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HUMAN CHAIN GRAPH

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Abstract: In this paper, a new graph called Human chain graph is introduced and some of its characteristics are discussed. We also provide an algorithm to determine the eccentricity of a graph vertex.

AMS Subject classification : 05C78

Key words: Human chain, Girth, Eccentricity

1. Introduction

Let G= $HC_{n,m}(p,q)$, $n\in N$, $m\geq 3$ be a simple, finite and undirected human chain graph with p= 2mn+n+1 vertices and q=2mn+2n edges. If n=1, $HC_{1,m}(p,q)$ may be considered as human graph which means that only one man is in the chain. Let $\Delta(G)$ be the maximum degree of vertices and $\delta(G)$ be the minimum degree of vertices of a graph G. Let E(V) be the eccentricity of a graph vertex and r(G) denote the radius of G and D(G) denote the diameter of G. Graph theoretical ideas are highly utilized in computer science application. Human chain graph is also utilized in networking, open banking, telecommunication and mobile application. In this paper, we introduce human chain graph and also discuss some of its characterizations.

2. Preliminaries

In this section, we provide some basic definitions relevant to this paper.

2.1 Bipartite graph: A bipartite graph is one whose vertex set can be partitioned into two subsets x and y, so that each edge has one end in x and one end in y such a bipartition of the graph.

2.2 Y-tree graph: A Y-tree Y_{n+1} is a graph obtained from a path P_n by joining an edge to a vertex of the path P_n adjacent to an end point.

2.3 Girth: The girth of a graph is the length of a shortest cycle contained in the graph.

2.4 Eccentricity: The eccentricity of a graph vertex in a connected graph is the maximum graph distance between v and any other vertex u of G and it is denoted by E(v).

2.5 Degree: The degree d(v) of a vertex v is its number of incident edges.

2.6 Chromatic number: The smallest number of colors needed to color a graph G is called its chromatic number and it is denoted by χ (G).

3. Main Results

In this section, we provide definition of human chain graph, some of its characterizations and an algorithm to determine the eccentricity of a graph vertices.

3.1 Human chain graph

A human chain graph $HC_{n,m}(p,q)$ is obtained by a path $u_1, u_2, \ldots, u_{2n+1}$, $n \in N$ joining a cycle of length m (C_m) and Y-tree (Y_{m+1}, $m \geq 3$) to each u_{2i} for $1 \leq i \leq n$. The vertices of C_m and Y-tree Y_{m+1} are $v_1, v_2, \ldots, v_{(m-1)n}$ and $w_1, w_2, \ldots, w_{mn}$ respectively.

Illustration: 1 ($HC_{1,3}$)

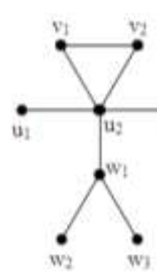


Illustration: 2 ($HC_{2,3}$)

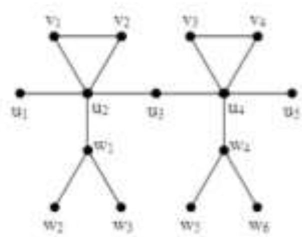
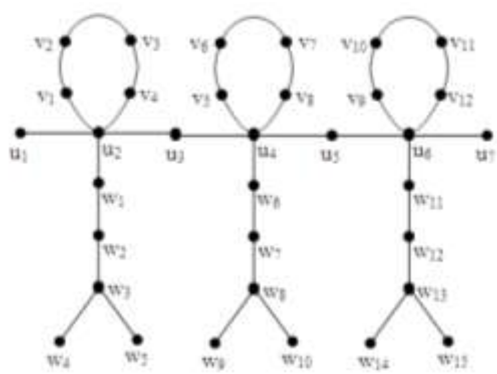


Illustration: 3 ($HC_{3,5}$)



