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Modeling and Simulation of Proportional-Integral-Derivative (PID) Temperature Controller for an Electric Kettle

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Abstract

Proportional-Derivative-Integral control has been a dominant strategy used in industry to control temperature and other processes. However, their use in simple appliances such as electric kettle has not been widespread. With the increasing concern for energy and convenience, attention is being focused on how to develop simple and cost efficient controllers that will reduce energy waste. In this study, a mathematical model of an electric kettle is developed using the lumped parameter model. The characteristics transfer function of the system was obtained using the open-response characteristics. A PID controller was then designed for the process while MATLAB Simulink model was developed. The model was then simulated. The results show increase in overshoot from 5%−12.5%, rise time 230−300 seconds, settling time 720−750 seconds and zero steady-state error between 50 ^{o}C−80 ^{o}C.

Keywords:

PID Controller;
Open-loop;
Closed-loop;
Temperature
Response;
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1. Introduction

Electric kettle is a type of pot, typically metal, specialized for boiling water with a lid spout and handle that functions in a self-contained manner. Electric kettles are widely used for boiling water, making Maggie, tea, baby bottle water, hand-boiled eggs, and hot chocolate. The fastest electric kettle would heat water at nearly twice the speed of stovetop. Thus, the use of electric kettle is versatile.

The problem of temperature control has always been a challenging issue especially for control engineer [1]. While most electrical appliances have one form of temperature control or the other, they have retained the use of simple control strategies such as on-off control. For electric kettle, the dominant control strategy is basically on-off control utilizing thermostats. Since the energy required to boil water with an electric kettle is high than the cost of doing same with a stovetop, there is need to develop appropriate control system that will reduce energy waste, and hence cost.

Proportional-Integral-Derivative (PID) control has been one of the control systems design of the longest history and is still extensively used [2]. Control engineers prefer Proportional-Integral-Derivative (PID) controllers for their application due to its simplicity and better performance in majority of cases [3]. For this study, a prototype of the electric kettle was locally fabricated and lagged to minimize energy loss to the surrounding. The desire is to be able to achieve the same function with the ones obtainable in the market at a lower cost. The main challenge is that temperature set points are maintained at a constant value over desired range.

The significant problems in temperature control are overshooting, long settling time, steady-state error, and the need for robustness under varying conditions and external disturbances. The design requirement is that, the simulated results should have less than 10% overshoot, rise time of 100 seconds, and settling time of 125 seconds which is the average settling period of electric kettles.

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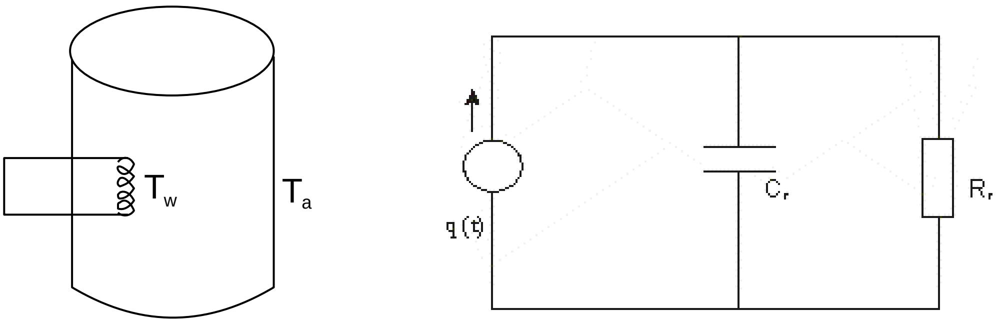
2.1 Research Method

An electric kettle (reactor) was constructed using locally available sheet metals. An immersion heater was also fixed inside which serves as the heating element. The cylindrical kettle was lagged to minimize energy loss from the kettle to the environment. It was also coated with Aluminum paint to minimize corrosion. The volume of the kettle is 1.7 litters, which is the average of available ones in the market An open-loop response test was carried out on the reactor by switching on the heater while the temperature of the water inside was measured and recorded per second with a digital multi-meter (MAS-345) connected to a laptop computer. The recorded temperature against time was then plotted in MATLAB as in Figure 3.1. This was necessary so as to determine the open-loop response characteristics of the process such as time constant, static gain and time delay.



