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ON FUZZY WARD CONTINUITY IN 2- FUZZY 2 - ANTI NORMED LINEAR SPACE.

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Abstract

In this paper the fuzzy ward continuity with some other kinds of continuities are investigated in 2- fuzzy 2-anti normed linear space. Further some theorems are developed. It turns out that uniform limit of fuzzy ward continuous functions is again fuzzy ward continuous

Keywords:

Quasi-Cauchy,
fuzzy ward continuity,
fuzzy ward compactness,
sequential continuity

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1. Introduction

The concept involving continuity plays a major role in all branches of sciences, and also pure mathematics. In 1965 Zadeh[11] introduced the concept of the fuzzy sets in his seminal paper. The concept of 2-normed spaces was developed by Gahler [4] in 1964. Menger [5] introduced the notion called a generalized metric in 1928. Recently many mathematicians came out with the results in 2- normed linear spaces and banach spaces [3,6,7] the concept of ward continuity of real functions and ward compactness of a subset E of R are introduced by Caaklli [2]. Using the main idea in the definition of sequential continuity and many kinds of continuities were introduced and investigated [1,2, 9]. R.M.Somasundaram and Thangaraj beaula[8] have newly coined 2-fuzzy normed liner space and proved many important theorems.

In this paper the authors aimed to introduce the concept of fuzzy ward continuity in 2-fuzzy 2- anti normed linear space. Also it is proved that the image of a fuzzy ward compact space under a uniform continuous map is fuzzy ward compact.

2. Preliminaries

Definition 2.1

A binary operation *: [0,1] x [0, 1]→ [0,1] is a continuous t-norm if itsatisfies the following conditions

1. * is commutative and associative
2. * is continuous
3. a * 1 = a, for all a∊ [0,1]
4. a * b ≤ c * d whenever a c and b d and a,b,c,d∊ [0,1]

Definition 2.2

A binary operation◊ : [0,1] x [0,1] →[0,1] is a continuous t – conorm ifit satisfies the following conditions

1.◊ is commutative and associative
2.◊ is continuous
3. a◊ 0 = a, for all a ∊ [0,1]
4. a◊ b≤ c◊ d whenever a ≤ c andb ≤ d and a, b, c, d∊ [0,1]

Definition 2.3

An intuitionistic fuzzy 2- normed linear space (i.2f-2-NLS) is of the form $A = \{F(X), N(f_1, f_2, t), M(f_1, f_2, t) / (f_1, f_2) F[(X)]^2\}$where F(X) is a linear space over afield K, * is a continuous t-norm,◊ is a continuous t-conorm, N and M are fuzzy sets on$[F(X)]^2$ (0,∞) such that N denotes the degree of membership and M denotes the degree ofnon-membership of (f_1, f_2, t)∊ $[F(X)]^2$x (0, ∞) satisfying the following conditions

(1) $N(f_1, f_2, t) + M(f_1, f_2, t) \leq 1$
(2) $N(f_1, f_2, t) > 0$
(3) $N(f_1, f_2, t) = 1$ if and only if $f_1, f_2$are linearly dependent
(4) $N(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2
(5) $N(f_1, f_2, t) : (0, \infty) \to [0,1]$ is continuous in t.
(6) $N(f_1, cf_2, t) = N(f_1, f_2, |\frac{t}{|c|}|)$, if $c \neq 0$, $c \in K$
(7) $N(f_1, f_2, s) * N(f_1, f_3, t) \leq N(f_1, f_2 + f_3, s + t)$
(8) $M(f_1, f_2, t) > 0$
(9) $M(f_1, f_2, t) = 0$ if and only if $f_1, f_2$are linearly dependent
(10) $M(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2
(11) $M(f_1, cf_2, t) = M(f_1, f_2,|\frac{t}{|c|}) |)$, if $c \neq 0$, $c \in k$
(12) $M(f_1, f_2, s) \lozenge M(f_1, f_3, t) \geq M(f_1, f_2 + f_2, s + t)$
(13) $M(f_1, f_2, t) : (0, \infty) \to [0,1]$ is continuous in t.

