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INTUITIONISTIC FUZZY RGW- CLOSED SETS

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Abstract

The aim of this paper is to introduce the new class of intuitionistic fuzzy closed sets called intuitionistic fuzzy rgw-closed sets in intuitionistic fuzzy topological space. The class of all intuitionistic fuzzy rgw-closed sets lies between the class of all intuitionistic fuzzy swg-closed sets and class of all intuitionistic fuzzy rwg-closed sets. We also introduce the concepts of intuitionistic fuzzy rgw- open sets, intuitionistic fuzzy rgw-continuous mappings in intuitionistic fuzzy topological spaces.

Keywords:

Intuitionistic fuzzy rgw-closed sets;
Intuitionistic fuzzy rgw-open sets:
Intuitionistic fuzzy rgw-connectedness;
Intuitionistic fuzzy rgw-compactness;
intuitionistic fuzzy rgw-continuous mappings.

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1. Introduction

After the introduction of fuzzy sets by Zadeh [24] in 1965 and fuzzy topology by Chang [4] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. In 2008 Thakur and Chtuvedi introduced the concepts of intuitionistic fuzzy generalized closed sets [14] in intuitionistic fuzzy topology. After that many weak and strong forms of intuitionistic fuzzy g-closed sets such as intuitionistic fuzzy rg-closed sets[15],intuitionistic fuzzy sg-closed sets[16], intuitionistic fuzzy gα-closed sets[8], intuitionistic fuzzy w-closed sets[17], intuitionistic fuzzy rw-closed sets[18], intuitionistic fuzzy gpr-closed sets[19], intuitionistic fuzzy rgα-closed sets[20], intuitionistic fuzzy gsp-[13]closed sets, intuitionistic fuzzy gp[11] , intuitionistic fuzzy strongly g*-closed sets [3] intuitionistic fuzzy sgp-closed sets[2], intuitionistic fuzzy swg-closed sets[9] and intuitionistic fuzzy rwg-closed sets[12] have been appeared in the literature.

In the present paper we extend the concepts of fuzzy rgw-closed sets due to Mishra S. and Bhardwaj N.[10] in intuitionistic fuzzy topological spaces . The class of intuitionistic fuzzy rgw-closed sets is properly placed between the class of intuitionistic fuzzy swg-closed sets and intuitionistic fuzzy rwg-closed sets. We also introduced the concepts of intuitionistic fuzzy rgw-open sets, intuitionistic fuzzy rgw $T_{1/2}$-space, intuitionistic fuzzy rgw-continuous mappings and intuitionistic fuzzy rgw-irresolute mappings and obtain some of their characterization and properties.

2. Preliminaries

Let X be a nonempty fixed set. An intuitionistic fuzzy set A[1] in X is an object having the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\Upsilon_A: X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$. The intutionistic fuzzy sets $\mathbf{0} = \{< x, 0, 1 > : x \in X\}$ and $\mathbf{1} = \{< x, 1, 0 > : x \in X\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set $A = \{< x, \mu_A(x), \gamma_A(x) > : x \in X\}$ is called a subset of an intuitionistic fuzzy set $B = \{< x, \mu_B(x), \gamma_B(x) > : x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for each $x \in X$. The complement of an intuitionistic fuzzy set $A = \{< x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is the intuitionistic fuzzy set $A^c = \{<x, \gamma_A(x), \mu_A(x)>: x \in X\}$. The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets $A_i = \{< x, \mu_{Ai}(x), \gamma_{Ai}(x) > : x \in X, (i \in \wedge)\}$ of X be the intuitionistic fuzzy set $\cap A_i = \{<x, \wedge \mu_{Ai}(x), \vee \gamma_{Ai}(x) > : x \in X\}$ (resp. $\cup A_i = \{<x, \vee \mu_{Ai}(x), \wedge \gamma_{Ai}(x) >: x \in X\}$). Two intuitionistic fuzzy sets $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ and $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ are said be q-coincident (A_qB for short) if and only if $\exists$ an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$. A family $\Im$ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology [6]on X if the intuitionistic fuzzy sets $\mathbf{0},\mathbf{1} \in \Im$, and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair $(X,\Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set .The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted int(A) [5].

Lemma 2.1 [5]: Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space $(X, \Im)$. Then:

(a) $(A_qB) \Leftrightarrow A \subseteq B^c$.
(b) A is an intuitionistic fuzzy closed set in X $\Leftrightarrow$ cl (A) = A
(c) A is an intuitionistic fuzzy open set in X $\Leftrightarrow$ int (A) = A.
(d) $cl(A^c) = (int(A))^c$.
(e) $int(A^c) = (cl(A))^c$.
(f) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$.
(g) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$.
(h) $cl(A \cup B) = cl(A) \cup cl(B)$.
(i) $int(A \cap B) = int(A) \cap int(B)$

Definition 2.1 [6]: Let X is a nonempty set and $c \in X$ a fixed element in X. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha + \beta \leq 1$ then:

(a) $c(\alpha,\beta) = < x, c_\alpha, c_{1-\beta} >$ is called an intuitionistic fuzzy point in X, where α denotes the degree of membership of $c(\alpha,\beta)$, and β denotes the degree of non membership of $c(\alpha,\beta)$.

(b) $c(\beta) = < x, 0, 1 - c_{1-\beta} >$ is called a vanishing intuitionistic fuzzy point in X, where β denotes the degree of non membership of $c(\beta)$.

Definition 2.2[7] : A family $\{ G_i : i \in \wedge \}$ of intuitionistic fuzzy sets in X is called an intuitionistic fuzzy open cover of X if $\cup \{ G_i : i \in \wedge \} = \mathbf{1}$ and a finite subfamily of an intuitionistic fuzzy open cover $\{ G_i : i \in \wedge \}$of X which also an intuitionistic fuzzy open cover of X is called a finite sub cover of $\{ G_i : i \in \wedge \}$.

Definition 2.3[7]: An intuitionistic fuzzy topological space $(X,\Im)$ is called intuitionistic fuzzy compact if every intuitionistic fuzzy open cover of X has a finite sub cover.

Definition 2.4 [23]: An intuitionistic fuzzy topological space X is called intuitionistic fuzzy connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy open and intuitionistic fuzzy closed .

Definition 2.5[5]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) An intuitionistic fuzzy semi open of X if there is an intuitionistic fuzzy set O such that $O \subseteq A \subseteq cl(O)$

(b) An intuitionistic fuzzy semi closed if the compliment of A is an intuitionistic fuzzy semi open set.

(c) An intuitionistic fuzzy regular open of X if $int(cl(A)) = A$.

(d) An intuitionistic fuzzy regular closed of X if $cl(int(A)) = A$.

(e) An intuitionistic fuzzy pre open if $A \subseteq int(cl(A))$.

(f) An intuitionistic fuzzy pre closed if $cl(int(A)) \subseteq A$

Definition 2.6 [22]: An intuitionistic fuzzy set A in intuitionistic fuzzy topological space $(X, \Im)$ is called intuitionistic fuzzy regular semi open if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.

Remark 2.1[21]: An intuitionistic fuzzy set is intuitionistic fuzzy regular semi open if and only if it is both intuitionistic fuzzy semi open and intuitionistic fuzzy semi closed.

