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A STUDY ON CLIQUE NUMBER OF POPPED FIBONACCI-SUM SET-GRAPHS

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Abstract— In this paper we study the Popped Fibonacci-sum set-graphs, its clique number and the chromatic number. The aforesaid graphs are an extension of the notion of Fibonacci-sum set-graphs to the notion of set-graphs. This paper is an attempt to solve the problem stated in [8].

Keywords— Clique number, Chromatic number, Fibonacci-Sum, Fibonacci-Sum Set-graphs, Popped Fibonacci-Sum Set-graphs, Set-Graphs.

I. INTRODUCTION

For general notation and concepts in graphs and digraphs see [2, 4, 7]. Unless stated otherwise, all graphs will be finite connected simple graphs.

Recall that the sequence of Fibonacci numbers $F = \{f_n\}_{n\geq 0}, n \in \mathbb{N}_\circ$ is defined recursively as $f_\circ = 0, f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$. As defined in [3], a Fibonacci-sum graph is defined for a finite set of the first *n* consecutive positive integers $\{1,2,3,\ldots,n\}$ as G_n^F with $V(G_n^F) = \{v_i : 1 \leq i \leq n\}$ and $E(G_n^F) = \{v_i v_j : i \neq j, i + j \in F\}$

In this paper, we study the notion of a new class of graph, namely the Popped Fibonacci-sum set-graphs its Clique number and the chromatic number. Popped Fibonacci-sum set-graphs are an extension of the notion of Fibonacci-sum Set-graphs to the notion of set-graphs.

II. DERIVATIVE SET-GRAPHS

The notion of Set-graph was introduced in [5] as explained below.

Let $A^{(n)} = \{a_1, a_2, a_3, \ldots, a_n\}, n \in \mathbb{N}$ be a non-empty set and the *i*-th *s*-element subset of $A^{(n)}$ be denoted by $A_{s,i}^{(n)}$. Now, consider $S = \{A_{s,i}^{(n)} : A_{s,i}^{(n)} \subseteq A^{(n)}, A_{s,i}^{(n)} \neq \phi\}$. The **set-graph** corresponding to set $A^{(n)}$, denoted $G_{A^{(n)}}$, is defined to be the graph with $V(G_{A^{(n)}}) = \{v_{s,i} : A_{s,i}^{(n)} \in S\}$ and $E(G_{A^{(n)}}) = \{v_{s,i} v_{t,j} : A_{s,i}^{(n)} \cap A_{t,j}^{(n)} \neq \phi\}$, where $s \neq t$ or $i \neq j$.

Note that the definition of vertices implies, $v_{s,i} \mapsto A_{s,i}^{(n)} \in S$.

The notion of Fibonacci-Sum set-graphs was introduced in [1] as explained below.

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Let $A^{(n)} = \{a_1, a_2, a_3, ..., a_n\}, n \in \mathrm{N}$ be a non-empty set and the *i*-th *s*-element subset of $A^{(n)}$ be denoted by $A^{(n)}_{s,i}$. Now, consider $S = \{A^{(n)}_{s,i} : A^{(n)}_{s,i} \subseteq A^{(n)}, A^{(n)}_{s,i} \neq \phi\}$. The **Fibonacci-Sum Set-graph** corresponding to set $A^{(n)}$, denoted $G^F_{A^{(n)}}$, is defined to be the graph with $V(G^F_{A^{(n)}}) = \{v_{s,i} : A^{(n)}_{s,i} \in S\}$ and $E(G^F_{A^{(n)}}) = \{v_{s,i}v_{t,j} : \forall (i', j'), i' \in A^n_{s,i}, j' \in A^n_{t,j}, i' \neq j' \text{ and the sum } i' + j' \in F\}$. Since $A^{(n)}_{s,i}$ and $A^{(n)}_{t,j}$ are not necessarily distinct, loops are permitted.
Note that the Fibonacci-Sum Set-graphs are finite connected graphs with multiple edges and loops.

III.CLIQUE NUMBER OF POPPED FIBONACCI-SUM SET-GRAPH

The notion of Popped Fibonacci-Sum set-graphs was introduced in [1] is as follows.