(a) Electric kettle (reactor) with Heating element

(b) Simplified thermal model of the water tank with heating element

Figure 2.1: *Diagram of the reactor*

2.2 Mathematical Model

The initial phase of the design process was to establish and quantify the desired response characteristics of the system. The process model was then obtained using the so called lumped parameter model [4]. It was assumed that the temperature of the water inside the kettle is uniform and that the external boundary of the system is at a fixed temperature. Then, using the first Law of thermodynamics [5],

$$C_T \frac{dT_W}{dt} = q(t) - q_o(t) \qquad 2.1$$

where C_T = thermal capacity of the system (mass x specific heat capacity)

$q_O(t) = \frac{T_w - T_a}{R_T}(J)$; (Heat loss to the environment) at time t.

T_a = ambient temperature (^oC)

R_T = thermal resistance ($^oC/J$) at time, t.

T_w = temperature of the water

$q(t)$ = heat supplied by the heater (J) at time t.

Substituting $q_O(t) = \frac{T_w - T_a}{R_T}$ into (1), we have:

$$C_T \frac{dT_w}{dt} = q(t) - \frac{(T_w - T_a)}{R_T} \qquad 2.2$$

$$\Rightarrow \frac{dT_w}{dt} + \frac{T_w}{R_T C_T} = \frac{q(t)}{C_T} + \frac{T_a}{R_T C_T} \qquad 2.3$$

Taking the Laplace transform of (3) and re-arranging:

$$sR_T C_T T_w(s) + T_w(0) + T_w(s) = Q(s)R_T + T_a(s) \qquad 2.4$$

But at constant ambient temperatures,

$T_W(0) = T_a(s)$ which are the initial conditions, and

$$F(s) = \frac{R_T}{\tau S + 1} \qquad 2.5$$

So that,

$$F(s) = \frac{K}{\tau S + 1} \qquad 2.6$$

where $\tau = R_T C_T \cong$ time constant of the process

$K = R_T \cong$ Steady-state gain or static gain

s = frequency.

The time constant, τ, of the process is a measure of the time necessary for the process to adjust to a change in its input. The value of the response reaches 63.2% of its final value when the time elapsed equals 1τ. Equation 2.6 implies that this is a first order system [6].

From the experimental results:

dead time, $t_d = 10s$

step-input (power input to the plant) $\approx 1181W$,

steady-state change in output = $97^o C - 28^o C = 69^o C$

Process gain,

$$K = \frac{69^O C}{1181W} = 0.058^o C / W \approx 0.06$$

The temperature corresponding to the time constant of the process is, thus:

$$63.21\% \times (97 - 28)^o C + 28^o C = 71^o C$$

from which (Figure 3.1),

$$\tau = 555s$$

Equation 2.6 becomes,

$$G(s) = \frac{0.06e^{-10s}}{555s + 1} \qquad 2.7$$

The overall control of the process is substantially improved by using a correcting signal whose value, and hence the control action has been determined by the Proportional-Integral-Derivative (PID) control algorithm. This three term control is given as [7],

$$U(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} = K_p \left(e(t) + \frac{k_i}{k_p} \int e(t)dt + K_d \frac{de(t)}{dt} \right)$$

$$\therefore \quad U(t) = K_P \left(e(t) + \frac{1}{T_I} \int e(t)dt + T_d \frac{de(t)}{dt} \right) \qquad 2.8$$

where U(t) is the manipulated variable.

The values of K_p, K_i, and K_d, are obtained as proposed by Ziegler and Nichols [8]

$$K_p = \frac{1.2\tau}{Kt_d} = \frac{1.2 \times 555}{0.06 \times 10} = 1110$$

$$K_i = \frac{1}{2t_d} = \frac{1}{2 \times 10} = 0.05$$

$$K_d = 0.05 t_d = 0.5 \times 10 = 5$$

3. Results and Analysis

In this study, the open loop response was obtained from the time-temperature data recorded by the digital meter. The closed-loop response for 80 °C, 50 °C, 60 °C and 70 °C temperature set-points were then obtained from MATLAB Simulink model on Figure 3.2 (a) and (b).

3.1. Closed-loop Response

PID is a linear controller, hence their transient characteristics are usually evaluated by studying the responses to a unit input signal [9] Simulation was run on the control law developed in equation 2.8 using the MATLAB model in Figure 3.2. From Figures 3.3, it can be noted that the system show high oscillation, high step input, and very fast rise time. The PID gains were then tuned and are shown in Figure 3.4 to 3.7. Sufficient gain to meet the performance specifications will require larger heat outputs than the heater is capable of producing. This was the case for this system and the result is that the rise time specification cannot be met.