Definition 2.7[5]: If A is an intuitionistic fuzzy set in intuitionistic fuzzy topological space$(X,\Im)$ then

(a) $scl(A) = \cap \{ F : A \subseteq F$, F is intuitionistic fuzzy semi closed$\}$

(b) $pcl(A) = \cap \{ F : A \subseteq F$, F is intuitionistic fuzzy pre closed$\}$

Definition 2.8*:* An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) Intuitionistic fuzzy g-closed if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy open.[14]

(b) Intuitionistic fuzzy rg-closed if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open.[15]

(c) Intuitionistic fuzzy sg-closed if scl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[16]
(d) Intuitionistic fuzzy gα-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy α- open.[8]
(e) Intuitionistic fuzzy w-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[17]
(f) Intuitionistic fuzzy rw-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular semi open.[18]
(g) Intuitionistic fuzzy gpr-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[19]
(h) Intuitionistic fuzzy rgα-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular α-open.[20]
(i) Intuitionistic fuzzy gsp-closed if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy - open.[13]
(j) Intuitionistic fuzzy gp-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[11]
(k) Intuitionistic fuzzy sgp closed set if if pcl $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi-open in X.[2].
(l) Intuitionistic fuzzy rwg-closed set if cl (int(A)) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular- open.[12]
(m) Intuitionistic fuzzy swg-closed set if cl (int(A)) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[9]

The complements of the above mentioned closed set are their respective open sets.

Remark 2.2:
(a) Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy g-closed but its converse may not be true.[17]
(b) Every intuitionistic fuzzy g- closed set is intuitionistic fuzzy rwg-closed but its converse may not be true.[12]
(c) Every intuitionistic fuzzy w-closed (resp. Intuitionistic fuzzy w-open) set is intuitionistic fuzzy rw-closed (intuitionistic fuzzy g-open) but its converse may not be true.[18]
(d) Every intuitionistic fuzzy swg-closed (resp. Intuitionistic fuzzy swg-open) set is intuitionistic fuzzy rw-closed (intuitionistic fuzzy g-open) but its converse may not be true.[9]

***Definition 2.9**: [**5**]:* Let X and Y are two nonempty sets and f: X $\rightarrow$ Y is a function. :
(a) If B = {<y, $\mu_B(y)$, $\gamma_B(y)$> : y $\in$ Y}is an intuitionistic fuzzy set in Y, then the pre image of B under f denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = <x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)>: x \in X\}.$$

(b) If A = {<x, $\lambda_A(x)$, $\nu_A(x)$> : x $\in$ X}is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by

$$f(A) = \{<y, f(\lambda_A)(y), f(\nu_A)(y)>: y \in Y\}$$

Where $f(\nu_A) = 1 - f(1- \nu_A)$.

Definition 2.10[5]: Let (X,$\Im$) and (Y, σ) be two intuitionistic fuzzy topological spaces and let *f*: X$\rightarrow$Y be a function. Then *f* is said to be, Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.

Definition 2.11[21]: Let (X,ℑ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let *f*: X→Y be a function. Then *f* is said to be intuitionistic fuzzy almost continuous if inverse image of every intuitionistic fuzzy regular closed set of Y is intuitionistic fuzzy closed in X.

Definition2.12 [21]: Let (X,ℑ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let *f*: X→Y be a function. Then *f* is said to be intuitionistic fuzzy almost irresolute if inverse image of every intuitionistic fuzzy regular semi open set of Y is intuitionistic fuzzy semi open in X.

Definition 2.13: Let (X,ℑ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: X→Y be a function. Then *f* is said to be

(a) Intutionistic fuzzy w-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy w –closed in X.[17]
(b) Intutionistic fuzzy rw-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rw –closed in X. [18]

Remark 2.4

(a) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy w-continuous, but the converse may not be true [17].
(b) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy rw-continuous, but the converse may not be true [178.
(c) Every intuitionistic fuzzy w- continuous mapping is intuitionistic fuzzy rw-continuous, but the converse may not be true [18].

***Definition 2.14*:** An intuitionistic fuzzy topological space (X,ℑ) is said to be :

(a) Intuitionistic fuzzy $T_{1/2}$ space if every intuitionistic fuzzy g-closed set is closed in (X,ℑ).[14]
(b) Intuitionistic fuzzy w-$T_{1/2}$ space if every intuitionistic fuzzy w-closed set is closed in (X,ℑ).[17]
(c) Intuitionistic fuzzy rw-$T_{1/2}$ space if every intuitionistic fuzzy rw-closed set is closed in (X,ℑ).[18]
(d) Intuitionistic fuzzy sw$T_{1/2}$ space if every intuitionistic fuzzy swg-closed set is intuitionistic fuzzy closed in (X,ℑ).[9]

3. Intuitionistic fuzzy rgw-closed set

Definition 3.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,ℑ) is called an intuitionistic fuzzy rgw-closed if cl (int(A)) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy regular semi open in X.

First we prove that the class of intuitionistic fuzzy rgw- closed sets properly lies between the class of intuitionistic fuzzy swg-closed sets and the class of intuitionistic fuzzy rwg-closed sets.

Theorem 3.1: Every intuitionistic fuzzy closed set is intuitionistic fuzzy rgw-closed.

Proof : Let A be an intuitionistic fuzzy closed set in (X, τ). Let U be an intuitionistic fuzzy regular semi open set in (X, τ) such that A ⊆ U. Since A is an intuitionistic fuzzy closed, cl(A) = A and hence cl(A) ⊆ U. But cl(int(A)) ⊆ cl(A) ⊆ U. Therefore, cl (int (A)) ⊆ U. Hence A is an intuitionistic fuzzy rgw-closed in X.

Remark 3.1: The converse of the Theorem 3.1 need not be true, as seen from the following example

Example 3.1: Let X = {a, b, c, d, e} and intuitionistic fuzzy sets O, U, V defined as follows

O = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0,1>,<d,0,1> <e,0,1>}
U = { < a,0,1,> ,<b,0,1>,<c,0.8,0.1>,<d,0.7,0.2> <e,0,1>}

V = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0.8,0.1>, <d,0.7,0.2>,<e,0,1>}

Let ℑ = {***0***, O, U, V, ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.9 ,0.1 >,<b ,0, 1 >,<c, 0, 1>,<d, 0, 1>,< e,0,1 >} is intuitionistic fuzzy rgw -closed but it is not intuitionistic fuzzy -closed.

Theorem 3.2: Every intuitionistic fuzzy pre-closed set is intuitionistic fuzzy rgw-closed.

Proof **:** Let A be an intuitionistic fuzzy pre-closed set in (X, τ). Let U be an intuitionistic fuzzy regular semi open set in (X, τ) such that A ⊆ U. By def of intuitionistic fuzzy pre-closed cl(int(A)) ⊆ A So we have cl(int(A)) ⊆ A ⊆ U that is cl(int(A)) ⊆ U Hence A is an intuitionistic fuzzy rgw-closed in X.

Remark 3.2: The converse of the theorem 3.2 need not be true, as seen from the following example

Example 3.2: Let X = {a, b, c} and intuitionistic fuzzy sets O, U defined as follows

O = {< a, 0.9, 0.1 > ,<b, 0,1 > <c,0,1>},
U= {< a, 0, 1 > ,<b, 0.8,0.1 > ,<c,0,1>}

Let ℑ = {***0*** O, U, ***1***} be an intuitionistic fuzzy topology on X. Then intuitionistic fuzzy set A= {< a, 0.6, 0.1 >,< b, 0.8, 0.2 > ,<c, 0, 1 >} is intuitionistic fuzzy rgw -closed set in X , but it is not intuitionistic fuzzy pre-closed.