The **Popped Fibonacci-Sum Set-graph** denoted by $G^{F^\ni}_{A^{(n)}}$ is obtained by deleting all loops and all multiple edges except one edge (to retain adjacency) from $G^F_{A^{(n)}}$.

The following Simple Graph represents Popped Fibonacci-Sum Set-Graph $G^{F^\ni}_{A^{(4)}}$ corresponding to set $A^{(4)} = \{1,2,3,4\}$ with $F = \{0,1,1,2,3,5,8,13,21,34,55,89,144,233,377\}$

$V(G^{F^\ni}_{A^{(4)}}) = \{v11, v12, v13, v14, v21, v22, v23, v24, v25, v26, v31, v32, v33, v34, v41\}$

$E(G^{F^\ni}_{A^{(4)}}) = \{v_{s,i}, v_{t,j} : a \in v_{s,i} \text{ and } b \in v_{t,j}; a + b \in F\}$. Here the multiple edges except one (to retain adjacency) are deleted.

For understanding, Consider the vertices *v*11 with element {1} and *v*12 with element {2}, here there exist an edge between them, since the sum of the elements {1} and {2} belongs to Fibonacci sequence, *F.*

Consider the vertices *v*11 with element {1} and v13 with element {3}, there exist an no edge between them, since the sum of the elements {1} and {3} does not belong to Fibonacci sequence, *F* and so on.

Similarly, Consider the vertices *v*11 with element {1} and *v*14 with element {1, 2, 3, 4}, there exist three edges between them, since the sum of the element of v11 (i.e)., {1} with elements of *v*14: {1}, {2}, {4} belongs to Fibonacci sequence, *F.* As we do not consider multiple edges here for adjacency we retain one edge and delete remaining two edges.

On repeating this process the following figure was obtained.

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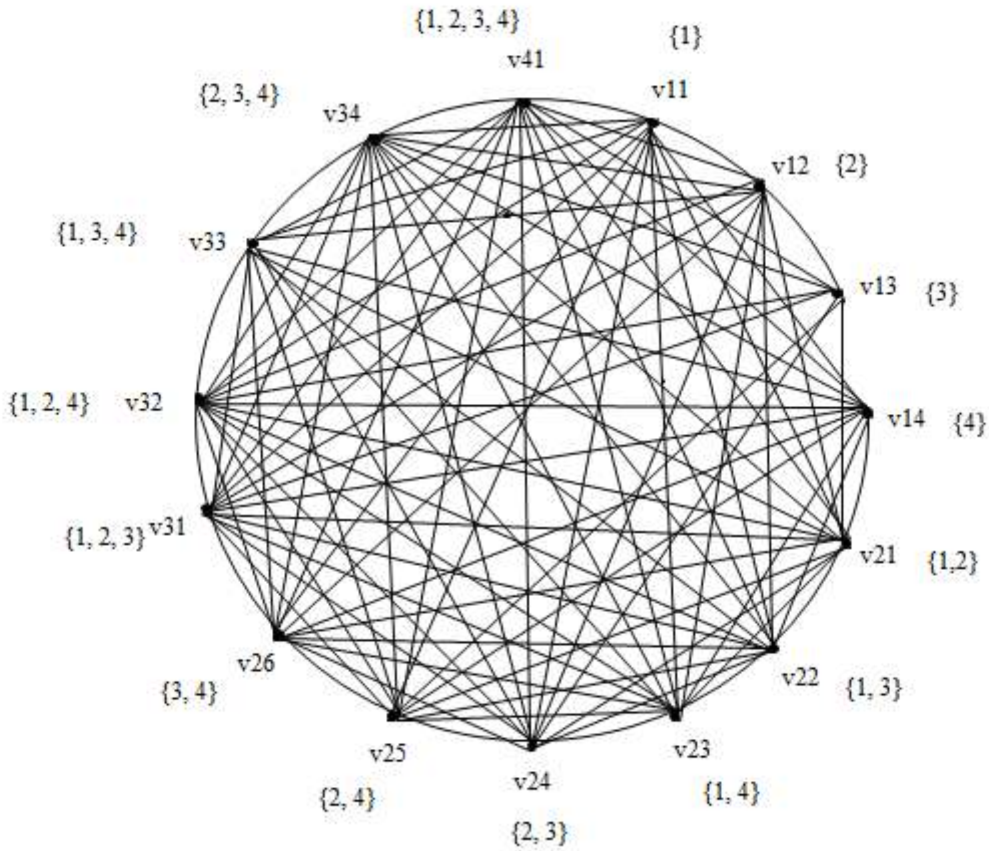