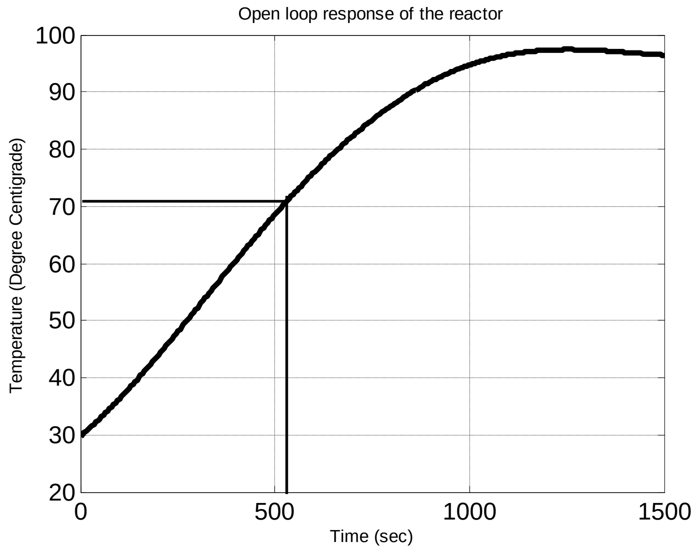


Figure 3.1: *Open-loop Response of the reactor*

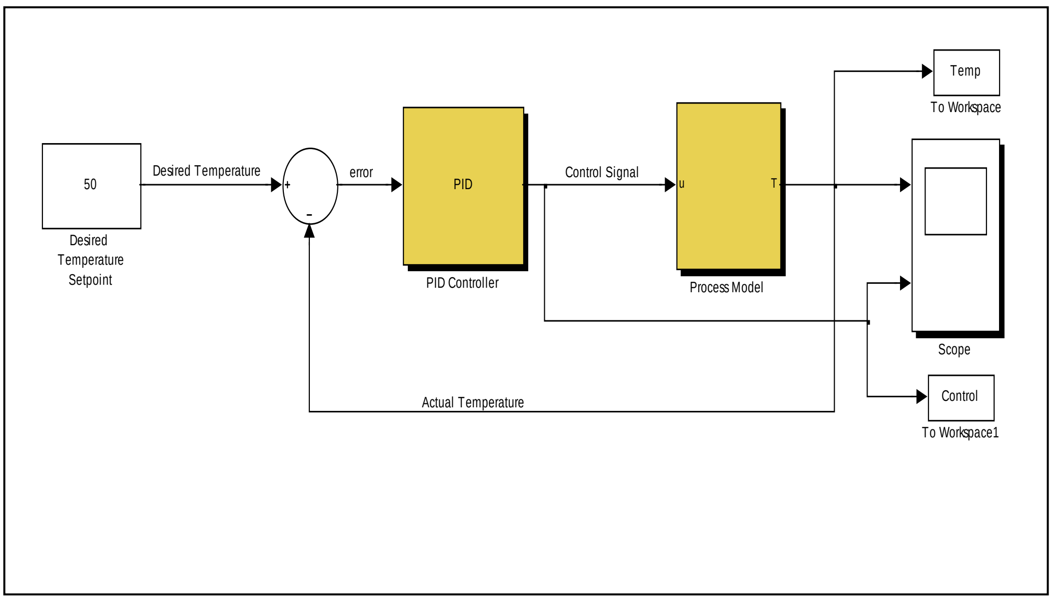


Figure 3.2: *Simulink model of the closed-loop*

The PID tuned parameters used for the simulation were P = 90; I = 0.5; D = 5; the simulation was carried out for the temperatures 50°C, 60°C, 70°C, and 80°C degrees Celsius. Each of the responses is shown with their control signal in Figures 3.3 to 3.7. The results show that, though the oscillation and instability shown in Figure 3.3 are eliminated, it still exhibit low oscillation, low overshoot and fast transient response. However, the step input appears higher than the practical heater could deliver. These were the case with this model. The result is that, the desired criteria could not be wholly met.