Theorem 3.3: Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy rgw-closed.

Proof: Let *A* be an intuitionistic fuzzy w-closed set in *X*. Suppose *A* ⊆ *U*, where U is intuitionistic fuzzy regular semi-open in X. Since every intuitionistic fuzzy regular semi open set is intuitionistic fuzzy semiopen i.e. *A* ⊆ *U*, and *U* is intuitionistic fuzzy semi-open in *X*. By definition of intuitionistic fuzzy *w*-closed set we have cl(A) ⊆ U. But cl(int(A)) ⊆ cl(A) ⊆ U. Therefore, cl (int (A)) ⊆ U whenever *A* ⊆ *U* and *U* is intuitionistic fuzzy regular semi-open in *X*. Hence *A* is intuitionistic fuzzy *rgw*-closed set.

Remark 3.3: The converse of the theorem 3.3 need not be true, as seen from the following example

Example 3.3: Let X = {a, b} and Á = {***0***, U, ***1***} be an intuitionistic fuzzy topology on X, where U ={< a, 0.5 ,0.5>,< b, 0.4, 0.6 >}.Then the intuitionistic fuzzy set A ={ <a, 0.5, 0.5 >,<b,0.3,0.7>} is intuitionistic fuzzy rgw -closed but it is not intuitionistic fuzzy w-closed.

Theorem 3.4: Every intuitionistic fuzzy rw-closed set is intuitionistic fuzzy rgw-closed.

Proof: Let *A* be an intuitionistic fuzzy rw-closed set in *X*. Suppose *A* ⊆ *U*, where U is intuitionistic fuzzy regular semi-open in X. By definition of intuitionistic fuzzy r*w*-closed set we have cl(A) ⊆ U. But cl(int(A)) ⊆ cl(A) ⊆ U. Therefore, cl (int (A)) ⊆ U whenever *A* ⊆ *U* and *U* is intuitionistic fuzzy regular semi-open in *X*. Hence *A* is intuitionistic fuzzy *rgw*-closed set.

Remark 3.4: The converse of theorem 3.4 need not be true as from the following example.

Example 3.4: Let X = {a, b, c, d } and intuitionistic fuzzy sets O,U, V,W defined as follows

O = { < a,0.9,0.i> ,<b,0,1>,<c,0,1>,<d,0,1>}

U = { < a,0,1,> ,<b,0.8,0.1>,<c,0,1>,<d,0,1>}

V = { < a,0.9,0.1> ,<b,0.8,0.1,>,<c,0,1>,<d,0,1>}

W = { < a,0.9,0.1,> ,<b,0.8,0.1>,<c,0.7,0.2>,<d,0,1>}

ℑ = {***0***, O, U, V, W, ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {< a,0, 1>,< b, 0, 1> <c, 0.7, 0.2>,< d, 0, 1 >} is intuitionistic fuzzy rgw -closed but it is not intuitionistic fuzzy rw-closed.

Theorem 3.5: Every intuitionistic fuzzy swg-closed set is intuitionistic fuzzy rgw-closed.

Proof: Let *A* be an arbitrary intuitionistic fuzzy swg-closed set in *X*. Suppose *A* ⊆ *U*, where U is intuitionistic fuzzy regular semi-open in X. Since every intuitionistic fuzzy regular semi open set is intuitionistic fuzzy semi open i.e. *A* ⊆ *U*, and *U* is intuitionistic fuzzy semi-open in *X*. By definition of intuitionistic fuzzy *swg*-closed set we have *cl*(*int*(*A*)) ⊆ *U* whenever *A* ⊆ *U* and *U* is intuitionistic fuzzy regular semi-open in *X*. Hence *A* is intuitionistic fuzzy *rgw*-closed set.

Remark 3.5: The converse of theorem 3.5 need not be true as from the following example.

Example 3.5: Let X = {a, b} and intuitionistic fuzzy sets O and U are defined as follows

O = { < a,0..2,0.8.> ,<b,0.1,0.9>}

U = { < a,0.5,0.4 > ,<b,0, 1>}

Let ℑ = {***0***, O, U, ***1***} be an intuitionistic fuzzy topology on X.

Then the intuitionistic fuzzy set A = {< a, 0.9, 0.1>,<b ,0.6, 0.3>} is intuitionistic fuzzy rgw-closed but it is not intuitionistic fuzzy swg -closed .

Theorem 3.6: Every intuitionistic fuzzy rgw-closed set is intuitionistic fuzzy rwg-closed.

Proof: Let *A* be an intuitionistic fuzzy rgw-closed set in *X*. Suppose *A* ⊆ *U*, where *U* is intuitionistic fuzzy regular open. Since every intuitionistic fuzzy regular open set is intuitionistic fuzzy regular semi-open in *X*. So, we can say that *A* ⊆ *U*, where *U* is regular semi-open. Hence by definition 3.1 , we have *cl*(*int*(*A*)) ⊆ *U* whenever *A* ⊆ *U*. Finally *A* is intuitionistic fuzzy *rwg*-closed set.

Remark 3.6: The converse of the Theorem 3.6 need not be true, as seen from the following example:

Example 3.6: Let X = {a, b, c, d, e} and intuitionistic fuzzy sets P, Q and R defined as follows

P = { < a,0.9,0.1> ,<b,0,1>,<c,0,1>,<d,0,1> <e,0,1>}

Q = { < a,0,1,> ,<b,0,1>,<c,0,1>,<d,0.9,0.1 > <e,0,1>}

R = { < a,0,1> ,<b,0,1>,<c,0,1>, <d,0,1>,<e,0.8,0.2>}

Let ℑ = {***0***, P, Q, R ***1***} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0, 1 >,< b, 0.9, 0.1 >,< c, 0, 1>,<d ,0, 1>,<e,0,1 >} is intuitionistic fuzzy rwg -closed but it is not intuitionistic fuzzy rgw-closed.

Remark 3.7: The concept of intuitionistic fuzzy g-closed sets and intuitionistic fuzzy rgw-closed sets are independent. For,

Example 3.7: Let X = {a, b} and intuitionistic fuzzy sets U and A on X are defined as follows:

U = {< a, 0.4, 0.6 >, < b, 0.4, 0.6 > }

A = {< a, 0.1, 0.9 >, < b, 0.5, 0.5 > }

Let $\Im$ = {***0***, U, ***1***} be an intuitionistic fuzzy topology on X . Then intuitionistic fuzzy set A is intuitionistic fuzzy rgw - closed, but it is not intuitionistic fuzzy g- closed .

Example 3.8: Let X = {a, b} and intuitionistic fuzzy sets U and A on X are defined as follows:

U= {< a, 0.5, 0.5 >, < b, 0.4, 0.6 >}

A= {< a, 0.5, 0.5 >, < b, 0.3, 0.7 >}

Let $\Im$ = {***0***, U, ***1***} be an intuitionistic fuzzy topology on X .Then intuitionistic fuzzy set A is intuitionistic fuzzy g - closed, but it is not intuitionistic fuzzy rgw- closed.

