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IFS and Fuzzy Information Measure: A Mathematical tool for Medical Diagnosis

D.P. Semwal
DAV College, Sector 10, Chandigarh

Abstract
In present scientific era of medical sciences, the major task is to diagnose the problems related to health of human beings. It's a tedious task as information available for the patients is uncertain for their medical relationships. IFS and Fuzzy Information measure is used for the improvement of the problem in this study. A hypothetical case study is considered in this paper which contains medical information with assigned degree of membership / non-membership and hesitation index.
Keywords: Fuzzy Sets, Intuitionistic Fuzzy Sets, Fuzzy Relations, Fuzzy Information Theory.

Introduction and Related Work
Medical diagnosis investigations are multifaceted and difficult. Fuzzy sets are mainly used for the qualitative analysis of data that show the degree of membership and non-membership elements. The process for diagnosis of human problem of glaucoma using fuzzy sets [9], the fuzzy set theory in medical diagnosis [1], different measures for inuitionistic fuzzy sets [10], Intuitionistic fuzzy sets for medical diagnosis of headache [2], interval-valued fuzzy sets in supporting medical diagnostic reasoning [6], method in the medical diagnosis of headache and procedure of the types of Glaucoma, the medical diagnosis of Diabetes [4,5,8] are the various studies by the eminent researchers in the area of problem identification in human beings. The data and information is used by the doctor to infer a diagnosis from the symptoms displayed by the patient, depending upon reports of laboratory tests and medical history. Initially most common cause of blindness was conventionally treated with surgery for Cataract. Loss of vision arises due to opacification of the lens focused on the retina at the back to the eye. The persons suffer from cataract experience difficulty in recognizing colors, changes in contrast, driving, reading and recognizing faces and problems coping with glare from bright lights. A mathematical model is proposed for diagnosing the types of cataract by using IFS theory and Shannon's Entropy Measure in the form of treatment recommendation system. The results of these two methods are also compared. There are no warning signs or symptoms of cataract. A comprehensive medical history is important in identifying the disease. There are many types of cataract but the major types are as: nuclear sclerotic cataract (due to natural aging), cortical cataract (due to diabetic), posterior sub-capsular cataract (due to use of steroids for long term), radiation cataract (due to ultraviolet light) and traumatic cataract (due to direct injury or trauma). The important factors which influence the disease are as: age, family background, medical problems, steroids, exposure to the sun, x-rays or radiation treatments, smoking and alcohol etc. On the basis of availability of information, a list of diagnostic possibilities is found by the doctor. Table (1.1) shows the procedure using Intuitionistic Fuzzy relation between patients and symptoms of cataract with assigned degree of membership / non-membership elements whereas Intuitionistic Fuzzy relation between symptoms of cataract and the types of cataract with assigned degree of membership / non-membership elements as explained in table (1.2). A new measure for diagnosis of the type of cataract on the basis of max-min-max composition of Intuitionistic Fuzzy relations as explained in table (1.3). Table 1.4 shows the maximum weight for each

patient from probable diagnosis. The Fuzzy Information Measure of information theory for the purpose of comparison, in this diagnosis procedure is discussed in table 1.5.

Intuitionistic Fuzzy Sets (IFS) and Intuitionistic Fuzzy Relations (IFR)

Intuitionistic fuzzy sets (IFS), is an important tool to deal with vagueness [3]. There are some situations where fuzzy set theory is not applicable in that case we use the Intuitionistic fuzzy set (IFS) theory.

IFS of A is for a fixed set X is defined as:

$A = \{ < x, u_A(x), v_A(x) > | \, x \in X \}$; Where $u_A(x): X \to [0, 1]$ & $v_A(x): X \to [0, 1]$

defines the degree of membership / non-membership of the element x $\in$ X to the set A. Intuitionstic index or hesitancy degree π for each element x in a finite set X [11].