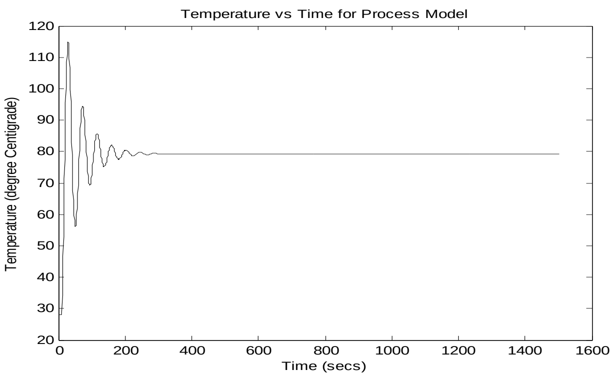


Figure 3.3 (a): *Step response*

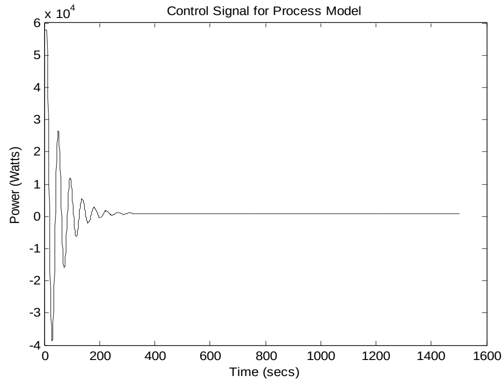


Figure 3.3 (b): *Control signal*

Figure 3.3 (a & b) is the unturned closed-loop response for P = 1110; I = 0.05; D = 5; 80^{o}C. The results shows that at these computed values from the mathematical model, the system would oscillate and eventually stabilize after a period of 400 sec. These oscillations are undesirable for most systems. The values were then manually tuned and the responses are given in Figures 3.4 to 3.7. It could be seen also, that the rise time, overshoot, and the settling time of the responses increases as the temperature set point of the system is increased. For instance, the response from Figure 3.4 (a) shows about 2.5^{o}C overshoot, 230 sec rise time and 720 sec settling time

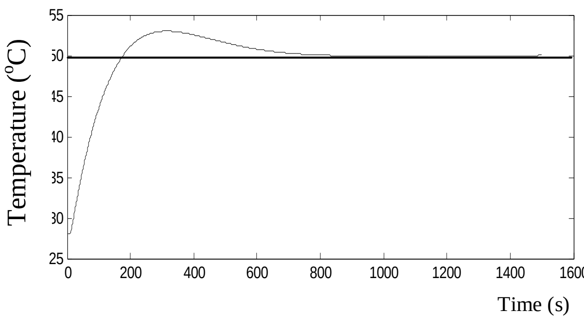


Figure 3.4 (a): *Step response at 50^{o}C*

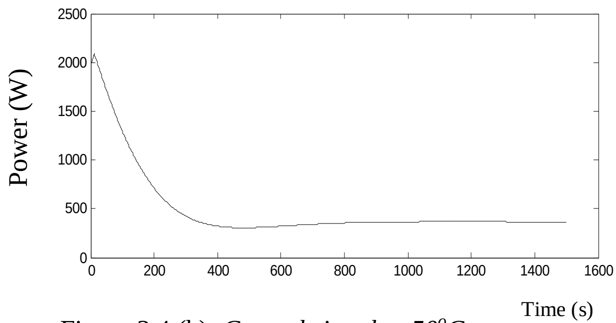


Figure 3.4 (b): *Control signal at 50^{o}C*

It could also be seen from the control signals that there is progressive increase of the power input to the system as the temperature set-points are increased from 50^{o}C to 80^{o}C. This impies that the heater must deliver appropriate power into the system in order to be able to achieve the desired response. The need for this large power means that thesystem inertia must be overcomed. Water, with its characteristic high heat capacity, will exhibit slow response response. The results showan increase of 1000 W of the heat input per 10^{o}C increase in set-point temperature. It also shows zero steady-state error.

