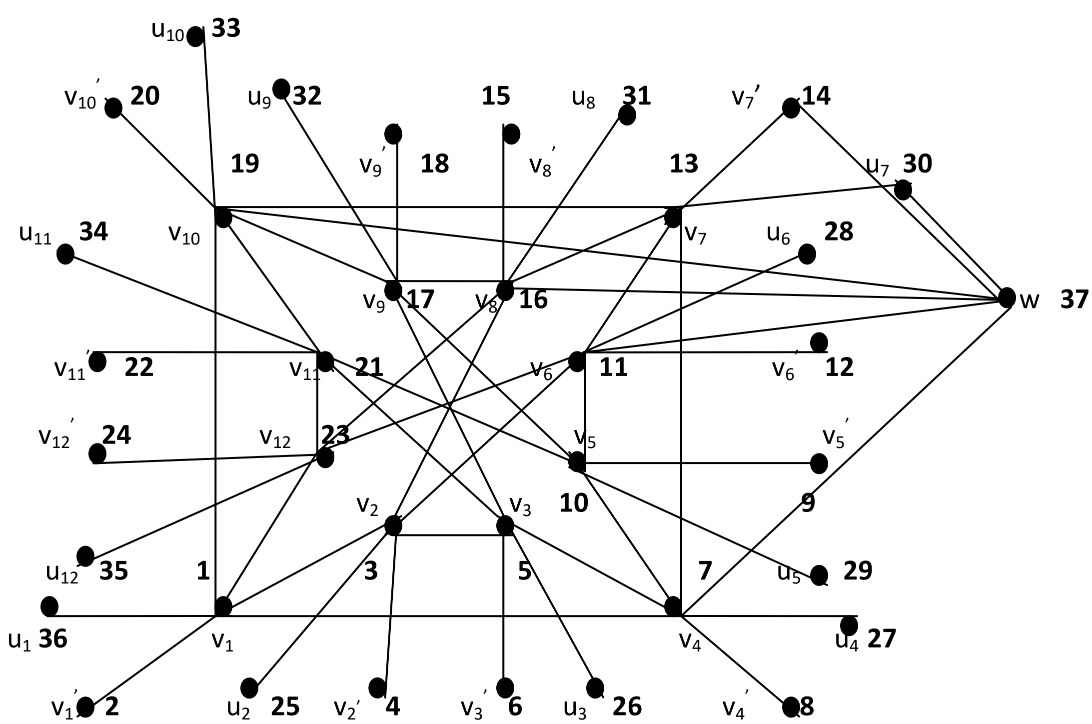

Figure: 4 Duplication of a vertex v_7 of a graph $G\odot {K_2}^C$ where G is a chavatal graph

Theorem 3.6

The graph $< G, k_{1,m} >, m \geq 6$, is a prime graph where G is a chavatal graph.

Proof:

Let G be a chavatal graph. Let $G^* =< G, k_{1,m} >\ , m \geq 6$. The vertex set of G^* is

$V(G^*) = \{v_1, v_2, \dots \dots \dots . v_{12}. u_1, u_2, \dots \dots \dots, u_m\}$ .Let v_1 be the common vertex of G and $K_{1,m}$.

Then $|V(G^*)| = 12 + m$ and $(G^*) = 24 + m\ , m \geq 6$ .The edge set of $G^*$is

$E(G^*) = \{v_i v_{i+1}/1 \leq i \leq 11\} \cup \{v_i v_{i+3} | i = 1,4,7\} \cup \{v_i v_{12} | i = 1,6,8\} \cup \{v_2 v_i\ ,\ i = 6,8\} \cup \{v_3 v_i | \ i = 9,11\} \cup \{v_5 v_i | i = 9,11\} \cup \{v_1 v_{10}\} \ \cup \{v_1 v_i | 1 \leq i \leq m\}$.

Define a labeling $f: V(G^*) \to \{1,2, \dots .. |V(G^*)|\}$ as follows.

$f(v_i) = 2i - 1, i = 1,2,3,4,5,7,9$;

$f(v_6) = 14$; $f(v_8) = 16$; $f(v_{10}) = 18$; $f(v_{11}) = 11$; $f(v_{12}) = 15$;

$f(u_i) = 2i, i = 1,2,3,4,5,6$; $f(u_i) = 12 + \mathrm{i}\ , 7 \leq i \leq m$.

