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ON POSITIVE BOUNDED IMPLICATIVE PSEUDO FUZZY SOFT KU-MODULES

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Abstract: In this paper, we introduce the concept of pseudo fuzzy soft KU- sub module (PFSKU-sub module) and some properties, related results and equivalent conditions. Also, we investigate every imaginable PFSKU-sub module is a sub module, onto homomorphic image and its characterization with supremum property.

Keywords: bounded, KU-sub module, soft set, fuzzy set, pseudo fuzzy soft KU-sub module, homomorphism, supremum property, characterstic, sum.

Introduction: Any bounded implicative BCK-algebra can be realized as an algebra, with the supremum operation, which acts over itself, as an abelian group with suitable operations. This motivates that the concept a BCK-module is studied, which Aslam et.al considered in [1]. In 1966, Imai and Iseki [5] defined a new class of algebras, which generalizes on one hand the notion of the algebra of sets with the set subtraction as the only fundamental non-nullary operation and, on the other hand, the notion of implication algebra, called BCK-algebra. Since then a great deal of literature has been produced on this theory. Some other subclasses of BCK-algebras such as positive implicative BCK-algebra, implicative BCK-algebra and commutative BCK-algebra were introduced in [4, 6, 21] and some properties and important

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results were investigated. In 1991, Xi [22] applied fuzzy set theory to BCK-algebras and introduced the notion of fuzzy sub algebras (ideals) of the BCK-algebras, and since then some authors studied fuzzy sub algebras and fuzzy ideals (see [3] and [7-18]). They considered chain conditions, exact sequences, projective and injective BCK-modules and gave some related results. The limitations of classical methods in dealing with uncertainties in economics, environmental sciences, engineering models and other fields persuade researchers to think otherwise. This result in development of fuzzy sets [23], rough set theory [19], probability theory and other mathematical tools. However, these methods inherited their own difficulties and limitations. Consequently, in [17], Molodtsov proposed a new approach to deal with these difficulties, which is referred as the soft set theory. The idea attracted many researchers and the theory developed rapidly. A detailed theoretical study of soft sets and their implementation on decision making is discussed by Maji et.al in [16]. The application of soft sets is not limited to these areas only but it also motivated people working in more abstract areas of mathematics to apply soft sets in their areas. In this regard, Aktas et.al [3], introduced the notion of soft group and developed its basic theory. Jun et.al applied soft set theory to BCK/BCI-algebras in [11]. Soft rings were introduced by Acar et.al in [2]. In this article, we introduce the concept of pseudo fuzzy soft KU- sub module (PFSKU-sub module) and some properties, related results and equivalent conditions. Also, we investigate every imaginable PFSKU-sub modules is a sub module, onto homomorphic image and its characterization with supremum property.

Section-2 Preliminaries

In this section, some definitions and results from the literature are given, for more details, see the references.

Definition 2.1 [22]: By a KU-algebra, we mean an algebra (X, *, 0) of type (2, 0) satisfying the following axioms;

(KU-1) (x*y) * [(x*z) * (z*y)] = 0

(KU-2) 0*x = 0

(KU-3) x *0 = x

(KU-4) x *y = 0 and y *x = 0 imply x = y, for all x, y ∈ X.

An algebra (X, *, 0) satisfying, the conditions (KU-1), (KU-3) and (KU-4) is called pseudo KU- algebra.

A partial ordering ≤ is defined on X by $x \leq y \leftrightarrow x * y = 0$.

Definition 2.2 [22]: A pseudo KU-algebra X is said to be bounded if there is an element $I \in X$ such that $x \leq I$, for all $x \in X$.

Definition 2.3 [22]: A pseudo KU-algebra X is said to be commutative if it satisfies the identity $x \cap y = y \cap x$, where $x \cap y = y * (y * x)$, for all $x, y \in X$.

Definition 2.4 [22]: A pseudo KU-algebra X is said to be implicative if $x * (y * x) = x$. for all $x,y \in X$.