3. On fuzzy ward continuity in 2- fuzzy 2 - anti normed linear space.

Definition 3.1

A sequence $\{f_n\}$ of a points in a fuzzy 2- anti normed linear space (F(X), N*) is said to be quasi-Cauchy if $N^*(\Delta f_n, g, t) < r$ for every g ∊ F(X), t ∊ (0,1) where $\Delta f_n = f_{n+1} - f_n$

Definition 3.2

A subspace A of F(X) is said to be fuzzy ward compact if any sequence in A has a quasi- Cauchy subsequence.

Definition 3.3

Let $(F(X),N_1^*)$ and $(F(Y), N_2^*)$ be fuzzy 2- anti normed linear space. A function φ: F(X)→F(Y) is said to be fuzzy ward continuous if it preserves quasi- Cauchy property

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that is, $N_2^* (\Delta \varphi(f_n), g, t) < r_2$ for every $g \in F(Y)$
whenever $N_1^*(\Delta(f_n), h, t) < r$ for every $h \in F(X)$ where $r_1, r_2 \in (0,1)$

Definition 3.4

A function ϕ on a subspace A of a fuzzy 2 - anti normed linear space (F(X), N) is said to be sequentially continuous at f_0 if for any sequence $\{f_n\}$ in A converges to f_0 then sequence $\phi \{(f_n)\}$ converges to $\phi\{(f_o)\}$ in (F(Y), N)

Definition 3.5

A function $\varphi:F(X)\rightarrow F(Y)$ is said to be uniformly continuous on a subspace A of F(X) if for any $\varepsilon \in(0,1)$there exist a $\delta \in(0,1)$ such that

$$N_2^* (\varphi(f_1)- \varphi(f_2) ,h , t) < \varepsilon \text{ for any } h\in F(Y), t \in (0,1)$$

whenever $N_1^*((f_1 - f_2), g, t) < \delta$ for every $f_1 ,f_2 \in A$ and $g \in F(X)$

Theorem3.1

If φ: F(X)→F(Y) is fuzzy ward continuous on A of F(Y) then it is sequentially continuous on A.

Proof

If $\{f_n\}$ be a convergent sequence in A, then $N^*((f_n - f_0), h, t) < r$ where $h \in F(X); t\in (0,1)$

Construct a sequence $\{g_n\}$ as,

$$g_n = \begin{cases} f_n, & \text{if } n = 2k-1 \text{ where k is a positive integer} \\ f_0, & \text{if n is even} \end{cases}$$

Consider, $N^*(g_n - f_0 , h , t) = N^*((g_n - f_n + f_n - f_0), h,\frac{t}{2} + \frac{t}{2})$

$$\leq \max \{ N^* ((g_n - f_n , h ,\tfrac{t}{2}) , N^* ((f_n - f_0 , h ,\tfrac{t}{2})\} \quad \text{-------(1)}$$

When n is odd, $N^* ((g_n - f_0 , h ,t) \leq N^*(f_n - f_0 , h , t) < r$
When n is even $N^*(g_n - f_0 , h , t) = N^*((f_0 - f_n + f_n - f_0), h \frac{t}{2} + \frac{t}{2})$

$$\leq \max \{ N^* ((f_0 - f_n , h ,\tfrac{t}{2}) , N^* ((f_n - f_0 , h ,\tfrac{t}{2})\}$$
$$= N^* (f_0 - f_n , h , t) < r$$

Now to prove that $\{ g_n \}$ is a quasi- Cauchy sequence that is, $\Delta g_n = g_{n+1} - g_n$
When n is odd, the above equation becomes $\Delta f_n = f_{n+1} - f_n$
Therefore $N^*(\Delta g_n , , h , t) = N^*(f_{n+1} - f_n), h, t)$

$$= N^*((f_{n+1} - f_0 + f_0 - f_n), h , t)$$

$$= \max \{N^*((f_{n+1} - f_0), h , \tfrac{t}{2}),N^*(f_0 - f_n , h , \tfrac{t}{2})\}$$

$< r.$ (since $\{ f_n \}$converges to f_0)

Hence $\{ g_n \}$ is a quasi Cauchy sequence when n is even the case is trivial.
Given φ is fuzzy ward continuous define the transformed sequence $\varphi (g_n)$

As $$\varphi (g_n) = \begin{cases} \varphi(f_n), & \text{if } n = 2k-1 \text{ where } k \text{ is a positive integer} \\ \varphi(f_0), & \text{if } n \text{ is even} \end{cases}$$