Clearly all the vertices have distinct labels and $g, c. d\{f(x), f(y)\} = 1 \forall xy \in E(G^*)$.

Then f admits a prime labeling .Hence $< G, k_{1,m} >,\ m \geq 6$, is a prime graph.

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Illustration of theorem 3.6

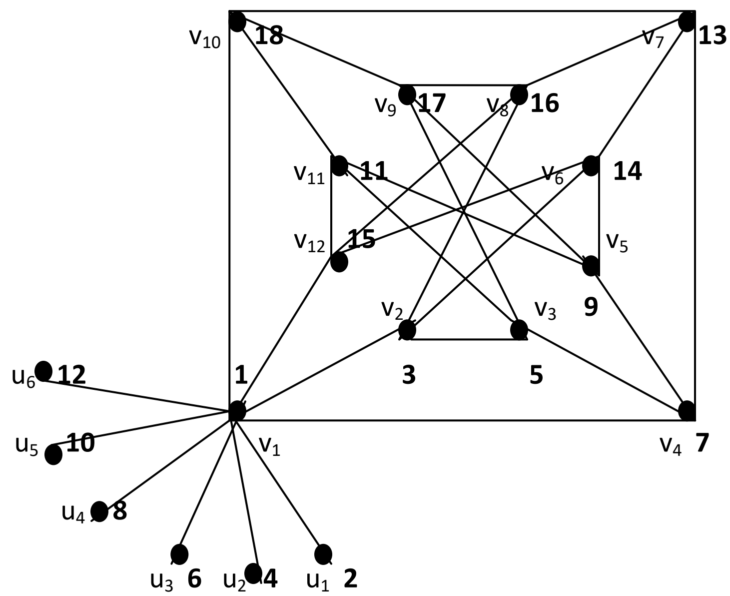

Figure: 5 Prime labeling of the graph $< G, K_{1,m} >$ $, m \geq 6$ **where G is chavatal graph.**

Theorem 3.7

Let G be a chavatal graph .The graph obtained by duplicating a vertex of $< G, k_{1,m} >$, $6 \leq m \leq 24$ is a prime graph.

Proof:

Let G^* be a graph obtained by duplicating a vertex of $< G, k_{1,m} >, 6 \leq m \leq 24$ where G is a chavatal graph . Let $G^* =< G, K_{1,m} >$ $, 6 \leq m \leq 24$. Let w be the new vertex by duplicating a vertex of the graph G^*.

The vertex set of G^* is $V(G^*) = \{v_1, v_2 \ldots \ldots \ldots v_{12}, u_1, u_2, \ldots \ldots, u_m, w\}$. Then $|V(G^*)| = 13 + m$ $, m \geq 6$.

Define a Labeling $f: V(G^*) \rightarrow \{1,2, \ldots \ldots . |V(G^*)|\}$ as follows.

$f(v_i) = 2i - 1, i = 1,2,3,4,5,7,9$;

$f(v_6) = 14$; $f(v_8) = 16$; $f(v_{10}) = 18$; $f(v_{11}) = 11$; $f(v_{12}) = 15$;

$f(u_i) = 2i$, $i = 1,2,3,4,5,6$; $f(u_i) = 13 + i$, $7 \leq i \leq m$; $f(w) = 19$.

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Clearly all the vertices have distinct labels and $g.c.d\{f(x), f(y)\} = 1 \forall xy \in E(G^*)$.

Hence the duplication of a vertex of $< G, k_{1,m} >$,$6 \leq m \leq 24$ is a prime graph.

Example 3.8

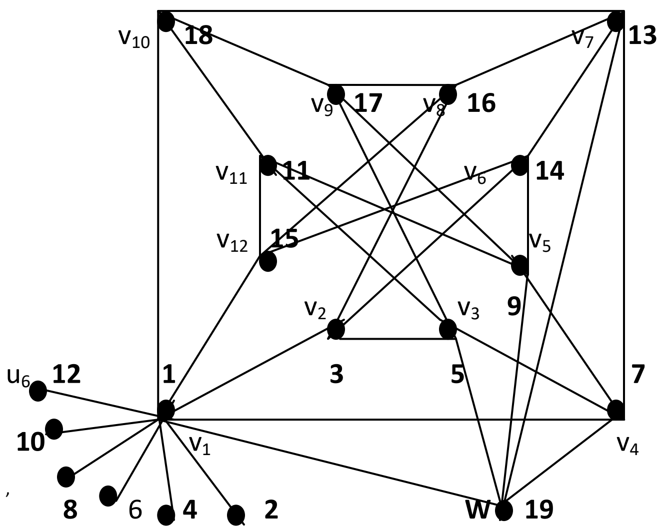

Figure: 6 Duplication of a vertex v_1 of $< G, K_{1,6} >$

Theorem 3.9

Let G be a chavatal graph .The graph $< G, k_{1,m} > +k_1, 6 \leq m \leq 24$ is a prime graph.