Structural properties of $HC_{n,m}$

1. The vertex set of $HC_{n,m} = \{u_i, v_j, w_k / 1 \le i \le 2n+1,\ 1 \le j \le (m-1)n,\ 1 \le k \le mn\}$.

2. The total number of vertices of $HC_{n,m} = |V| = 2mn+n+1$.

3. The edge of set of $HC_{n,m} = |E|\ \{u_i u_{i+1} / 1 \le i \le 2n\} \cup$

$\{u_{2i} w_{m(i-1)+1};\ u_{2i} v_{(m-1)i};\ u_{2i} v_{(m-1)(i-1)+1};\ w_{mi} w_{mi-2} / 1 \le i \le n\} \cup \{ w_{mi+j} w_{mi+j+1};$

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$v_{(m-1)i+j}\, v_{(m-i)+j+1}$/ $0\le i\le n-1,\ 1\le j\le m-2\}$.

4. The total number of edges of $HC_{n,m}=|E|= 2mn+2n$.

5. The maximum degree of $HC_{n,m}= \Delta=5$.

6. The minimum degree of $HC_{n,m}= \delta=1$.

Theorem 3.1

If $m\geq3$ and $n\in N$, then the girth of the human chain graph is m.

Proof:

Let $HC_{n,m}$ be a human chain graph. To construct the human chain graph, we consider a path $u_1, u_2, \ldots, u_{2n+1}$ and adding a cycle of length m and Y-tree Y_{m+1} to each u_{2i} for $1\le i\le n$. Clearly path and Y-tree does not contain any cycle of length m to each u_{2i} for $1\le i\le n$. From the structure of $HC_{n,m}$, all the cycles does not intersect each other. Hence the shortest distance of cycles of $HC_{n,m}$ is m. Thus the girth of the human chain graph is m.

Theorem 3.2

If $m\geq3$ then $\chi(HC_{n,m})=\begin{cases}2 \text{ if } m \text{ is even}\\ 3 \text{ if } m \text{ is odd}\end{cases}$

Proof:

From the structure of $HC_{n,m}$, $\{u_1, u_2, \ldots, u_{2n+1}\} \cup \{w_1, w_2, \ldots, w_{mn}\}$ is acyclic and clearly it is 2-colorable. In $HC_{n,m}$, the length of each cycle is m and does not intersect each other.

Case(i): if m is odd, the length of each cycle of $HC_{n,m}$ is odd. Clearly odd length of cycle is 3-colorable.

Case(ii): if m is even, the length of each cycle of $HC_{n,m}$ is even. Clearly even length of cycle is 2-colorable.

Hence $\chi(HC_{n,m})=\begin{cases}2 \text{ if } m \text{ is even}\\ 3 \text{ if } m \text{ is odd}\end{cases}$

Theorem 3.3

If m is even then $HC_{n,m}$ is bipartite.

Proof:

Let $HC_{n,m}$, m≥3 be a human chain graph. By the theorem 3.2, $HC_{n,m}$ is 2-colorable only when m is even. In this case, V(G) can be partitioned into two color classes. These color classes are independent sets and hence form a bipartition of $HC_{n,m}$. Hence $HC_{n,m}$ is bipartite only if m is even.

Observation 3.1

If m≥3, the number of vertices of $HC_{n,m}=\begin{cases} even \ \ if\ n\ is\ even \\ odd \ \ if\ n\ \ is\ odd \end{cases}$

Proof

Let $HC_{n,m}$, m≥3 be a human chain graph. The total number of vertices of $HC_{n,m}$= $|V|$= 2mn+n+1.If m≥3 and n∈N , 2mn is always even and 2mn+1 is odd. Therefore $|V|$= odd+n. If n is even, $|V|$= odd +even =odd and if n is odd, $|V|$= odd+odd = even. Hence the number of vertices of $HC_{n,m}$ is even if n is odd and odd if n is even.