Remark 3.8*:* From the above discussions and known results we have the following diagram of implication

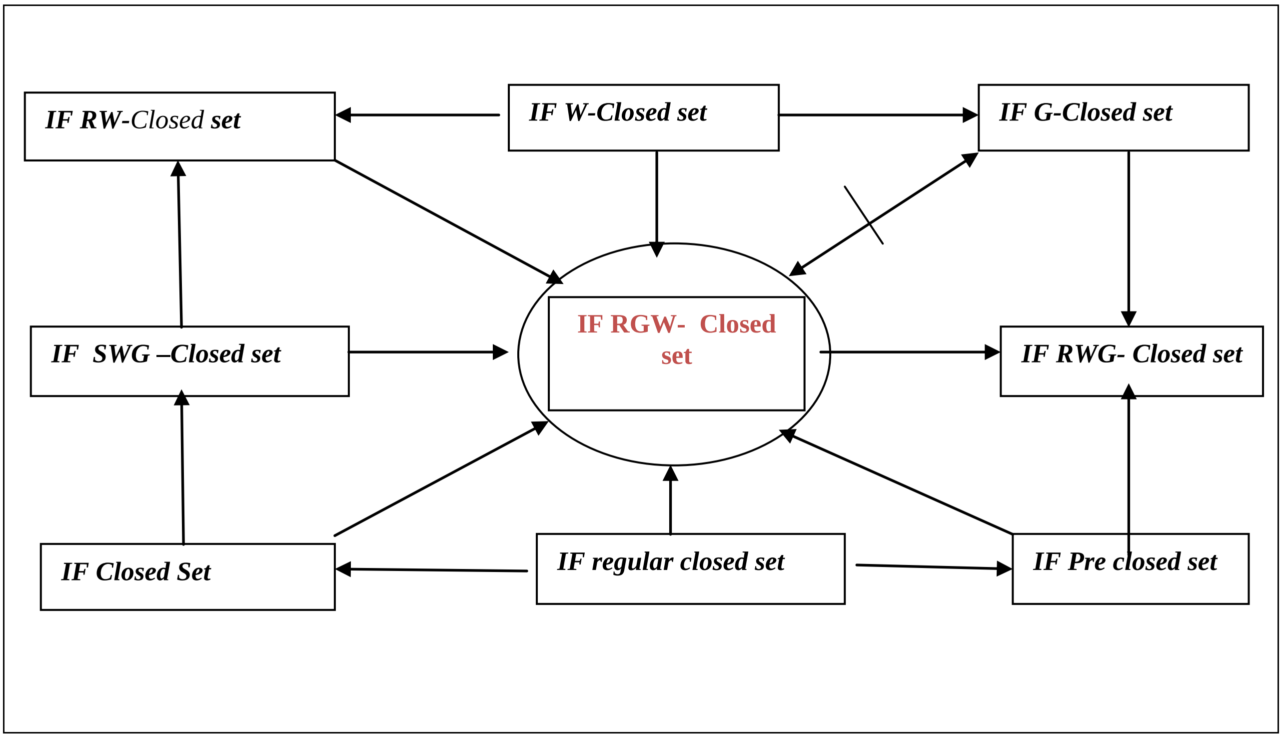

Fig.1 Relations between intuitionistic fuzzy rgw-closed set and other existing intuitionistic fuzzy closed sets

Theorem 3.7*:* Let (X,$\Im$) be an intuitionistic fuzzy topological space and A is an intuitionistic fuzzy set of X. Then A is intuitionistic fuzzy rgw-closed if and only if $\rceil$(AqF) $\Rightarrow$ $\rceil$(cl (int(A))qF) for every intuitionistic fuzzy regular semi closed set F of X.

Proof: Necessity*:* Let F be an intuitionistic fuzzy regular semi closed set of X and $\rceil$(AqF). Then by Lemma 2.1(a), A $\subseteq$ F^c and F^c intuitionistic fuzzy regular semi open in X.

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Therefore $cl(int(A)) \subseteq F^c$ by Def 3.1 because A is intuitionistic fuzzy rgw-closed. Hence by lemma 2.1(a), $\rceil(cl(int(A))qF)$.

***Sufficiency**:* Let O be an intuitionistic fuzzy regular semi open set of X such that $A \subseteq O$ i.e. $A \subseteq ((O)^c)^c$ Then by Lemma 2.1(a), $\rceil(A_q O^c)$ and O^c is an intuitionistic fuzzy regular semi closed set in X. Hence by hypothesis $\rceil(cl(int(A))_q O^c)$. Therefore by Lemma 2.1(a), $cl(int(A)) \subseteq ((O)^c)^c$ i.e. $cl(int(A)) \subseteq O$ Hence A is intuitionistic fuzzy rgw-closed in X.

Remark 3.9: The intersection of two intuitionistic fuzzy rgw-closed sets in an intuitionistic fuzzy topological space $(X,\Im)$ may not be intuitionistic fuzzy rgw-closed. For,

Example 3.9: Let X = {a, b, c, d } and intuitionistic fuzzy sets O,U, V,W defined as follows

$O = \{ < a,0.9,0.1> ,<b,0,1>,<c,0,1>,<d,0,1>\}$

$U = \{ < a,0,1,> ,<b,0.8,0.1>,<c,0,1>,<d,0,1>\}$

$V = \{ < a,0.9,0.1> ,<b,0.8,0.1,>,<c,0,1>,<d,0,1>\}$

$W = \{ < a,0.9,0.1,> ,<b,0.8,0.1>,<c,0.7,0.2>,<d,0,1>\}$

$\Im = \{\mathbf{0}, O, U, V, W, \mathbf{1}\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set $A = \{<a,0.9,0.1>,<b,0.8,0.1> <c,0,1>,<d,0,1>\}$ and $B = \{<a, 0.9, 0.1 >, < b, 0, 1>, < c, 0.7, 0.2> ,<d,0.9,0.1>\}$ are intuitionistic fuzzy rgw-closed in $(X,\Im)$ but $A \cap B$ is not intuitionistic fuzzy rgw-closed.

Theorem 3.8: Let A be an intuitionistic fuzzy rgw-closed set in an intuitionistic fuzzy topological space $(X,\Im)$ and $A \subseteq B \subseteq cl(int(A))$. Then B is intuitionistic fuzzy rgw-closed in X.

Proof: Let O be an intuitionistic fuzzy regular semi open set in X such that $B \subseteq O$. Then $A \subseteq O$ and since A is intuitionistic fuzzy rgw-closed, $cl(int(A)) \subseteq O$. Now $B \subseteq cl(int(A)) \Rightarrow cl(int(B)) \subseteq B \subseteq cl(int(A)) \subseteq O$ Consequently B is intuitionistic fuzzy rgw-closed.

Theorem 3.9: If A is intuitionistic fuzzy regular open and intuitionistic fuzzy rwg-closed then A is intuitionistic rgw-closed in X.

