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APPLICATIONS OF M -DIMENSIONAL FLEXIBLE FUZZY SOFT ALGEBRAIC STRUCTURES

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Abstract: In this paper, we introduce the concept of m-dimensional structures on flexible fuzzy soft subgroups, and investigate some of its properties. we also obtain the characterisations of normal flexible fuzzy soft subgroup with illustrative examples.

Keywords: Soft set, relation, fuzzy soft set, pre-image, flexible fuzzy subset, m-level subset, m-dimensional flexible subgroup, cosset.

Section-1: INTRODUCTION: In classical mathematics, the notion of exact solution of a mathematical model is defined. However, in general, mathematical models are quite complicated and it becomes an arduous task to define exact solution of these models. As a result, the notion of approximate solutions is introduced. Such introduction included the emergence of soft set theory where an approximate description of the object is provided. In fact, in soft set theory, there is no restriction on the parameterization tools which makes it very convenient and easily applicable in real life. Thus, soft set approach has come to be recognised as fundamentally important. Aktas and Cagman [1] introduced the basic concepts of soft groups, soft subgroups, normal soft subgroups and soft homomorphism and discussed their basic properties. Jun [4] also in another paper, introduced the notion of soft p- ideals , p-idealistic soft BCI- algebras and discussed their basic properties. The algebraic structures of soft set theory have also been studied extensively. Feng et.al [2, 3] considered the algebraic structure of semi ring and introduced the notion of soft semi ring. Some basic algebraic properties of soft semi ring and some related notions such as soft ideals, idealistic soft semi rings and soft semi ring homomorphism were defined and investigated with illustrative examples .Jun [5] applied the notion of soft sets to the theory of BCK/BCI- algebras and introduced the notion of soft BCK/ BCI- algebras, soft sub algebras and then derived their basic properties. It was proved that soft equality relation is a congruence relation with respect to some operations. The notions of soft sub rings, soft ideal of a soft ring, idealistic soft rings and soft ring homomorphism were introduced with some corresponding example. Atagun and Sezgin [13] introduced and studied some sub structures such as soft sub rings and soft ideals of a ring, soft subfield of a field and soft sub module of a module with several illustrative examples. Complex intuitionstic flexible fuzzy soft interior ideals and M-structures defined various algebraic structure in [15,16] . By introducing the concept of normalistic soft group, normalistic soft group homomorphism, and establishing that the normalistic soft group isomorphism is an equivalence relation on normalistic soft groups which defined in [1]. On flexible fuzzy subgroups with flexible fuzzy order discussed by [14-16] .Maji et.al [8,9,10] introduced the notion of fuzzy soft sets. In 2011, Neog and Sut [12] put forward some propositions regarding fuzzy soft set theory. In this paper, we introduce the concept of m-dimensional structures on flexible fuzzy soft subgroup, and investigate some of its properties. we also obtain the various structures of flexible fuzzy soft subgroup with illustrative examples.

Section-2 Preliminaries:

We review basic definitions that we are necessary for this paper.

Definition 2.1:[18]: A fuzzy set μ in a universe X is a mapping $\mu : X \to [0,1]$.

Definition 2.2:[9] Let U be any Universal set, E set of parameters and $A \subseteq E$. Then a pair (K,A) is called soft set over U, where K is a mapping from A to 2^U , the power set of U.

Example 2.3: Let X={c_1,c_2,c_3} be the set of three cars and E = {costly(e_1), metallic colour(e_2), cheap(e_3)}be the set of parameters, where A={e_1,e_2} $\subset$ E. Then (K,A)={K(e_1)={c_1,c_2,c_3},K(e_2)={c_1,c_2} } is the crisp soft set over X.

Definition 2.4:[11]: Let U be an initial universe. Let P (U) be the power set of U, E be the set of all parameters and A$\subseteq$ E. A soft set (f_A, E) on the universe U is defined by the set of order pairs (f_A, E) = {(e, f_A (e)): e $\in$ E, $f_A \in$ P(U)} where f_A : E $\rightarrow$ P(U) such that f_A (e) = ϕ if e $\notin$ A. Heref_A is called an approximate function of the soft set.