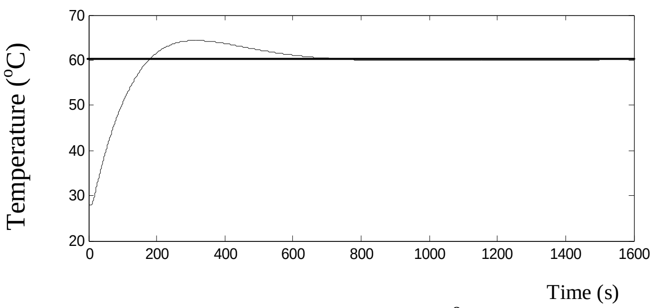


Figure 3.5 (a): Step response at 60^{o}C

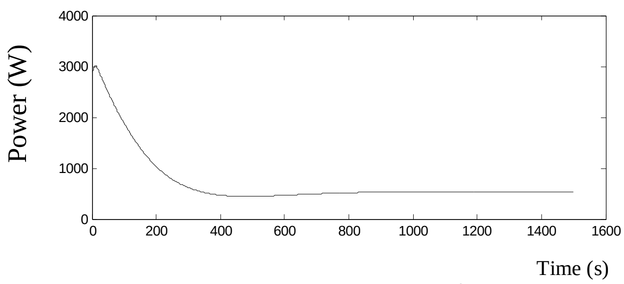


Figure 3.5 (a): Control signal at 60^{o}C

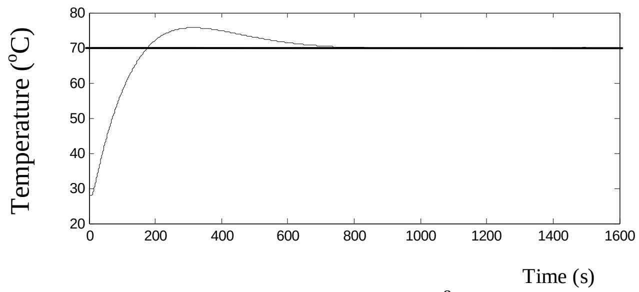


Figure 3.6 (a): Step response at 70°C

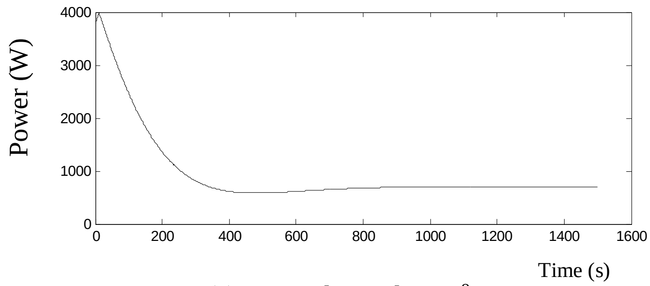


Figure 3.6 (a): Control signal at 70°C

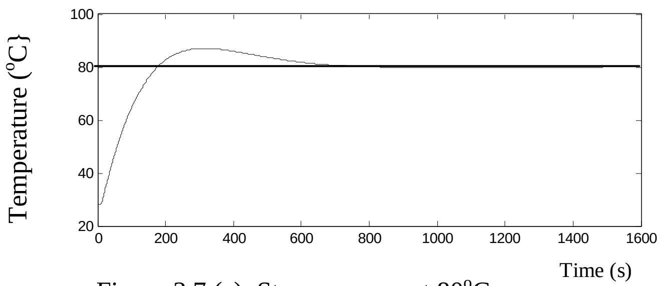


Figure 3.7 (a): Step response at 80°C

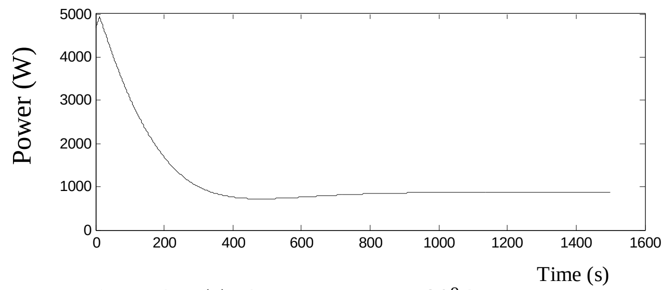


Figure 3.7 (a): Step response at 80°C

4. Conclusion

Figures 3.3 to 3.7 are plots of the simulated results at the temperatures indicated. These models show that at the tuned parameters, there is reduced overshoot, increased rise time, and small overshoot. This also implies that, while the performance requirement of 10% overshoot was met, the requirement for 100 seconds rise time and 125 seconds settling time was unrealistic given the capacity (1181 W) of the heater.

References

The main references are international journals and proceedings. All references should be to the most pertinent and up-to-date sources. References are written in APA style of Roman scripts. Please use a consistent format for references – see examples below (9 pt):

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