Figure 1: Popped Fibonacci-Sum Set-Graph $G^{F^{\ni}}_{A^{(4)}}$.

Note: The subsets of the set (i.e)., the elements in each vertex of the graph was obtained by using the MATLAB software.

Proposition-1

The Clique number and the Chromatic number of the Popped Fibonacci-Sum Set-Graphs $G^{F^{\ni}}_{A^{(n)}}$ are as follows:

(i) For *n=2*, $\omega = \chi = 1$.

(ii) For *n=3*, $\omega = \chi = 6$

(iii) For *n=4*, $\omega = \chi = 13$

Proof:

The results are obvious by the definition popped Fibonacci-Sum set-graphs.

Proposition-2

The Clique number of the Popped Fibonacci-Sum Set-Graphs $G_{A^{(n)}}^{F^{\ni}}$ are as follows:

(i) For *n*=5, $\omega(G_{A^{(5)}}^{F^{\ni}})=23$

(ii) For *n*=6, $\omega(G_{A^{(6)}}^{F^{\ni}})=51$

(iii) For *n*=7, $\omega(G_{A^{(7)}}^{F^{\ni}})=104$

Proof:

For *n*=5

$A^{(5)}=\{1,2,3,4,5\}$ with $f_n=f_{n-1}+f_{n-2}, n=31$

(i.e.)., $F=\{0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,...\}$

$$V(G_{A^{(5)}}^{F^{\ni}})=\{v11,v12,v13,v14,v15,v21,v22,v23,v24,v25,v26,v27,v28,v29,v210,$$
$$v31,v32,v33,v34,v35,v36,v37,v38,v39,v310,v41,v42,v43,v44,v45,v51\}$$

$E(G_{A^{(5)}}^{F^{\ni}})=\{v_{s,i},v_{t,j}: a\in v_{s,i} \text{ and } b\in v_{t,j}; a+b\in F\}$. Here the multiple edges except one (to retain adjacency) are deleted.

Now, Consider the vertices *v*11 with element {1} and *v*12 with element {2}, there exist an edge between them, since the sum of the elements {1} and {2} belongs to Fibonacci sequence, *F.* Consider the vertices *v*11 with element {1} and *v*13 with element {3}, there exist an no edge between them, since the sum of the elements {1} and {3} does not belong to Fibonacci sequence, *F* and so on. Similarly, Consider the vertices *v*11 with element {1} and *v*15 with element {1, 2, 3, 4, 5}, there exist three edges between them, since the sum of the element of *v*11 (i.e)., {1} with elements of *v*15: {1}, {2}, {4} belongs to Fibonacci sequence, *F.* As we do not consider multiple edges here for adjacency we retain one edge and delete remaining two edges. (i.e)., there exists an edge between *v*11 and *v*15.

On repeating this for remaining edges we obtain the graph $G_{A^{(5)}}^{F^{\ni}}$. And then by the definition of Popped Fibonacci sum set graph and clique number the result was obtained.

Similarly, the result was obtained for *n*=6, 7.

IV. CONCLUSION

In this paper the clique number of popped Fibonacci- Sum set-graphs corresponding to $A^{(n)}$, for $1\le n\le 7$ was obtained. The further work can be done by finding the clique number for any *n*.

Problem-1. If possible, determine the clique number of the popped Fibonacci-Sum set-graph $G_{A^{(n)}}^{F^{\ni}}$ for any $n\ge 7$.

Problem-2. If possible, determine the chromatic number of the popped Fibonacci-Sum set-graph $G_{A^{(n)}}^{F^{\ni}}$ for any $n\ge 5$.

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