Example 2.5: Let U = {u_1,u_2,u_3,u_4} be a set of four shirts and E = {white(e_1),red(e_2),blue (e_3)} be a set of parameters. If A = {e_1 ,e_2} $\subseteq$ E. Let $f_A(e_1)$ = {u_1, u_2 ,u_3, u_4} and $f_A(e_2)$= {u_1,u_2,u_3}. Then we write the soft set (f_A , E) = {(e_1, {u_1, u_2 ,u_3, u_4}), (e_2,{ u_1,u_2,u_3})} over U which describe the "colour of the shirts" which Mr. X is going to buy. We may represent the soft set in the following form: U = { (e_1,u_1), (e_2,u_1), (e_1, u_2), (e_2,u_2), (e_1,u_3), (e_2, u_3), (e_1, u_4)}

Definition 2.6:[7] Let U be the universal set, E be the set of parameters and A$\subset$ E. Let K(X) denote the set of all fuzzy subsets of U. Then a pair (K,A) is called fuzzy soft set over U, where K is a mapping from A to K(U).

Example 2.7:Let U={c_1,c_2,c_3} be the set of three cars and E={costly(e_1),metallic color(e_2), cheap(e_3)} be the set of parameters, where A={e_1,e_2} $\subset$ E. Then (K,A)={K(e_1)={c_1/0.6,c_2/0.4,c_3/0.3} , K(e_2)={ c_1/0.5,c_2/0.7,c_3/0.8}} is the fuzzy soft set over U denoted by K_A.

Definition 2.8:[7] Let K_A be a fuzzy soft set over U and α be a subset of U. Then upper α -inclusion of K_A denoted by $K^\alpha{}_A$ = { x$\in$A /K(x) $\geq \alpha$ }. Similarly $K^\alpha{}_A$ = { x$\in$ A / K(x) $\leq \alpha$ } is called lower α-inclusion of K_A.

Definition 2.9:[11] Let K_A and G_B be fuzzy soft sets over the common universe U and ψ : A $\rightarrow$ B be a function. Then fuzzy soft image of K_A under ψ over U denoted by $\psi(K_A)$ is a set-valued function, where $\psi(K_A)$:B$\rightarrow 2^U$ defined by $\psi(K_A)$ (b)={$\cup${K(a) / a$\in$A and ψ (a)=b} , if ψ^{-1}(b)$\neq \varphi$} for all b$\in$ B, the soft pre-image of G_B under ψ over U denoted by ψ^-1(G_B) is a set-valued function, where$\psi^{-}1(G_B)$: A $\rightarrow$ 2^U defined by $\psi^{-1}(G_B)(b) = G(\psi(a))$ for all a $\in$ A. Then fuzzy soft anti-image of K_A under ψ over U denoted by $\psi(K_A)$ is a set-valued function, where $\psi(K_A)$:B $\rightarrow 2^U$ defined by$\psi^{-1}(K_A)(b) =\{\cap\{K(a) / a\in A$ and $\psi(a) = b\}$, if $\psi^{-1}(b) \neq \varphi\}$ for all b $\in$ B.

Definition 2.10:[14] Let X be a set. Then a mapping μ: X$\rightarrow$ $P^*([0,1])$ is called flexible subset of X, where $P^*([0,1])$ denotes the set of all non empty subset of [0,1]

Definition 2.11:[14] Let X be a non empty set .Let μ and λ be two flexible fuzzy subset of X. Then the intersection of μ and λ denoted by $\mu\cap\lambda$ and defined by $\mu\cap\lambda=\{\min\{a,b\}/ a\in\mu(x),b\in\lambda(x)\}$ for all $x\in X$. The union of μ and λ denoted by $\mu\cup\lambda$ and defined by $\mu\cup\lambda=\{\max\{a,b\}/ a\in \mu(x),b\in\lambda(x)\}$ for all $x\in X$.

Definition 2.12 [14] Let U be an initial universe, E be the set of all parameters and A $\subset$ E. A pair (F, A) is called a flexible fuzzy soft set over U where F: A $\rightarrow$ $\tilde{P}$(U) is a mapping from A into $\tilde{P}$(U), where $\tilde{P}$(U) denotes the collection of all subsets of U.

Example 2.13: Consider the example 2.5, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp numbers 0 and 1, which associate with each element a real number in the interval [0,1].Then

$(f_A, E) = \{f_A(e_1) = \{(u_1, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)\}$,

$f_A(e_2) = \{(u_1, 0.5), (u_2, 0.1), (u_3, 0.5)\}\}$ is the fuzzy soft set representing the "colour of the shirts" which Mr. X is going to buy.