Proof : Let *A* be intuitionistic fuzzy regular open and intuitionistic fuzzy rwg-closed in *X*. We prove that *A* is an intuitionistic fuzzy *rgw*-closed set in *X*. Let *U* be any intuitionistic fuzzy regular semi-open set in *X* such that $A \subseteq U$. Since *A* is intuitionistic fuzzy regular open and intuitionistic fuzzy *rwg*-closed, by definition of intuitionistic fuzzy rwg-closed set, we have $cl(int(A)) \subseteq A$. Then $cl(int(A)) \subseteq A \subseteq U$ or $cl(int(A)) \subseteq U$. Hence *A* is intuitionistic fuzzy *rgw*-closed in *X*.

Theorem 3.10 : If an intuitionistic fuzzy set A of intuitionistic fuzzy topological space $(X, \Im)$ is both intuitionistic fuzzy regular semiopen and intuitionistic fuzzy rgw-closed then it is intuitionistic fuzzy regular closed.

Proof: Suppose a subset *A* of an intuitionistic fuzzy topological space $(X,\Im)$ is both intuitionistic fuzzy regular semiopen and intuitionistic fuzzy *rgw*-closed. Now $A \subseteq A$ then by definition of intuitionistic fuzzy *rgw*-closed we have $cl(int(A)) \subseteq A$. Since every intuitionistic fuzzy regular semi-open is intuitionistic fuzzy semi-opens, so $A \subseteq cl(int(A))$. Thus we have $A = cl(int(A))$. Finally, *A* is intuitionistic fuzzy regular closed.

Theorem 3.11: Let A be intuitionistic fuzzy regular semi-open set and intuitionistic fuzzy rgw-closed in X. Suppose that F is intuitionistic fuzzy regular closed in X. Then A ∩ F is an intuitionistic fuzzy rgw-closed set in X.

Proof: Let *A* be intuitionistic fuzzy regular semi-open and intuitionistic fuzzy *rgw*-closed in *X*, by theorem 3.10 *A* is intuitionistic fuzzy regular closed. By hypothesis *F* is intuitionistic fuzzy regular closed in *X* and we know every intuitionistic fuzzy regular closed is intuitionistic fuzzy closed i.e. both *A* and *F* are intuitionistic fuzzy closed, so *A* ∩ *F* is intuitionistic fuzzy closed. Hence by theorem 3.1 *A* ∩ *F* is an intuitionistic fuzzy *rgw*-closed set in *X*.

Theorem 3.12: If A is both intuitionistic fuzzy open and intuitionistic fuzzy wg-closed in X, then it is intuitionistic fuzzy rgw-closed in X.

Proof: Let *A* be an intuitionistic fuzzy open and intuitionistic fuzzy *wg*-closed in *X*. Let *A* ⊆ *U* and *U* be intuitionistic fuzzy regular semiopen in *X*. Now *A* ⊆ *A*. As *A* is intuitionistic fuzzy *wg*-closed so by definition of intuitionistic fuzzy wg-closed set *cl*(*int*(*A*)) ⊆ *A*. That is *cl*(*int*(*A*)) ⊆ *U*. Hence *A* is intuitionistic fuzzy *rgw*-closed in *X*.

Definition 3.2: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,ℑ) is called intuitionistic fuzzy rgw-open if and only if its complement A^c is intuitionistic fuzzy rgw-closed.

Remark 3.10*:* Every intuitionistic fuzzy w-open set is intuitionistic fuzzy rgw-open but its converse may not be true.

Example 3.10: Let X = {a, b} and ℑ = {**0**, U, **1**} be an intuitionistic fuzzy topology on X, where U ={< a, 0.7, 0.2 >,< b, 0.6, 0.3 >}.Then the intuitionistic fuzzy set A={< a, 0.2, 0.7 >,<b, 0.1, 0.8 >} is intuitionistic fuzzy rgw-open in (X, ℑ) but it is not intuitionistic fuzzy w-open in (X, ℑ).

Remark 3.11*:* Every intuitionistic fuzzy rw-open set is intuitionistic fuzzy rgw-open but its converse may not be true.

Example 3.11: Let X = { a, b, c } and intuitionistic fuzzy sets O, U, V on X are defined as follows:

O = { < a, 0.9, 0.1 > ,< b, 0,1 > < c, 0, 1 > }

U = { < a, 0, 1 > ,< b, 0.8, 0.1 > ,< c, 0, 1 > }

V = { < a, 0.9, 0.1 >,< b, 0.8, 0.1 > ,< c, 0,1 > }

Let ℑ = {***0***, O, U, V, ***1***} be an intuitionistic fuzzy topology on X. Then intuitionistic fuzzy set A = {< a, 0.1, 0.9 >, < b, 0.1, 0.8 > ,< c ,1 ,0 > } is intuitionistic fuzzy rgw-open set in X, but it is not intuitionistic fuzzy rw-open.

Theorem 3.13*:* An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,ℑ) is intuitionistic fuzzy rgw-open if and only if F⊆ int (cl(A)) whenever F is intuitionistic fuzzy regular semi closed and F ⊆ A.

Proof: Necessity: Suppose A is an intuitionistic fuzzy rgw-open in X. Let F be an intuitionistic fuzzy regular semi closed in X and F ⊆ A. Then F^c is an intuitionistic fuzzy regular semi open set in X such that A^c ⊆ F^c. Since A^c is an intuitionistic fuzzy rgw-closed set, we have cl(int(A^c)) ⊆ F^c.Hence $(int(cl(A)))^c$ ⊆ F^c. This implies F ⊆ int(cl(A)).

Sufficiency: Let A be an Intuitionistic fuzzy open set of X and let F ⊆ int(cl(A)) whenever F is an F is an intuitionistic fuzzy regular semi closed and F ⊆ A. Then A^c ⊆ F^c and F^c is an F is intuitionistic fuzzy regular semi open set in X,. By hypothesis, $(int(cl(A)))^c \subseteq F^c$. Hence $cl(int(A^c)) \subseteq F^c$. Therefore, A^c is intuitionistic fuzzy rgw-closed. Hence A is intuitionistic fuzzy rgw-closed set.

Theorem 3.14: Let A be an intuitionistic fuzzy w-closed set in an intuitionistic fuzzy topological space (X,ℑ) and f: (X,ℑ) → (Y, ℑ*) is an intuitionistic fuzzy almost irresolute and intuitionistic fuzzy closed mapping then f (A) is an intuitionistic rgw-closed set in Y.

Proof: Let A be an intuitionistic fuzzy w-closed set in X and f: (X,ℑ) → (Y, ℑ*) is an intuitionistic fuzzy almost irresolute and intuitionistic fuzzy closed mapping. Let f(A) ⊆ G where G is intuitionistic fuzzy regular semi open in Y then A ⊆ $f^{-1}(G)$ and $f^{-1}(G)$ is intutionistic fuzzy semi open in X because f is intuitionistic fuzzy almost irresolute .Now A be an intuitionistic fuzzy w-closed set in X , cl(A) ⊆ $f^{-1}(G)$. Thus f(cl(A)) ⊆ G and f(cl(A)) is an intuitionistic fuzzy closed set in Y(since cl(A) is intuitionistic fuzzy closed in X and f is intuitionistic fuzzy closed mapping). It follows that cl (f (A)) ⊆ cl (f (cl (A))) = f (cl (A)) ⊆ G. Hence cl (f (A)) ⊆ G. Now cl(int(f(A)) ⊆ cl (f (A)) ⊆ G , whenever f (A) ⊆ G and G is intuitionistic fuzzy regular semi open in Y. Hence f (A) is intuitionistic fuzzy rgw-closed set in Y.