Again $\varphi (g_n)$ is quasi-Cauchy,

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For $N^*(\varphi(g_{n+1}) - \varphi(g_n), h, t)$

$$= N^*(\varphi(g_{n+1}) - \varphi(f_{n+1}) + \varphi(f_{n+1}) + \varphi(f_n) - \varphi(f_n) - \varphi(g_n), h, \tfrac{t}{3} + \tfrac{t}{3} + \tfrac{t}{3})$$
$$\leq \max\{N^*(\varphi(g_{n+1}) - \varphi(f_{n+1}) h, \tfrac{t}{3}), N^*(\varphi(f_n) - \varphi(f_n), h, \tfrac{t}{3}) N^*(\varphi(f_{n+1}) - \varphi(g_n), h, \tfrac{t}{3})\}$$
$$< r$$

If n is odd,

$$N^*(\varphi(g_{n+1}) - \varphi(g_n), h, t) = N^*(\varphi(f_{n+1}) - \varphi(g_n), h, t) < r$$

If n is even,

$$N^*(\varphi(g_{n+1}) - \varphi(g_n), h, t)$$
$$= N^*(\varphi(f_0) - \varphi(f_{n+1}) + \varphi(f_{n+1}) + \varphi(f_n) - \varphi(f_n) - \varphi(f_0), h, \tfrac{t}{3} + \tfrac{t}{3} + \tfrac{t}{3})$$
$$= \max\{N^*(\varphi(f_0) - \varphi(f_{n+1}) h, \tfrac{t}{3}), N^*(\varphi(f_n) - \varphi(f_0), h, \tfrac{t}{3}), (N^*(\varphi(f_{n+1}) - \varphi(f_n), h, \tfrac{t}{3}),\}$$
$$\leq (N^*(\varphi(f_{n+1}) - \varphi(f_n), h, t)$$
$$< r$$

Hence $\{\varphi(f_n)\}$ is a quasi - cauchy sequence and $N^*(\varphi(g_{n+1}) - \varphi(g_n), h, t) < r$.
By construction of g_n it follows that $N^*(\varphi(f_{n+1}) - \varphi(f_0), h, t) < r$. Thus $\varphi(f_{n+1})$ converges to $\varphi(f_0)$. andso φ is sequentially continuous on A.

Theorem3.2

Let $N^*(F(X), N_1^*)$ and $N^*(F(Y), N_2^*)$ be a 2- fuzzy 2- anti normed linear space and A be a fuzzy ward compact subspace of F(X). If $\varphi: F(X)\rightarrow F(Y)$ is fuzzy ward continuous on A then $\varphi(A)$ is fuzzy ward compact.

Proof

Given A be a fuzzy ward compact subspace of F(X), then, there exist a subsequence$\{g_{n_k}\}$ of $\{g_n\}$ satisfying $N^*(\Delta(g_{n_k}), h, t) < r$. for every $h \in F(X)$ and $t \in (0,1)$

Let $\{\varphi(g_{n_k})\} = \{f_{n_k}\}$,then $\{f_{n_k}\}$ is a subsequence of a sequence $\{\varphi(g_{n_k})\}$

Consider $N^*(\Delta f_{n_k} h, t) = N^*(f_{n_{k+1}} - f_{n_k} h, t)$

$$= N^*(\varphi(g_{n_{k+1}}) - \varphi(g_{n_k}), h, t)$$
$$< r$$

Because φ is fuzzy ward continuous.$\{\varphi(g_{n_k})\}$is quasi-Cauchy. Hence $N^*(\Delta f_{n_k}, h, t) < r$ which implies $\{f_{n_k}\}$ is quasi-Cauchy

Theorem3.3

If the function $\varphi : F(X)\rightarrow F(Y)$ is uniformly continuous on a subspace A of F(X) then it is fuzzy ward continuous on A

Proof

Given φ is uniformly continuous on A and let $\{f_n\}$ be quasi- Cauchy sequence in A For given $\epsilon > 0$ there exist $\delta > 0$ such that $N_2^*(\varphi(f_1) - \varphi(f_2), h, t) < \epsilon$ for every $h \in F(Y)$ and $t \in (0,1)$ whenever $N_1^*(f_1 - f_2), g, t) < \delta$ for every $f_1, f_2 \in A$ and $g \in F(X)$.
Now to prove that φ is fuzzy ward continuous on A.