For every x $\in$ X, $0 \le u_A(x) + v_A(x) \le 1$ and the amount $\pi_A(x) = 1 - u_A(x) - v_A(x)$ is the intuitionistic / hesitation index, that is needed both to membership / non-membership value.

Let A be Intuitionistic Fuzzy Sets of the set X & R be an Intuitionistic Fuzzy relation from X $\to$ Y, Then max-min-max composition of Intuitionistic Fuzzy Sets X with Intuitionistic Fuzzy relation R (X $\to$ Y) is defined as B = RoA with membership / non-membership function defined as:

$u_B(y) = \max_{x \in X} \{\min[u_A(x), u_R(x, y)]\}$ and $v_B(y) = \min_{x \in X} \{\max[v_A(x), v_R(x, y)]\}$

Let S = $\{s_1, s_2, \ldots\ldots s_m\}$; D=$\{d_1, d_2 \ldots\ldots d_n\}$; P = $\{p_1, p_2, \ldots\ldots p_q\}$; Finite set of Symptoms, Types of Diseases & Patients respectively.

Two fuzzy relations Q & R defined by [7] are as under:

$Q = \{<(p, s), u_Q(p, s), v_Q(p, s) > | (p, s) \in P \times S\}$

$R = \{<(s, d), u_R(s, d), v_R(s, d) > | (s, d) \in S \times D\}$,

Where $u_Q(p,s)$ and $v_Q(p,s)$ is the degree for the symptom s that appear and does not appear respectively in patient p. Similarly $u_R(s,d)$ and vR(s, d) is the degree to which the symptom s confirm and does not confirm the disease d respectively.

The composition T of IFR R & Q(T=RoQ) describe the state of patient p_i in terms of the diagnosis of disease cataract from P to D given by membership / non-membership as:

$$\mu_T(p_i, d) = \max_{s \in S} \{\min[\mu_Q(p_i, s), \mu_R(s, d)]\} \text{ and } v_T(p_i, d) = \min_{s \in S} \{\max[v_Q(p_i, s), v_R(s, d)]\}$$

From Q & R, one may complete new measure of IFR T for which, in general the diagnostic labels of patient p for any disease d such that the following is to be satisfied:

(a1) $S_T = u_T - v_T * \pi_T$ is greatest &

(a2) The equality T = R o Q is retained.

Higher degree of association of symptoms and lower degrees of intuitionistic index to the diagnosis will be translated by measure of T. In case of approximate values for different diagnosis in T, the case for which intuitionistic index is least is considered.

Case Study

Consider the procedure by taking a hypothetical case study as:

Let P=$\{P_1, P_2, P_3, P_4, P_5\}$; a set of patients & S=$\{S_1, S_2, S_3, S_4, S_5\}$ be the set of symptoms present in the patients.

Let IFR: Q(P$\to$S) is given by

Table – 1.1

Q	S_1		S_2		S_3		S_4		S_5	
Patients	u_Q	v_Q	u_Q	v_Q	u_Q	v_Q	u_Q	v_Q	u_Q	v_Q
P_1	0.6	0.2	0.7	0.2	0.5	0.4	0.4	0.0	0.0	0.5
P_2	0.3	0.4	0.4	0.7	0.2	0.5	0.5	0.1	0.6	0.2
P_3	0.5	0.5	0.2	0.1	0.6	0.3	0.3	0.6	0.3	0.7
P_4	0.7	0.1	0.0	0.9	0.1	0.1	0.1	0.3	0.1	0.4
P_5	0.1	0.8	0.9	0.0	0.3	0.5	0.5	0.4	0.8	0.2

Let D = {Nuclear Sclerotic Cataract, Cortical Cataract, Posterior Subcapsular Cataract, Radiation Cataract, Traumatic Cataract,} be the set of diseases with which the patient suffers.