Example 2.5: Let A be a non empty set and X = P(A), the power set of A. Then (X, − , Φ) is a pseudo KU-algebra.

Definition 2.6: Let X be a pseudo KU-algebra. Then by a left X-module (abbreviated by X-module), we mean an abelian group M with an operation $X \times M \rightarrow M$ with $(x,m) \dashv xm$ satisfies the following conditions; for all $x, y \in X$ and $m, n \in M$,

(i) $(x \cap y)\, m = x\,(ym)$

(ii) $x\,(m + n) = xm + xn$

(iii) $0m = 0$

Moreover if X is bounded and M satisfies $1m = m$, for all $m \in M$, then M is said to be unitary.

Example 2.7: If A is a non empty set, then X = P(A), the power set of A , is an X-module with $xm = x \cap m$, for any $x, m \in X$.

Example 2.8: Let X be a bounded implicative pseudo KU-algebra. Then (X, + , 0) is an X-module, where " + " is defined as $x + y = (x*y) \cup (y*x)$ and $xy = x \cap y$.

Theorem 2.9: A subset A of a pseudo KU-module M is a pseudo KU-sub module only if $a - b,\ xa \in A$, for every $a, b \in A$ and $x \in X$.

From now on in this paper, X will denote a bounded pseudo KU-algebra, ᑍ(M) denotes the set of all fuzzy soft subsets of pseudo KU-modules of M and ᑍƤ(M) denotes the set of all fuzzy soft pseudo KU-sub modules of M.

Section-3 Pseudo fuzzy soft KU-sub module

We first recall some fuzzy logic concepts.

Definition 3.1[17]: Let U be an initial universe set and E be a set of parameter. A pair (F, E) is called a soft set (over U) and if and only if F is a mapping of E into the set of all subsets of the set U.

Example 3.2: A soft set (F,E) describe the attractiveness of the houses which Mr. X is going to buy

U = is the set of houses under consideration.

E = is the set of parameters. Each parameter is a word or sentence.

E = {expensive, beautiful, wooden, cheep in the green surroundings, modern in good repair, in bad repair}.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on.

Definition 3.3 [23]: Let X be a non empty set. A fuzzy set A in X is a function from X into [0,1].

Example 3.4: Let X = {a, b, c, d} be a non empty set. Then the fuzzy set A in X is defined by A = {[a, 0.4], [b, 0.8], [0. 1], [0, 0.6]}.

Definition 3.5: A pair (Λ, Σ) is called a fuzzy soft set over X, where $\Lambda: \Sigma \to P(X)$ is a mapping from Σ into P(X).

Definition 3.6: Let X be a universe and E a set of attributes. Then the pair (X, E) denotes the collection of all fuzzy soft sets on X with attributes from E and is called a fuzzy soft class.

Definition 3.7: For two fuzzy soft sets (Λ, Σ) and (Δ, Ω) in a fuzzy soft class (X, E), we say that (Λ, Σ) is a fuzzy soft subset of (Δ, Ω), if (i) $\Sigma \subseteq \Omega$, (ii) For all $\mathcal{E} \in \Sigma, \Lambda(\mathcal{E}) \leq \Delta(\mathcal{E})$, and is written as $(\Lambda, \Sigma) \subseteq (\Delta,\Omega)$.

Definition 3.8: A fuzzy soft set A of M is said to be a pseudo fuzzy soft KU-sub module if for all $m, m_1, m_2 \in M$, $x \in X$, the following axioms hold;

(PFSKU-1) $A(m_1 + m_2) \geq T\{A(m_1), A(m_2)\}$

(PFAKU-2) $A(-m) = A(m)$

(PFSKU-3) $A(xm) \geq A(m)$.

Example 3.9: Let X = {0, 1, 2, 3}, and consider the following table

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

Then (X, *) is a bounded implicative pseudo KU-algebra and so is a pseudo KU-module over itself.