Definition 2.14: An m- dimensional flexible fuzzy soft set (or a $P^*[0,1]^m$- set) on X is a mapping A: X$\rightarrow$ P* $[0,1]^m$. we denote the set of all m-dimensional flexible fuzzy soft sets on X by m(X).

Note that $[0,1]^m$ (m-power of [0,1]) is considered a posset with the point wise order $\leq$, where m is an arbitrary ordinal number (we make an appointment that m = [n/n < m] when m > 0), $\leq$ is defined by $x \leq y \leftrightarrow u_i(x) \leq u_i(y)$ for each i $\in$ m (x,y $\in$ $P^*[0,1]^m$), and ui: $P^*[0,1]^m$ $\rightarrow$ [0,1] is the i- th projection mapping (i $\in$ m). Also, 0 = (0,0,0, ……… ,0) is the smallest element in $P^*[0,1]^m$ and 1 = (1,1,………. , 1) is the largest element in $P^*[0,1]^m$.

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SECTION-3: M-DIMENSIONAL FLEXIBLE FUZZY SOFT SUBGROUP

Definition-3.1: An m-dimensional flexible fuzzy soft set A on a group G is called an m-dimensional flexible fuzzy soft subgroup if the following conditions hold:

(MFFSG1) $\inf\{A(x*y)\} \geq \min\{\inf A(x), \inf A(y)\}$ and $\inf\{A(x^{-1})\} \geq \inf\{A(x)\}$,

(MFFSG2) $\sup\{A(x*y)\} \leq \max\{\sup A(x), \sup A(y)\}$ and $\sup A(x^{-1}) \leq \sup A(x)$. That is

(MFFSG1) $\inf\{x_i \circ A(x*y)\} \geq \min\{\inf(x_i \circ A(x)), \inf(x_i \circ A(y))\}$ and $\inf\{x_i \circ A(x^{-1})\} \geq \inf\{x_i \circ A(x)\}$,

(MFFSG2) $\sup\{x_i \circ A(x*y)\} \geq \min\{\sup(x_{i\circ} A(x)), \sup(x_i \circ A(y))\}$ and $\sup\{x_{i\circ} A(x^{-1})\} \geq \sup\{x_i \circ A(x)\}$, for all $x, y \in G$, $i = 1,2,3$ ----------, m. we denote the set of all m-dimensional flexible fuzzy soft subgroup of a group G by $F_m(G)$.

Example-3.2: Let S = {e,a,b,ab}be the non-cyclic group of order 4. We define an m-dimensional flexible fuzzy subset $A : S \longrightarrow P^*[0,1]^m$ by

$$A(q) = \begin{cases} (0.7, 0.7, \ldots\ldots ,0.7), \text{ if } q = e_i \\ (0.2, 0.2 \ldots\ldots ,0.2), \text{ otherwise.} \end{cases}$$

By direct calculations, It is easy to see that A is an m-dimensional flexible fuzzy soft subgroup of S.

Now we state the following lemma's without proof.

Lemma-3.3: Let $A \in F_m(G)$.Then for all $x \in G$

(i) $\inf A(e) \geq \inf A(x)$ and $\inf A(x) = \inf A(x^{-1})$

(ii) $\sup A(e) \leq \sup A(x)$ and $\sup A(x) = \sup A(x^{-1})$.

Lemma-3.4: Let A be an m-dimensional flexible fuzzy soft subgroup of G. Then $A_\# = \{ x / x \in G, \inf A(x) = \inf A(e)\}$ and $A^\# = \{ x / x \in G, \inf A(x) \geq \{0,0,0,\ldots\ldots\ldots\ldots, 0\}$ are subgroups of G.

Definition-3.5: Let A_1 and A_2 be two m-dimensional flexible fuzzy soft subsets of a group. Then the intersection is defined as $\inf(A_1 \cap A_2)(xy^{-1}) = \min\{\inf A_1(xy^{-1}), \inf A_2(xy^{-1})\}$

Theorem-3.6: Let A_1 and A_2 be two m-dimensional flexible fuzzy soft subsets of a group. Then $A_1 \cap A_2$ is an m-dimensional flexible fuzzy soft subgroup of G.