Definition 3.3: A mapping f: (X,ℑ) → (Y,ℑ*) is said to be intuitionistic fuzzy regular semi irresolute if inverse image of every regular semi open set of Y is intuitionistic fuzzy regular semi open in X.

Theorem 3.15: Let A be an intuitionistic fuzzy rgw-closed set in an intuitionistic fuzzy topological space (X,ℑ) and f: (X,ℑ) → (Y,ℑ*) is an intuitionistic fuzzy regular semi irresolute and intuitionistic fuzzy closed mapping then f (A) is an intuitionistic rgw-closed set in Y.

Proof: Let A be an intuitionistic fuzzy rw-closed set in X and f: (X,ℑ) → (Y,ℑ*) is an intuitionistic fuzzy regular semi irresolute and intuitionistic fuzzy closed mapping. Let f(A) ⊆ G where G is intuitionistic fuzzy regular semi open in Y then A ⊆ $f^{-1}(G)$ and $f^{-1}(G)$ is intutionistic fuzzy regular semi open in X because f is intuitionistic fuzzy regular semi irresolute .Now A be an intuitionistic fuzzy rgw-closed set in X , cl(int(A)) ⊆ $f^{-1}(G)$. Thus f(cl(int(A)) ⊆ G and f(cl(int(A)) is an intuitionistic fuzzy closed set in Y. (since cl(int(A)) is intuitionistic fuzzy closed in X and f is intuitionistic fuzzy closed mapping). It follows that cl (f (A) ⊆ cl (f (cl (int(A))) = f (cl (int(A)) ⊆ G. Hence cl (f (A)) ⊆ G . Now cl(int(f(A)) ⊆ cl (f (A)) ⊆ G ,whenever f (A) ⊆ G and G is intuitionistic fuzzy regular semi open in Y. Hence f (A) is intuitionistic fuzzy rgw-closed set in Y.

Definition 3.4: An intuitionistic fuzzy topological space (X ℑ)is called intuitionistic fuzzy rgw – connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy rgw- open and intuitionistic fuzzy rgw- closed .

Theorem 3.16: Every intuitionistic fuzzy rgw-connected space is intuitionistic fuzzy connected.

Proof: Let (X, ℑ) be an intuitionistic fuzzy rgw –connected space and suppose that (X, ℑ) is not intuitionistic fuzzy connected .Then there exists a proper intuitionistic fuzzy set A(A≠ **0,** A≠ ***1***) such that A is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Since every intuitionistic fuzzy open set (resp. intuitionistic fuzzy closed set) is intuitionistic rgw-open ((resp. intuitionistic fuzzy rgw-closed), X is not intuitionistic fuzzy rgw-connected, a contradiction.

Remark 3.12: Converse of theorem 3.16 may not be true for;

Example 3.12: Let X = {a, b} and $\Im$ = {**0**, U, **1**} be an intuitionistic fuzzy topology on X, where U ={< a,0.5,0.5>,< b, 0.4, 0.6 > }.Then intuitionistic fuzzy topological space (X, $\Im$) is intuitionistic fuzzy connected but not intuitionistic fuzzy rgw-connected because there exists a proper intuitionistic fuzzy set A={<a, 0.5, 0.5>,<b, 0.5, 0.5 >} which is both intuitionistic fuzzy rgw -closed and intuitionistic rgw-open in X.

Theorem 3.17*:* An intuitionistic fuzzy topological (X,$\Im$) is intuitionistic fuzzy rgw-connected if and only if there exists no non zero intuitionistic fuzzy rgw-open sets A and B in X such that $A=B^c$.

Proof: Necessity: Suppose that A and B are intuitionistic fuzzy rgw-open sets such that A≠ **0**≠ B and A = B^c. Since $A=B^c$, B is an intuitionistic fuzzy rgw-open set which implies that B^c = A is intuitionistic fuzzy rgw-closed set and B≠ **0** this implies that B^c≠ **1** i.e. A≠ **1** Hence there exists a proper intuitionistic fuzzy set A(A≠ **0,** A≠ **1**) such that A is both intuitionistic fuzzy rgw- open and intuitionistic fuzzy rgw-closed. But this is contradiction to the fact that X is intuitionistic fuzzy rgw- connected.

Sufficiency: Let (X,$\Im$) is an intuitionistic fuzzy topological space and A is both intuitionistic fuzzy rgw-open set and intuitionistic fuzzy rgw-closed set in X such that ***0* ≠** A ≠ *1***.** Now take B = A^c .In this case B is an intuitionistic fuzzy rgw-open set and A≠ **1.**This implies that B =A^c ≠ 0 which is a contradiction. Hence there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy rgw- open and intuitionistic fuzzy rgw- closed. Therefore intuitionistic fuzzy topological (X,$\Im$) is intuitionistic fuzzy rgw-connected

Definition 3.5: A collection { A_i : i∈ Λ} of intuitionistic fuzzy rgw- open sets in intuitionistic fuzzy topological space (X,$\Im$) is called intuitionistic fuzzy rgw- open cover of intuitionistic fuzzy set B of X if B ⊆ ∪{ A_i : i∈ Λ}

Definition 3.6: An intuitionistic fuzzy topological space (X,$\Im$) is said to be intuitionistic fuzzy rgw-compact if every intuitionistic fuzzy rgw- open cover of X has a finite sub cover.

Definition 3.7: An intuitionistic fuzzy set B of intuitionistic fuzzy topological space (X,$\Im$) is said to be intuitionistic fuzzy rgw- compact relative to X, if for every collection { A_i : i∈ Λ} of intuitionistic fuzzy rgw- open subset of X such that B ⊆ ∪{ A_i : i∈ Λ} there exists finite subset Λ_o of Λ such that B ⊆ ∪{ A_i : i∈ Λ_o}.

Definition 3.8: A crisp subset B of intuitionistic fuzzy topological space (X,$\Im$) is said to be intuitionistic fuzzy rgw- compact if B is intuitionistic fuzzy rgw- compact as intuitionistic fuzzy subspace of X .

Theorem 3.18: A intuitionistic fuzzy rgw-closed crisp subset of intuitionistic fuzzy rgw- compact space is intuitionistic fuzzy rgw- compact relative to X.

Proof: Let A be an intuitionistic fuzzy rgw- closed crisp subset of intuitionistic fuzzy rgw- compact space (X, $\Im$). Then A^c is intuitionistic fuzzy rgw- open in X. Let M be a cover of A by intuitionistic fuzzy rgw- open sets in X. Then the family {M, A^c } is intuitionistic fuzzy rgw- open cover of X. Since X is intuitionistic fuzzy rgw-compact, it has a finite sub cover say {G_1, G_2, G_3 ...., Gn}. If this sub cover contains A^c, we discard it. Otherwise leave the sub cover as it is. Thus we obtained a finite intuitionistic fuzzy rgw – open sub cover of A. Therefore A is intuitionistic fuzzy rgw – compact relative to X.

4: Intutionistic fuzzy rgw- continuous mappings

Definition 4.1:A mapping *f* : (X, $\Im$) →(Y, σ) is intuitionistic fuzzy rgw- continuous if inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy rgw-closed set in X.

Theorem 4.1: A mapping *f* : (X, $\Im$)→(Y, σ) is intuitionistic fuzzy rgw- continuous if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy rgw- open in X.