Now $a_0, a_1, a_2 \in [0, 1]$ be such that $a_0 > a_1 > a_2$. Define A: X→ [0, 1] by $A(0) = a_0$, $A(1) = a_1$, $A(2) = A(3) = a_2$. Then A is a pseudo fuzzy soft KU-sub module of X (abbreviated as PFSKU-module).

4. SOME STANDARD RESULTS

The following theorem is easily proved.

Theorem-4.1: A fuzzy soft set A of M is a PFSKU-sub module if and only if

(i) $A(xm) \geq A(m)$ (ii) $A(m_1\text{-}m_2) \geq T\{A(m_1), A(m_2)\}$.

Theorem-4.2: Let A ∈ Ƥ(M) . Then A is a PFSKU-sub module of M if and only if

(i) $A(0) \geq A(m)$ (ii) $A(xm\text{- } yn) \geq T\{A(m_1), A(m_2)\}$.

Proof: (→) It follows from theorem - 4.1 and that 0m = 0, for all m ∈ M.

(←) we have $A(xm) = A(xm\text{-}y0) \geq T\{A(m), A(0)\} = A(m)$ and

$A(m\text{-}n) = A(1m\text{-}1n) \geq T\{A(m), A(0)\}$ proving A is a PFSKU-sub module of M.

Theorem-4.3: Let A ∈ Ƥ(M). Then A is a PFSKU-sub module if and only if for all t ∈ [0,1] , $\Phi \neq A_t$ is a PFSKU-sub module of M.

Proof: (→) Let $A_t \in \Phi$, for $t \in [0, 1]$ and m, n ∈ A. Then $A(m), A(n) \geq t$.

Since A is a PFSKU-sub module,

$A(m\text{-}n) \geq T\{A(m), A(n)\} \geq t$ and so $m\text{-}n \in A_t$. This shows that A_t is a sub module of M. Now, let $m \in A_t$ and x ∈ X. Then $A(xm) \geq A(m) \geq t$, that is $xm \in A_t$.

Therefore A_t is PFSKU- sub module of M.

(←) Let $t = T\{A(m), A(n)\}$, for m, n ∈ M. Then $m, n \in A_t$ and so $m\text{-}n \in A_t$ which means that $A(m\text{-}n) \geq t = T\{A(m), A(n)\}$. Now, let S = A(m).

Therefore, A is a PFSKU-sub module of M.

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Theorem-4.4: Let A ∈ Ր(M). Then A is a PFSKU-sub module if and only if for all t ∈ [0, 1], $\Phi \neq A_t$ is a soft KU-sub module of M.

Proof: (→) Let $A_t \neq \Phi$, for t ∈ [0,1] and m, n ∈ A_t. Then A(m), A(n) ≥ t. Since A is a PFSKU-sub module, A(m-n) ≥ T {A(m), A(n)} ≥ t and so m-n ∈ A_t. This shows that A_t is a subgroup of M. Now, let m ∈ A_t and x ∈ X. Then A(xm) ≥ A(m) ≥ t, (i,e) xm ∈ A_t. Therefore A_t is a soft KU-sub module of M.

(←) Let t = T{A(m), A(n)}, for m, n ∈ M. Then m, n ∈ A_t and so m-n ∈ A_t which means that A(m-n) ≥ t = T{A(m), A(n)}. Now, let S = A(m). Then m ∈ A_s and so xm ∈ A_s which means that A(xm) ≥ S = A(m). Therefore, A is a PFSKU-sub module of M.

Theorem-4.5: Let {A_i / i ∈ ∧} be a non empty family of PFSKU-sub modules of M. Then ∩A_i is PFSKU-sub module of M.

Proof: The proof is easy.

Remark-4.6: Let K be a family of PFSKU-sub modules of M. The intersection of all PFSKU-sub modules of M containing K is called the PFSKU-sub module generated by K, denoted by <K>. If K = { v_1, v_2 ,………..,v_t}, then we write { v_1, v_2 ,…… ,v_t }. If K is finite and K = <K>, then we say that A is finitely generated. In particular, if A = < a_t >, then we say that A is cycle.