Proof: Here we show that

$\inf (A_1 \cap A_2)(x*y^{-1}) \geq \min \{\inf (A_1 \cap A_2)(x), \inf (A_1 \cap A_2)(y^{-1})\}$

By definition -3.5, we see that

$\inf (A_1 \cap A_2)(x*y^{-1}) = \min \{ \inf A_1(x*y^{-1}), \inf A_2(x*y^{-1})\}$

$\geq \min \{ \inf (A_1(x), A_1(y^{-1})), \inf (A_2(x), A_2(y^{-1}))\}$

$= \min \{ \inf (A_1(x), A_2(x)), \inf (A_1(y^{-1}), A_2(y^{-1}))\}$

$= \min \{\inf (A_1 \cap A_2)(x), \inf (A_1 \cap A_2)(y^{-1})\}$.

Hence $A_1 \cap A_2$ is an m-dimensional flexible fuzzy soft subgroup of G.

Definition-3.7: Let A ∈ m(U). For $t \in P^*[0,1]^m$, the set $A_t = \{ x \in U / A(x) \geq t\}$ is called an m- level subset of an m-dimensional flexible fuzzy soft subset A. Note that A_t is an ordinary subset of U.

Theorem-3.8: Let G be a group and let A be an m- dimensional flexible fuzzy soft subgroup of G. Then the m-level subset A_t, for $t \in P^*[0,1]^m$, $t \leq A(e)$, is a subgroup of G, where e is the identity of G.

Proof: Since $A_t = \{x \in G / A(x) \geq t\}$, then A_t is non-empty.

$e \in A_t$ for all $t \in P^*[0,1]^m$.

Let $x,y \in A_t$. Then $A(x) \geq t$ and $A(y) \geq t$. Since $A \in F_m(G)$, $\inf A(x*y) \geq \min \{\inf A(x), \inf A(y)\}$. This means that $A(x*y) \geq t$. Hence $x*y \in A_t$. Let $x \in A_t$ implies $A(x) \geq t$. Since A is an m-dimensional flexible fuzzy soft subgroup of G, $\inf A(x^{-1}) \geq \inf A(x)$ and hence $A(x) \geq t$. This implies that $x^{-1} \in A_t$. Therefore, A_t is a subgroup of G.

Example 3.9: Let Q = {e, a,b,ab} be the klein 4-group. we define an m-dimensional flexible fuzzy soft subgroup A: $Q \rightarrow P^*[0,1]^m$ of S by A (e) = t_0, A(a) = t_1, A(b) = A(ab) = t_2, where $t_0 > t_1 > t_2$ for all t_0 , t_1, $t_2 \in P^*[0,1]^m$. Since A is an m-dimensional flexible fuzzy soft subgroup of S. we note that, $A_{t0} = \{e\}$, $A_{t1} = \{e,a\}$ and $A_{t2} = \{ e,a,b,ab \}$ are the subgroups of S.

Theorem-3.10: Let G be a group and let A be an m-dimensional flexible fuzzy soft subset of G such that A_t is a subgroup of G for al $t \in P^*[0,1]^m$, $t \leq A(e)$. Then A is an m-dimensional flexible fuzzy soft subgroup of G.

Proof: Suppose that $x,y \in A_t$ and let $A(x) = t_1$ and $A(y) = t_2$. Then $x \in A_{t1}$, $y \in A_{t1}$. We assume that $t_1 \leq t_2$. Then it implies A_{t2} is subset of A_{t1}. So $y \in A_{t1}$.Thus $x,y \in A_{t1}$ and since A_{t1} is a subgroup of G, by hypothesis, $x*y \in A_{t1}$.Therefore, $\inf A(x*y) \geq t_1 = \min \{\inf A(x),$

inf A(y)}. Again, let x ∈ G and A(x) = t. Then x ∈ A_{t1}. Since A_t is a subgroup of G, x^{-1} ∈ A_t. Therefore $A(x^{-1}) \geq t$. Hence $A(x^{-1}) \geq A(x)$. Thus. A is an m-dimensional flexible fuzzy soft subgroup of G.

Example 3.11: Let S_3 = { e ,a,a^2,b,ab,a^2b}be the symmetric group with 6 elements. we define an m-dimensional flexible fuzzy subset A : $S_3 \to P^*[0,1]^m$ by

A(a) = A(b) = t_1, A (e) = A(a^2b) = t_0 , A(a^2) = A(ab) = t_2, for all t_0, t_1, t_2 ∈ $P^*[0,1]^m$, where $t_0 > t_1 > t_2$. From the theorem-3.10, it is easy to clear that A is not an m-dimensional flexible fuzzy soft subgroup of S_3 because A_{t1} = {e, a, b, a^2b} is not a subgroup of S_3.