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Let $\{ f_n\}$ be a quasi Cauchy sequence in A, then

$N_1^*(f_{n+1} - f_n), g , t) < \delta$. From the above condition,

$N_2^*(\varphi (f_{n+1})- \varphi (f_n) , g , t) < \epsilon$ which implies that, $N_2^*(\Delta \varphi (f_n) , g , t) < \varepsilon$ Hence $\Delta \varphi (f_n)$ is a quasi -Cauchy sequence and so φ is a fuzzy ward continuous on A.

Theorem3.4

If a function $\varphi: N_1^* \rightarrow N_2^*$ is uniformly continuous on a subspace A of F(X) then it is fuzzy ward continuous on A.

Proof

Given φ is uniform continuous on A. For given $\varepsilon \in (0,1)$ there exits $\delta >(0,1)$ such that $N_2^*(\varphi (f_1)- \varphi (f_2) , h , t) < \epsilon$ for every $h \in F(Y)$ and $t \in (0,1)$ whenever $N_1^*(f_1 - f_2), g , t) < \delta$ for every $f_1 , f_2 \in A$ and $t \in (0,1)$.

Now to prove that φ is fuzzy ward continuous on A.

Let $\{ f_n \}$ be a quasi Cauchy sequence in A. then $N_1^*((f_{n+1} - f_n) , g , t) < \delta$, then by the hypothesis $N_2^*(\varphi (f_{n+1})- \varphi (f_n) , g , t) < \epsilon$

Let $\{ \varphi (f_n) \} = \{ g_n \}$ and $\Delta (g_n) = g_{n+1} - g_n$

Consider
$$N_2^*(\Delta g_n , h , t) = N_2^*(g_{n+1} - g_n, , h , t) = N_2^*(\varphi (f_{n+1})- \varphi (f_n) , h , t) < \epsilon.$$

Therefore $N_2^*(\Delta \varphi (f_n), h , t)< \epsilon$ whenever $N_1^*(\Delta f_n , g , t) < \delta$ and so φ is fuzzy ward continuous on A.

Theorem3.5

The image of a fuzzy ward compact space under a uniform continuous map is fuzzy ward compact.

Proof

Letφ : F(X)→F(Y) be uniform continuous and let A be a fuzzy ward compact subspace of F(Y)

Consider a sequence $\{g_n\}$ in $\varphi(A)$ provided $g_n = \varphi(f_n)$ where $\{ f_n \}$ is a sequence in A. Since A is fuzzy ward compact,$\{ f_n \}$ has a quasi Cauchy subsequence $\{ f_{n_k} \}$ therefore

$N_1^*(\Delta f_{n_k} , h , t) < \varepsilon$ For given $\varepsilon \in (0,1)$, there exist $\delta \in (0,1)$ such that $N_2^*(\Delta \varphi(f_{n_k}), g , t) < \varepsilon$, since $\Delta \varphi(f_{n_k})$,is quasi-Cauchy subsequence.

Hence $\varphi(A)$ is a fuzzy ward compact.

Theorem3.6

Let $\{ \varphi_n\}$ be a sequence of uniform continuous functions defined on a subspace A of F(X) to F(Y) and if $\{ \varphi_n\}$ converges uniformly to φthen $\varphi$is uniformly continuous.

Proof

Using uniform convergence of $\{ \varphi_n\}$ choose $\varepsilon > 0$ then there exist a positive integer N such that $N^*(\varphi_n (f_1)- \varphi_n (f_2), h, t) < \varepsilon$ where $n \geq N$ and $f, h \in A$.

Using the uniform continuity of φ_n on A for a given $\varepsilon \in (0,1)$ there exists $\delta \in (0,1)$ such that $N_2^*(\varphi_N (f_1)- \varphi_N (f_2), h, t) < \varepsilon$ for $f_1 , f_2 \in A$ and $h \in F(Y)$ provided $N_1^*((f_1- f_2), g , t) < \delta$.