Lemma 4.7: Let A and B be pseudo fuzzy soft subsets of M. Then

(i) <A_s> + < A_t> is a subset of < $A_{s\wedge t}$ >, for all s,t ∈ [0,1].

(ii) A is subset of B →<A_t> is a subset of <B_t>, for all t ∈ [0, 1].

Proposition 4.8: If A is PFSKU-sub module of M with respect to a t-norm T, then M_1 = {x/ x ∈ M, A(-m) = A(m)} is a sub module of M and A is a PFSKU-sub module of M_1 , with respect to T.

Proof: Let m_1, m_2 ∈ M_1 and α ∈ R. Then, according to condition (PFSKU-1), A(m_1 +m_2) ≥ T{A(m_1),A(m_2)} = T(1,1) = 1.Thus A(m_1+m_2) = 1.Hence m_1 +m_2 ∈ M_1. According to condition PFSKU-3, A(xm) ≥ A(m) = 1.Thus, we have A(xm) = 1. From here it follows that xm ∈ M_1. Finally according condition PFSKU-2, A(-m) = 1.Therefore –m ∈ M. Thus M_1 is a PFSKU – sub module of M.

Lemma 4.9: A pseudo fuzzy soft subset A of M is a PFSKU- sub module of M if and only if each level subset A_t , t ∈ Im (A), is a sub module of M.

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Proposition 4.10: Let A be a PFSKU- sub module of M. Then the pseudo fuzzy soft subset $< A >$ is a PFSKU- sub module of M generated by A. More over $< A >$ is the smallest PFSKU- sub module containing A.

Proof: Let $x, y \in X$ and let $A(x) = t_1$, $A(y) = t_2$ and $A(x+y) = t$.

Let it possible $t = < A >(x+y)$

$\leq T\{<A>(x), <A>(y)\}$. So $T\{t_1, t_2\} = t_1$ (say)

Then $t_1 = <A>(x) = \sup\{ k / x \in <A_k> \} \geq t$. Therefore there exists $k_1 \geq t$ such that $x \in <A_{k1}>$. Also $t_2 = <A>(y) = \sup\{k / y \in <A_k>\} \geq t$. So there exists $k_2 \geq t$ satisfying $y \in <A_{k2}>$. Without loss of generality, assume that k_1 , k_2 with $<A_{k1}> \subseteq <A_{k2}>$. Then $x, y \in <A_k>$ implies $(x + y)$ which is a contradiction since $k_2 \geq t$. Thus $t \geq t_1$.

Consequently, $A(x + y) \geq T\{<A>(x), <A>(y)\}$ --- (1).

Now let $t_3 = <A>(mx) \leq <A>(x) = t_1$.Then $t_1 = <A>(x) = \sup\{k / x \in <A_k>\} \geq t_3$ (if possible). Thus there exists k such that $x \in <A_k>$ and $t_1 \geq k \geq t_3$. So that $mx \in <A_k> \subseteq \{A_{t1}\}$ which is a contradiction. So $t_3 = < A >(mx) \geq <A>(x) = t_1$ --- (2).

Consequently conditions (1) and (2) yield that $< A >$ is a PFSKU- sub module of M. Finally, the aim is to show that $< A>$ is the smallest PFSKU- sub module containing A.

Assume that θ is a PFSKU- sub module of M such that $A \subseteq \theta$ and claim that $< A> \subseteq \theta$. Let $t = <A>(x) \geq \theta(x)$ for some $x \in m$ (if possible). Let $\varepsilon > 0$ be given. Then $t = A_t = \sup\{k / x \in <A_k>\}$. Thus there exists k such that $x \in <A_k>$ and $t-\varepsilon \leq k \leq t$. So that $x \in <A_k> \subseteq$ $< A_{t-\varepsilon}>$ for all $\varepsilon > 0$.