Proof: It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$ for every intuitionistic fuzzy set U of Y.

Remark4.1: Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy rgw-continuous, but converse may not be true. For,

Example 4.1: Let X = {a, b}, Y ={x, y} and intuitionistic fuzzy sets U and V are defined as follows :

U= {< a, 0.7, 0.2>, < b, 0.6, 0.3>}

V= {<x, 0.7, 0.2>, <y, 0.8, 0.1>}

Let $\Im$ = {***0*, U, 1**} and σ = {**0 V, 1**} be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,$\Im$) →(Y, σ) defined by *f* (a) = x and *f* (b) = y is intuitionistic fuzzy rgw- continuous but not intuitionistic fuzzy continuous.

Remark4.2: Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy rgw-continuous, but converse may not be true. For,

Example 4.2: Let X = {a, b}, Y ={x ,y} and intuitionistic fuzzy sets U and V are defined as follows :

U= {< a, 0.5, 0.5>, < b, 0.4, 0.6>} , V= {<a, 0.5, 0.5>, <b, 0.3, 0.7>}

Let $\Im$ = { ***0*, U, *1*** } and σ ={ **0, V, 1** } be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,$\Im$) →(Y, σ) defined by *f* (a) = x and *f* (b) = y is intuitionistic fuzzy rgw- continuous but not intuitionistic fuzzy w-continuous.

Remark4.3: Every intuitionistic fuzzy rw-continuous mapping is intuitionistic fuzzy rgw-continuous, but converse may not be true. For,

Example 4.3: Let X = {a, b, c, d } Y ={p, q r, s} and intuitionistic fuzzy sets O,U,V,W,T are defined as follows:

O = { < a,0.9,0.1> ,<b,0,1>,<c,0,1>,<d,0,1>}

U = { < a,0,1,> ,<b,0.8,0.1>,<c,0,1>,<d,0,1>}

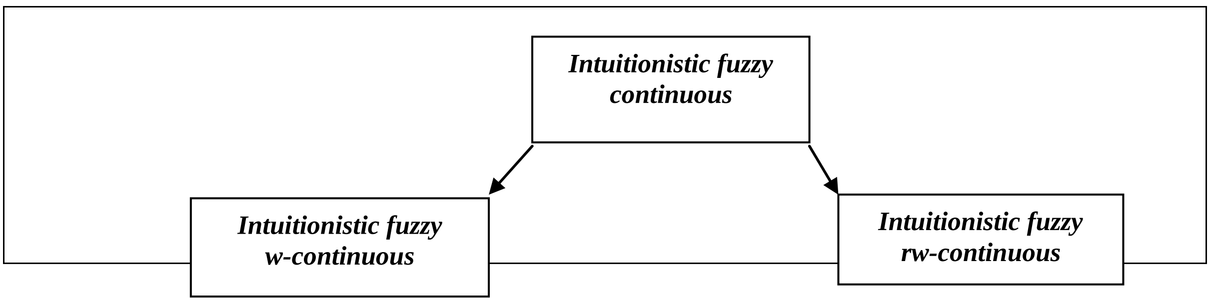
V = { < a,0.9,0.1> ,<b,0.8,0.1,>,<c,0,1>,<d,0,1>}

W = { < a,0.9,0.1,> ,<b,0.8,0.1>,<c,0.7,0.2>,<d,0,1>}

T = {<p,0,1>,<q,0,1> <r,0.7,0.2>,<s,0,1>}

Let $\Im$ = { **0,** O, U, V, W **1** } and σ ={ **0,** T,**1** } be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,$\Im$) →(Y, σ) defined by *f* (a) = p, *f* (b) = q , *f* (c) = r , *f* (d) = s is intuitionistic fuzzy rgw- continuous but not intuitionistic fuzzy rw- continuous.

Remark 4.4: From the above discussion and known results we have the following diagram of implication



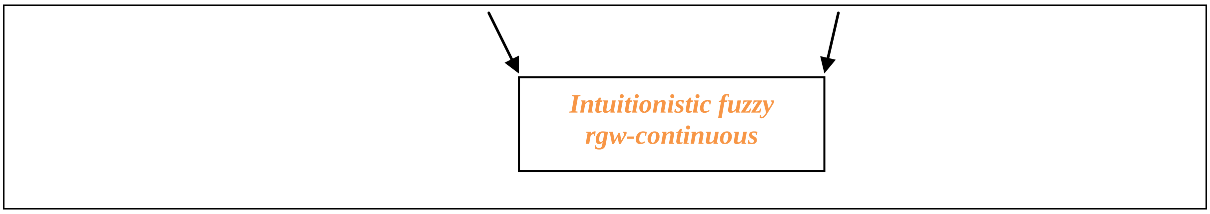


Fig.1 Relations between intuitionistic fuzzy rgw-continuous mappings and other existing intuitionistic fuzzy continuous mappings.

Theorem 4.2: If f: $(X,\Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy rgw- continuous then for each intuitionistic fuzzy point $c(\alpha,\beta)$ of X and each intuitionistic fuzzy open set V of Y such that $f(c(\alpha,\beta)) \subseteq V$ there exists an intuitionistic fuzzy rgw- open set U of X such that $c(\alpha,\beta) \subseteq U$ and $f(U) \subseteq V$

Proof : Let $c(\alpha,\beta)$ be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that $f(c(\alpha,\beta)) \subseteq V$. Put $U = f^{-1}(V)$. Then by hypothesis U is intuitionistic fuzzy rgw- open set of X such that $c(\alpha,\beta) \subseteq U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Theorem 4.3: Let f: $(X,\Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy rgw- continuous then for each intuitionistic fuzzy point $c(\alpha,\beta)$ of X and each intuitionistic fuzzy open set V of Y such that $f(c(\alpha,\beta))qV$, there exists an intuitionistic fuzzy rgw- open set U of X such that $c(\alpha,\beta)qU$ and $f(U) \subseteq V$.

Proof: Let $c(\alpha,\beta)$ be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that $f(c(\alpha,\beta))q\, V$. Put $U = f^{-1}(V)$. Then by hypothesis U is intuitionistic fuzzy rgw- open set of X such that $c(\alpha,\beta)q\, U$ and $f(U)= f(f^{-1}(V)) \subseteq V$.

Theorem 4.4: If f: $(X,\Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy rgw-continuous and $g :(Y, \sigma) \rightarrow (Z, \mu)$ is intuitionistic fuzzy continuous. Then $gof : (X,\Im) \rightarrow (Z,\mu)$ is intuitionistic fuzzy rgw-continuous**.**

Proof: Let A is an intuitionistic fuzzy closed set in Z. then $g^{-1}(A)$ is intuitionstic fuzzy closed in Y because g is intuitionistic fuzzy continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy rgw – closed in X. Hence gof is intuitionistic fuzzy rgw – continuous.

Theorem 4.5: If $f : (X, \Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy rgw-continuous and $g : (Y,\sigma)$. $\rightarrow (Z,\mu)$ is intuitionistic fuzzy g-continuous and (Y,σ) is intuitionistic fuzzy $T_{1/2}$ then $gof : (X,\Im) \rightarrow (Z,\mu)$ is intuitionistic fuzzy rgw-continuous.