Then
$$N_2^*(\varphi(f_1)- \varphi(f_2), h, t) = N_2^*(\varphi(f_1)- \varphi_N (f_1)+ \varphi_N (f_1)- \varphi_N (f_2)+ \varphi_N (f_2)- \varphi(f_2) , h , t)$$
$$\leq \max \{N_2^*(\varphi(f_1)- \varphi_N (f_1) , h , \tfrac{t}{3}), N_2^*(\varphi_N (f_1)- \varphi_N (f_2) , h , \tfrac{t}{3}), N_2^*(\varphi_N (f_2)- \varphi(f_2) , h ,\tfrac{t}{3})\}$$

$< \varepsilon$

$N_2^*(\varphi(f_1)- \varphi(f_2), h, t) < \varepsilon$ for every $h \in F(Y)$ whenever $N_1^*((f_1- f_2), g, t) < \delta$ so, φ is uniformly continuous on A.

Uniform limit theorem for fuzzy ward continuity

Theorem3.7

Let $\{ \varphi_n \}$ be a sequence of fuzzy ward continuous functions defined on a subspace A of a fuzzy 2- normed linear space F(X) to F(Y) and if $\{ \varphi_n \}$ is uniformly convergent to a function φ then φ is fuzzy ward continuous.

Proof

We assert that φ is fuzzy ward continuous on A. Let $\{f_n\}$be a quasi-Cauchy sequence in A. As $\{ \varphi_n \}$ uniformly converges to φ for $\varepsilon \in (0,1)$ there exist a positive integer N such that

$N^*(\varphi_n (f_n)- \varphi(f_n), h, t) <\frac{\varepsilon}{3}$. Since φ_N is fuzzy ward continuous on A there exist a positive integer $n_0 \geq N$ such that $N^*(\varphi_n (f_{n+1})- \varphi_N(f_n), h, t) < \frac{\varepsilon}{3}$ where every $n_0 \geq N$. and $h \in F(Y)$, $\varepsilon \in (0,1)$

$$N_2^*(\varphi(f_{n+1})- \varphi(f_n), h, t) = N_2^*(\varphi(f_{n+1})- \varphi_N (f_{n+1}) + \varphi_N(f_{n+1})- \varphi_N (f_n)+ \varphi_N (f_n)- \varphi(f_n), h,t)$$
$$\leq \max \{ N_2^*(\varphi(f_{n+1})- \varphi_N (f_{n+1}), h, \tfrac{t}{3}), N_2^*(\varphi_N (f_{n+1})- \varphi_N (f_n), h, \tfrac{t}{3}), N_2^*(\varphi_N (f_n)- \varphi(f_n), h, \tfrac{t}{3}) \}$$
$$< \varepsilon$$

$N_2^*(\varphi(f_{n+1})- \varphi(f_n), h, t) < \varepsilon$ for every $n_0 \geq N$ and hence φ is a fuzzy ward continuous on A.

References

[1] Burton, J.Coleman, Quasi-cauchy sequences, *American Mathematical Monthly*,117(4) (2010) 328-333.
[2] H.Cakalli, N -ward continuity, *Abstract and Applied Analysis*, (2012)80456.
[3] P.Das, E.Savas, S.Bhunia, Two valued measure and some new double sequence spaces in 2-normed spaces, *Czechoslovak Math. J.* 61(3)(2011),pp. 809-825.
[4]. S.Gahler, Linear 2-normierte Raume, *Mathematsche Nachrichten*,28(1964) 1-3.
[5]. K.Menger,Unter Suchungen Ueberallgeine Metrik, *Math.Ann*.100(1) (1928) 75-163.
[6]. S.P.Mohiuddine, Some new results on approximation in fuzzy 2-normed space, *Mathematical Computing Modelling*, 53(5-6) (2011) 574-580.
[7]. R.Pilakkt, S.Thirumangalath, Results in linear 2-normed spaces analogous to Baire's theorem and closed graph theorem, *International Journal of Pure and Applied Mathematics*,74(4) (2012) 509-517.
[8]. R.M. Somasundaram and Thangaraj Beaula, Some aspects of 2 fuzzy 2-normed linear space, *Bulletin of Malaysian Mathematical Society*, 32(2) (2009) 211-222.
[9]. Thangaraj Beaula and Lilly Esthar Rani, On fuzzy ward continuity in an intuitionstic 2-fuzzy 2-normed linerar space,*International journal of fuzzymathematical archive*.(2015) 6 (2), 207-213.
[10]. B.Vulich, Ona generalized notion of convergence in a Banach space, *Annals of Math*. 38 (1973) 156-174.
[11]. L.A.Zadeh, Fuzzy Sets, *Information and Control*, 8 (1965) 338-355.