Now $x = \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n$, $\alpha_i \in R$, x_i belongs to $t-\varepsilon$. $x_i \in A_{t-\varepsilon}$ implies $A(x_i) \geq t-\varepsilon$, that is $\theta(x_i) \geq t-\varepsilon$ for all $\varepsilon > 0$.

Thus $\theta(x) \geq T\{\theta(x_1), \theta(x_2), \ldots, \theta(x_n)\} \geq t-\varepsilon$ for $\varepsilon > 0$.

Hence $\theta(x) = t$ which is a contradiction to our supposition. Hence the proof.

Definition 4.11: Let A and θ be two PFSKU- sub modules of module M. Then the sum of A and θ denoted by $A+\theta$ is defined as

$$(A+\theta)(x) = \sup_{x_i = a + b} [T(A(a), A(b))] \text{ for all } x_i \in X.$$

Clearly $A+\theta$ is a pseudo fuzzy soft subset of X.

Proposition 4.12: Let A and θ be two PFSKU- sub modules of M. Then the sum A+θ is PFSKU - sub module of M.

Proposition 4.13: Let A and θ be two PFSKU- sub modules of M such that A (0) = θ (0). Then A⊆ A+θ, θ ⊆ A+θ.

Proof: Let x ∈X. Then

$$\begin{aligned}(A+\theta)(x) &= \sup_{x=a+b}\{T(A(a),\theta(b))\}\\ &\geq T(A(x),\theta(x))\\ &= T(\theta(x),A(0))\\ &= \theta(x).\end{aligned}$$

Similarly (A+θ) (x) ≥ A (x).

Lemma 4.14: (4.13) is not true if A (0) ≠ θ (0) < A (0). So A does not contain A+θ.

Proposition 4.15: Let A and θ be two PFSKU-sub modules of M such that A (0) = θ (0). Then A+θ = < A+θ >.

Proof: Let x ∈ X.

$$t_1 = (A+\theta)(x) = \sup_{x=a+b}\{T(A(a),A(b))\}$$

Let ε > 0 be given.

Then $t_{1-\varepsilon} \leq T(A(a),\theta(b))$ for some a,b∈ M such that x = a+b, so that $t_{1-\varepsilon} \leq A(a)$ and $t_{2-\varepsilon} < \theta(b)$ but A, θ ⊂ <A∪ θ> .

Therefore

$$\begin{aligned}t_{1-\varepsilon} &\leq T\{<A\cup\theta>(a),<A\cup\theta>(b)\}\\ &\leq <A\cup\theta>(a+b)\\ &= <A\cup\theta>(x), \text{ for all } \varepsilon>0.\end{aligned}$$

Hence $t_1 \leq <A+\theta>(x) = t$ (say).

Let it possible $(A+\theta)(x) = t_1 \leq <A\cup\theta>(x) = \sup\{k / x \in <(A\cup\theta)>\}$.

Thus there exists k such that $x \in <(A+\theta)k>$ and $t_1 \leq k \leq t$.

Then $(A\cup\theta)_t \subset (A\cup\theta)_{t1}$. so that $<(A\cup\theta)_t> \subset <(A\cup\theta)_k> \subset <(A\cup\theta)_{t1}>$ implying $x \in <(A+\theta)_{t1}>$ which is a contradiction. Hence $t_1 = (A+\theta)(x) = <A\cup\theta>(x) = t$.

Corollary 4.16: For any PFSKU- sub module A of X, A + A = A.

Lemma 4.17: (4.13) is not true if $A(0) \neq \theta(0)$, for let $A(0) > \theta(0)$ implies $< A \cup \theta > (0) = A(0) > (A \cup \theta)(0)$. So $< A \cup \theta > (0) \geq (A \cup \theta)(0)$ and so $A + \theta \neq < A \cup \theta >$.

Definition 4.18: APFSKU-sub module 'A' of a module M is said to be normal if A(0) = 1.

SECTION-5: PROPERTIES OF PFSKU- SUB MODULES OF MODULES

Proposition 5.1: Let A be a PFSKU- sub module of X and let A* be a pseudo fuzzy soft set in X defined by $A^*(x) = A(x) + 1 - A(0)$ for all $x \in X$. Then A* is a normal PFSKU- sub module of X containing A.