Proof:

Let G be a chavatal graph .Let $G^* =< G, k_{1,m} > +k_1$,$6 \leq m \leq 24$ .Let w be the new vertex of G^* .

Let the vertex set of G^*be $V(G^*) = \{v_1, v_2 \dots \dots \dots v_{12}, u_1, u_2 \dots \dots \dots u_m, w\}$. Then $|V(G^*)| = 19$.

The edge set $E(G) = \{v_i v_{i+1} / 1 \leq i \leq 11\} \cup \{v_i v_{i+3} | i = 1,4,7\} \cup \{v_i v_{12} | i = 1,6,8\} \cup \{v_2 v_i\ ,\ i = 6,8\} \cup \{v_3 v_i | \ i = 9,11\} \cup \{v_5 v_i | i = 9,11\} \cup \{v_1 v_{10}\} \cup \{w v_i | 1 \leq i \leq 12\} \cup \{v_1 u_i | 1 \leq i \leq m\} \cup \{w u_i | 1 \leq i \leq m\}$.

Define a labeling $f: V(G^*) \rightarrow \{1,2, \dots \dots \dots \dots . |V(G^*)|\}$ as follows

$f(v_i) = 2i - 1, i = 1,2,3,4,5,7,9$; $f(v_6) = 14$; $f(v_8) = 16$; $f(v_{10}) = 18$; $f(v_{11}) = 11$;

$f(v_{12}) = 15$; $f(u_i) = 2i, i = 1,2,3,4,5,6$; $f(w) = 19$; $f(u_i) = 13 + i$, $7 \leq i \leq m$.

Clearly all the vertices have distinct labels and $\gcd\{f(x), f(y)\} = 1 \forall\, xy \in E(G^*)$.

Hence $< G, k_{1,m} > +k_1$, $6 \leq m \leq 24$ is a prime graph.

Illustration of theorem 3.9

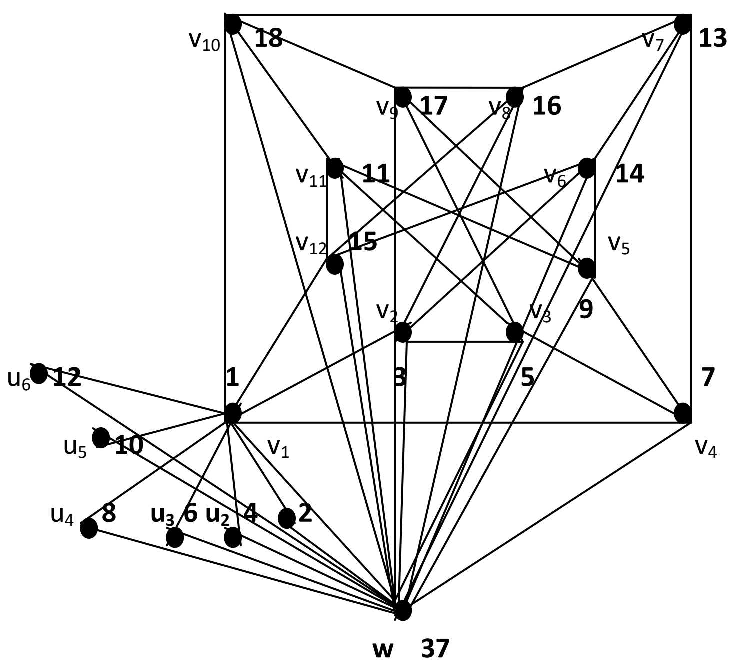

Figure :7 Prime labeling of the graph $< G, K_{1,6} > +k_1$, where G is chavatal

IV.CONCLUSION

In this paper, prime labeling for the special graph namely Chavatal graph is investigated .Extending the graph by the operation duplication is also discussed. Further discussion will be performed in this context.

V.ACKNOWLEDGEMENT

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