Let IFR: R(S→D) is given as

Table – 1.2

R	Nuclear Sclerotic Cataract		Cortical Cataract		Posterior Subcapsular Cataract		Radiation Cataract		Traumatic Cataract	
Symptoms	u_R	v_R	u_R	v_R	u_R	v_R	u_R	v_R	u_R	v_R
S_1	0.3	0.3	0.8	0.1	0.1	0.9	0.7	0.2	0.0	0.1
S_2	0.2	0.7	0.5	0.5	0.6	0.1	0.4	0.0	0.3	0.3
S_3	0.8	0.2	0.4	0.3	0.2	0.2	0.1	0.8	0.1	0.4
S_4	0.6	0.3	0.2	0.7	0.0	0.5	0.4	0.1	0.4	0.6
S_5	0.5	0.4	0.8	0.0	0.4	0.4	0.2	0.6	0.3	0.7

Let composition T = RoQ is given as:

Table – 1.3

T	Nuclear Sclerotic Cataract		Cortical Cataract		Posterior Subcapsular Cataract		Radiation Cataract		Traumatic Cataract	
Patients	u_Q	v_Q	u_Q	v_Q	u_Q	v_Q	u_Q	v_Q	u_Q	v_Q
P_1	0.4	0.3	0.6	0.2	0.6	0.3	0.6	0.1	0.3	0.2
P_2	0.7	0.2	0.6	0.2	0.4	0.2	0.4	0.1	0.4	0.4
P_3	0.3	0.5	0.5	0.5	0.3	0.6	0.5	0.5	0.3	0.5
P_4	0.8	0.2	0.7	0.1	0.2	0.2	0.7	0.2	0.4	0.1
P_5	0.5	0.3	0.8	0.2	0.6	0.1	0.4	0.0	0.3	0.3

Compute S_T as below

Table – 1.4

S_T	Nuclear Sclerotic Cataract	Cortical Cataract	Posterior Subcapsular Cataract	Radiation Cataract	Traumatic Cataract
P_1	0.31	0.56	**0.57**	**0.57**	0.20
P_2	**0.68**	0.56	0.32	0.35	0.32
P_3	0.20	**0.50**	0.24	**0.50**	0.20
P_4	**0.80**	0.68	0.08	0.68	0.35
P_5	0.44	**0.80**	0.57	0.40	0.18

From the table (1.4), we conclude that patients P_4 and P_2 are suffering from the Nuclear Sclerotic cataract, patient P_5 is suffering from Cortical cataract, patient P_1 is effecting from Posterior Subcapsular cataract and Radiation cataract, while the patient P_3 is suffering from the Cortical and Radiation cataract.

Again here we apply the Shannon's information measure as intuitionistic fuzzy measure in the form of

$$H(P) = -\sum_{i=1}^{n}(\mu_i log \mu_i) + (v_i log v_i) + (\pi_i log \pi_i),$$

Where μ_i , v_i and π_i represent degree of membership, degree of non-membership and hesitation index respectively. In case of Shannon's information measure the minimum weight for each patient will be considered as the solution. The information measure on the values of table (1.3) is represented as results in table (1.5):

Table – 1.5

S	Nuclear Sclerotic Cataract	Cortical Cataract	Posterior Subcapsular Cataract	Radiation Cataract	Traumatic Cataract
P_1	0.47	0.41	**0.38**	**0.38**	0.44
P_2	**0.34**	0.41	0.45	0.40	0.45
P_3	0.44	**0.30**	0.38	**0.30**	0.44
P_4	**0.21**	0.34	0.41	0.34	0.40
P_5	0.44	**0.21**	0.38	0.29	0.47

From the table (1.5), again we get the same result as table (1.4), we find that patients P_4 and P_2 are suffering from the Nuclear Sclerotic cataract, patient P_5 is suffering from Cortical cataract, patient P_1 is effecting from Posterior Subcapsular cataract and Radiation cataract, while the patient P_3 is suffering from the Cortical and Radiation cataract.

Conclusion

Both of the procedures that we have used give same results in the medical diagnosis of cataract as shown in tables (1.4) and (1.5). This shows these methods can be used as decision making tools for medical related problems and can be further used for medical related diagnosis for some other type of diseases and decision making.

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