Proof: For any $x, y \in X$, it follows that

$A^*(x + y) = A(x+y) + 1 - A(0) \geq T(A(x)+1- A(0) , A(y)+1- A(0)) = T (A^*(x) , A^*(y))$.

$A^*(mx) = A(mx) + 1 - A(0) = A(x) + 1 - A(0) = A(x)$.

∴ A* is a normal PFSKU- sub module of X containing A.

Proposition 5.2: Let T be a t- norm. Then every imaginable PFSKU-sub module A of a module X is a PFSKU-sub module of X.

Proof: Assume A is imaginable PFSKU- sub module of X. Then it gives

$A(x + y) \geq T\{ A(x), A(y) \}$ and $A(mx) \geq A(x)$ for all x, y in X.

Since A is imaginable, it implies that

$$\begin{aligned} \min\{ A(x), A(y) \} &= T\{ \min\{ A(x), A(y)\}, \min\{ A(x), A(y)\}\} \\ &\leq T(A(x), A(y)) \\ &\leq \min\{ A(x), A(y) \}. \end{aligned}$$

So $T(A(x), A(y)) = \min\{ A(x), A(y) \}$. It follows that

$A(x+y) \geq T(A(x), A(y)) = \min\{ A(x), A(y) \}$ for all $x, y \in X$. Hence A is a PFSKU- sub module of X.

Proposition 5.3: If A is a PFSKU- sub module of a module X and Θ is an endomorphism of X, then $A_{[\Theta]}$ is a PFSKU- sub module of X.

Proof: For any $x, y \in X$, it gives that

$$\begin{aligned} \text{(i) } A_{[\Theta]}(x+y) &= A(\Theta(x+y)) \\ &= A(\Theta(x), \Theta(y)) \\ &\geq T(A(\Theta(x)), A(\Theta(y))) \end{aligned}$$

$= T\{ A_{[\Theta]}(x), A_{[\Theta]}(y)\}$

(ii) $A_{[\Theta]}(mx) = A(\Theta(mx)) \geq A(\Theta(x)) \geq A_{[\Theta]}(x)$. Hence $A_{[\Theta]}$ is a PFSKU- sub module of X.

Proposition 5.4: An onto homomorphism of a PFSKU- sub module of module X is a PFSKU sub module of X.

Proof: Let $f : X \rightarrow X^1$ be an onto homomorphism of modules and let ξ be a PFSKU- subgroup of X^1 and 'A' be the pre image of ξ under 'f. Then we have

(i) $A(x + y) = \xi(f(x + y))$

$= \xi(f(x) . f(y))$

$\geq T(\xi(f(x)), \xi(f(y)))$

$\geq T(A(x), A(y))$.

(ii) $A(mx) = \xi(f(mx))$

$\geq \xi(f(x))$

$\geq A(x)$.

Proposition 5.5: An onto homomorphism image of a PFSKU- sub module with the sup property is a PFSKU- sub module.

Proof: Let $f: X \rightarrow X^1$ be an onto homomorphism of modules and let A be a sup property of PFSKU- sub module of X.

Let $x^1, y^1 \in X^1$ and $x_0 \in f^{-1}(x^1)$, $y_0 \in f^{-1}(y^1)$ be such that

$A(x_0) = \sup_{h\in f^{-1}(x^1)} [A(h)]$; $A(y_0) = \sup_{h\in f^{-1}(y^1)} [A(h)]$ respectively.

It deduces that

(i) $A^f(x^1 + y^1) = \sup_{z\in f^{-1}(x^1+y^1)} [A(z)]$

$\geq \min\{A(x_0), A(y_0)\}$

$= \min\left\{ \sup_{h\in f^{-1}(x^1)} [A(h)], \sup_{h\in f^{-1}(y^1)} [A(h)] \right\}$

$= \min\{A^f(x^1), A^f(y^1)\}$.