Algorithm 3.1

Procedure: (Eccentricity of $HC_{n,m}$, n≥ 2 $and\ m \geq 3$)

Input: V←{{ u_1 ,u_2 ,…,u_{2n+1}, w_1 ,w_2 ,…,w_{mn}, v_1 ,v_2 ,…,$v_{(m-1)\,n}$}

if n ≥ 2

for i=1to n+1 **do**

$f(u_i)$← m+2n-1-i

end for

for i=1 to $\left\lfloor \frac{n+1}{2} \right\rfloor$ **do**

for j=1 to (m-1) **do**

$f(w_{mi-m+j}) \leftarrow m+2n-1-2i+j$

end for

end for

for i=1 to $\left\lfloor \frac{n+1}{2} \right\rfloor$ **do**

$f(w_i) \leftarrow 2m+2n--2i-2$

end for

for i=1 to $\left\lfloor \frac{n}{2} \right\rfloor$ **do**

$f(w_{mn-mi+m}) \leftarrow 2m+2n-2-2i$

end for

for i=1 to $\left\lfloor \frac{n}{2} \right\rfloor$ **do**

for j=1 to (m-1)**do**

$f(w_{mi-mi+j}) \leftarrow m+2n-1-2i+j$

end for

end for

for i=1 to $\left\lfloor \frac{n+1}{2} \right\rfloor$ **do** for j=1 to $\left\lfloor \frac{m}{2} \right\rfloor$ **do**

$f(v_{(m-1)i-m+1+j}) \leftarrow m+2n-1-2i+j$

end for

end for

for i=1 to $\left\lfloor \frac{n+1}{2} \right\rfloor$ **do**

for j=1 to $\left\lfloor \frac{m-1}{2} \right\rfloor$ **do**

$f(v_{(m-1)i+1-j}) \leftarrow m+2n-1-2i+j$

end for

end for

for i=1 to $\left\lfloor \frac{n}{2} \right\rfloor$ **do**

for j=1 to $\left\lfloor \frac{m}{2} \right\rfloor$ **do**

$f(v_{(m-1)n-(m-1)i+j}) \leftarrow m+2n-1-2i+j$

end for

end for

for i=1 to $\left\lfloor \frac{n}{2} \right\rfloor$ **do**

for j=1 to $\left\lfloor \frac{m-1}{2} \right\rfloor$ **do**

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$f(v_{(m-1)n-(m-1)i+m-j}) \leftarrow m+2n-1-2i+j.$

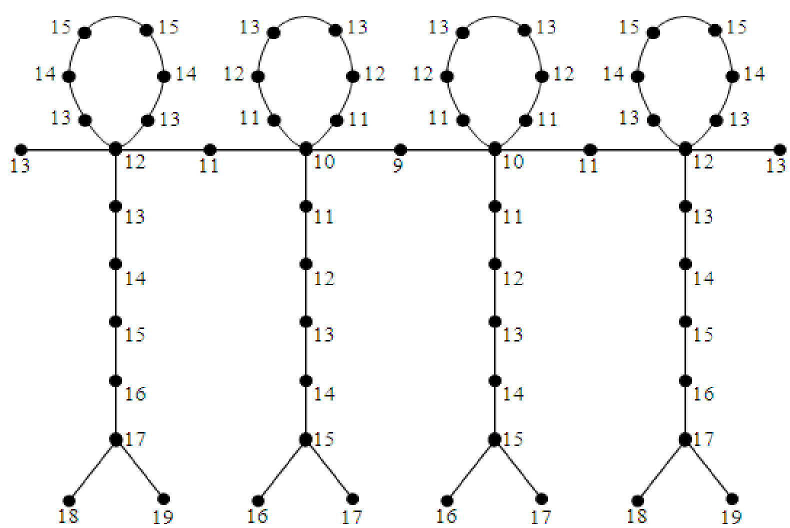
end for

end for

end if

end procedure

Illustration: 4 (Eccentricity for $HC_{4,7}$)



4. Conclusion

In this paper, we have discussed human chain graph , some of its characterization and also determined eccentricity of human chain graph by an algorithm.

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