Proof: Let A is an intuitionistic fuzzy closed set in Z, then $g^{-1}(A)$ is intuitionstic fuzzy g-closed in Y. Since Y is $T_{1/2}$, then $g^{-1}(A)$ is intuitionstic fuzzy closed in Y. Hence $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy rgw – closed in X. Hence gof is intuitionistic fuzzy rgw – continuous.

Theorem 4.6: An intuitionistic fuzzy rgw – continuous image of an intuitionistic fuzzy rgw-compact space is intuitionistic fuzzy compact.

Proof: Let. $f : (X,\Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy rgw-continuous map from a intuitionistic fuzzy rgw-compact space $(X,\Im)$ onto a intuitionistic fuzzy topological space (Y, σ). Let $\{Ai: i \in \Lambda\}$ be an intuitionistic fuzzy open cover of Y then $\{f^{-1}(Ai) : i \in \Lambda\}$ is a intuitionistic fuzzy rgw –open cover of X. Since X is intuitionistic fuzzy rgw-compact it has finite intuitionistic fuzzy sub cover say $\{ f^{-1}(A_1) , f^{-1}(A_2) ,----f^{-1}(An) \}$. Since f is onto $\{A_1, A_2, \text{-------------} A_n\}$ is an intuitionistic fuzzy open cover of Y and so (Y, σ) is intuitionistic fuzzy compact.

5: Application of intutionistic fuzzy rgw- closed sets

In this section we introduce intuitionistic fuzzy rgw$\mathbf{T_{1/2}}$-space as an application of intuitionistic fuzzy rgw-closed set. We have derived some characterizations of intuitionistic fuzzy rgw-closed sets.

Definition 5.1: An intuitionistic fuzzy topological space (X,$\Im$) is called a intuitionistic Fuzzy rgw-$T_{1/2}$ -Space if every intuitionistic fuzzy rgw-closet set is intuitionistic fuzzy closed.

Theorem 5.1: Every intuitionistic fuzzy rgw $T_{1/2}$ -space is intuitionistic fuzzy w$T_{1/2}$ space.

Proof: Let (X, $\Im$) be an intuitionistic fuzzy rgw$T_{1/2}$ space and let A be intuitionistic fuzzy w-closed set in (X, $\Im$).Then A is intuitionistic Fuzzy rwg-closed, by theorem 3.3, Since intuitionistic fuzzy topological space (X, $\Im$) is intuitionistic fuzzy rgw$T_{1/2}$ space, A is intuitionistic fuzzy closed in (X, $\Im$). Hence (X, $\Im$) is intutionistic fuzzy w$T_{1/2}$ space.

Remark5.1: The converse of the above theorem need not be true, as seen from the following example

Example 5.1: Let X = {a, b} and Let $\Im$ = {**0**, O, **1**} be an intuitionistic fuzzy topology on X , where O = {<a ,0.5, 0.5 >,<b ,0.4 ,0.6>}. Then intuitionistic fuzzy topological space (X, $\Im$) is intutionistic fuzzy w$T_{1/2}$ space but not intuitionistic fuzzy rgw$T_{1/2}$ -space.

Theorem 5.2: Every intuitionistic fuzzy rgw $T_{1/2}$ -space is intuitionistic fuzzy rw$T_{1/2}$ space.

Proof: Let (X, $\Im$) be an intuitionistic fuzzy rgw$T_{1/2}$ space and let A be intuitionistic fuzzy rw-closed set in (X, $\Im$).Then A is intuitionistic Fuzzy rwg-closed, by theorem 3.4, Since intuitionistic fuzzy topological space (X, $\Im$) is intuitionistic fuzzy rgw$T_{1/2}$ space, A is intuitionistic fuzzy closed in (X, $\Im$). Hence (X, $\Im$) is intutionistic fuzzy rw$T_{1/2}$ space.

Remark 5.2: The converse of the above theorem need not be true, as seen from the following example

Example 5.2: Let X = {a, b, c} and Let $\Im$ = {**0**, A, B, **1**} be an intuitionistic fuzzy topology on X where A={<a ,0.7, 0.5 >,<b ,0.3 ,0.6>, < c, 1, 0 >} and B={<a ,0.7, 0.3 >,<b ,0.0 ,0.1>, < c, 0, 1>} .Then iintuitionistic fuzzy topological space (X,$\Im$) is intutionistic fuzzy rw-$T_{1/2}$ space but not intuitionistic fuzzy rgw$T_{1/2}$ -space.

Theorem 5.3: Every intuitionistic fuzzy rgw $T_{1/2}$ -space is intuitionistic fuzzy sw$T_{1/2}$ space.

Proof: Let (X, $\Im$) be an intuitionistic fuzzy rgw$T_{1/2}$ space and let A be an intuitionistic fuzzy swg-closed set in (X, $\Im$).Then A is intuitionistic Fuzzy rwg-closed, by theorem 3.5, Since intuitionistic fuzzy topological space (X, $\Im$) is intuitionistic fuzzy rgw$T_{1/2}$ space, A is intuitionistic fuzzy closed in (X, $\Im$). Hence (X, $\Im$) is intutionistic fuzzy rw$T_{1/2}$ space.

Remark 5.3: The converse of the above theorem need not be true, as seen from the following example

Example 5.3: Let X = {a, b, c } and Let $\Im$ = {0□, A, 1□} be an intuitionistic fuzzy topology on X, where A={<a ,0.6, 0.4 >,<b ,0.3 ,0.6>, < c, 1, 0 >} . Then iintuitionistic fuzzy topological space (X, $\Im$) is intutionistic fuzzy sw$T_{1/2}$ space but not intuitionistic fuzzy rgw$T_{1/2}$-space.

Theorem 5.4: Let (X,ℑ) be an intuitionistic fuzzy rgw- $T_{1/2}$ space, then the following conditions are equivalent:

(a) (X,ℑ) is intuitionistic fuzzy rgw-connected.

(b) (X,ℑ) is intuitionistic fuzzy connected.

Proof: (a) ⇒(b) follows from Theorem 3.16.

(b) ⇒(a): Assume that (X,ℑ) is intuitionistic fuzzy rgw- $T_{1/2}$ space and intuitionistic fuzzy -connected space. Then we have to prove that (X,ℑ) is intuitionistic fuzzy rgw-connected. If possible, let X be not intuitionistic fuzzy rgw-connected, then there exists a proper intuitionistic fuzzy set A such that A is both intuitionistic fuzzy rgw-open and rgw-closed. Since X is intuitionistic fuzzy rgw-$T_{1/2}$, A is intuitionistic fuzzy open and intuitionistic fuzzy closed which implies that X is not intuitionistic fuzzy connected, a contradiction

VI. Conclusion

The theory of g-closed sets plays an important role in general topology. Since its inception many weak and strong forms of g-closed sets have been introduced in general topology as well as intuitionistic fuzzy topology. The present paper investigated a new class of intuitionistic fuzzy closed sets called Intuitionistic fuzzy rgw-closed sets which contain the classes of intuitionistic fuzzy closed sets and intuitionistic fuzzy w- closed sets , intuitionistic fuzzy rw- closed sets. intuitionistic fuzzy swg- closed sets and contained in the classes of intuitionistic fuzzy rgw-closed sets. Several properties and application of intuitionistic fuzzy rgw-closed sets are studied. Many examples are given to justify the result.

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