(ii) $A^f(mx) = \sup_{z \in f^{-1}(m^1x^1)} [A(z)]$

$\geq A(y_0)$

$= \sup_{h \in f^{-1}(y^1)} [A(h)]$

$= A^f(y^1)$.

Hence A^f is a PFSKU-sub module of X.

Proposition 5.6: Let T be a continuous t-norm and let f be a homomorphism on a module X. If A is a PFSKU- sub module of X, then A^f is a PFSKU-sub module of f(X).

Proof: Let $A_1=f^{-1}(y_1)$, $A_2 = f^{-1}(y_2)$ and $A_{12} = f^{-1}(y_1-y_2)$ where $y_1,y_2 \in f(X)$. Consider the set $A_1-A_2=\{x \in S / x = a_1-a_2\}$ for some $a_1 \in A_1$ and $a_2 \in A_2$.

If $x \in A_1-A_2$, then $x = x_1 - x_2$ for some $x_1 \in A_1$ and $x_2 \in A_2$. So that $f(x) = f(x_1) - f(x_2) = y_1- y_2$ implies $x \in f^{-1}(y_1 - y_2) = f^{-1}(y_1-y_2) = A_{12}$. Thus $A_1 - A_2 \subset A_{12}$. It follows that

$A^f(y_1+y_2) = \sup \{ A(x) / x \in f^{-1} (y_1- y_2)\}$

$= \sup\{ A(x) / x \in A_{12} \}$

$\geq \sup \{A(x) / x \in A_1-A_2\}$

$\geq \sup \{ A(x_1- x_2) / x_1 \in A_1 \text{ and } x_2 \in A_2\}$

$\geq \sup \{ T(A(x_1) , A(x_2)) / x_1 \in A_1 \text{ and } x_2 \in A_2\}$.

Since T is continuous, and for every $\varepsilon > 0$, there exists $\delta > 0$ such that

$\sup \{ A(x_1) / x_1 \in A_1\} - x_1^* \} \leq \delta$ and

$\sup \{ A(x_2) / x_2 \in A_2\} - x_2^* \} \leq \delta$.

$\therefore T\{\sup\{A(x_1) / x_1 \in A_1\} , \sup \{ A(x_2) / x_2 \in A_2 \}\} - T (x_1^*, x_2^*) \leq \varepsilon$.

Choose $a_1 \in A_1$ and $a_2 \in A_2$ such that

$\sup \{ A(x_1) / x_1 \in A_1\} - A(a_1) \leq \delta$ and

$\sup \{ A(x_2)/ x_2 \in A_2\} - A(a_2) \leq \delta$.

Then it implies that

$T\{\sup\{A(x_1) / x_1 \in A_1\}, \sup\{A(x_2) / x_2 \in A_2\}\} - T(A(a_1), A(a_2)) \leq \varepsilon$. Consequently, $A^f(y_1 - y_2) \geq \sup\{T(A(x_1), A(x_2)) / x_1 \in A_1, x_2 \in A_2\}$

$$\geq T(\sup\{A(x_1) / x_1 \in A_1\}, \sup\{A(x_2) / x_2 \in A_2\})$$

$$\geq T(A^f(y_1), A^f(y_2)).$$

Similarly $A^f(mx) \geq A^f(y)$. Hence A^f is a PFSKU- sub module of f(X).

Conclusion: Molodtsov proposed a new approach to deal with these difficulties, which is referred as the soft set theory. The idea attracted many researchers and the theory developed rapidly. A detailed theoretical study of soft sets and their implementation on decision making is discussed by Maji et.al in [16]. The application of soft sets is not limited to these areas only but it also motivated people working in more abstract areas of mathematics to apply soft sets in their areas.In this article, we introduce the concept of pseudo fuzzy soft KU- sub module (PFSKU-sub module) and some properties, related results and equivalent conditions. Also, we investigate every imaginable PFSKU-sub modules is a sub module , onto homomorphic image and its characterization with supremum property.

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