Definition 3.12: Let G be a group and A be an m-dimensional flexible fuzzy soft subgroup of G. The subgroups A_t = { x ∈ G / A(x) ≥ t }for t ∈ $P^*[0,1]^m$ are called m-dimensional level subgroups of A.

We now state the following theorem without its proof.

Theorem 3.13: Every subgroup H of a group G can be realized as an m-dimensional level subgroup of some m- dimensional flexible fuzzy soft subgroup of G.

Definition 3.14: Let A ∈ Fm(G). Then A is called an m-dimensional commutative flexible fuzzy soft subset of G if and only if A(x*y) = A(y*x) for all x, y ∈ X.

Definition 3.15: Let A ∈ F_m(G). Then A is called an m-dimensional normal flexible fuzzy soft subgroup of G if it is an m-dimensional commutative flexible fuzzy subset of G.

Let N_m(G) denotes the set of all m-dimensional normal flexible fuzzy soft subgroups of G.

Example 3.16: Let Φs = {± 1, ± i , ± j , ± k } be a group of quaternious with 8 elements. we define an m-dimensional flexible fuzzy soft subgroup A : Φs → $P^*[0,1]^m$ of Φs by

A(1) = t_0, A(-1) = A(± i) = t_1 , A(± j) = A(±k) = t_2, for all $t_0 > t_1 > t_2$ and t_0, t_1, t_2 ∈ $P^*[0,1]^m$. Then A is an m-dimensional flexible fuzzy soft subgroup of Φs.

Remark 3.17: Every m-dimensional flexible fuzzy soft subgroup of G is normal if G is an abelian group.

Example 3.18: Let Q = { e, a,b,ab} be the klein 4-group. We define an m-dimensional flexible fuzzy soft subgroup A: Q → $P^*[0,1]^m$ by A (e) = A(ab) = t_0 , A(a) = A(b) = t_1, for all $t_0 > t_1$ and t_0 , t_1 ∈ $P^*[0,1]^{m.}$ Since Q is an abelian group, so from the above remark, A is an m-dimensional normal flexible fuzzy soft subgroup of Q.

Remark 3.19: If $A_1, A_2 \in F_m(G)$ and A_1, A_2 does not belong to $N_m(G)$, then $A_1 \cap A_2$ is not an m-dimensional normal flexible fuzzy soft subgroup of G.

4. Properties of M-dimensional Normal flexible fuzzy subgroups

Definition 4.1: Let $A_1, A_2 \in Fm(G)$ and A_1 is subset of A_2 . Then A_1 is called an m-dimensional normal flexible fuzzy soft subgroup of an m-dimensional flexible fuzzy soft subgroup A_2 , if inf $A(xyx^{-1}) \geq \min$ {inf A(y), inf A(x)} for all x, y ∈ G.

From the above definition we see that

(1) If $A_1 \in N_m(G)$, $A_2 \in F_m(G)$, and A_1 is subset of A_2, then A_1 is m-dimension normal flexible fuzzy soft subgroup of A_2.

(2) Every m-dimensional flexible fuzzy soft subgroup is an m-dimensional normal flexible fuzzy soft subgroup of itself.

Definition 4.2: Let A be an m-dimension flexible fuzzy soft subgroup of a group G. For any x ∈ G, we define a map $Å_x : G \to P^*[0,1]^m$ by $Å_x(x) = A(xy^{-1})$ for all y ∈ G.

Theorem 4.3: If $A_1 \in N_m(G)$ and $A_2 \in F_m(G)$, then $A_1 \cap A_2$ is an m-dimensional normal flexible fuzzy soft subgroup of A_2.

Proof: Clearly, $A_1 \cap A_2 \in F_m(G)$ and $A_1 \cap A_2$ is subset of A_2. By definition- 4.2,

$$\begin{aligned}(A_1 \cap A_2)(xyx^{-1}) &= \min \{ \inf A_1(xyx^{-1}), \inf A_2(xyx^{-1})\} \\ &= \min \{\inf A_1(y), \inf A_2(xyx^{-1})\} \\ &\geq \min \{ \inf A_1(y) , \inf(A_2(x), A_2(y), A_2(x^{-1}))\} \\ &= \min \{ \inf (A_1 \cap A_2)(y), A_2(x)\}\end{aligned}$$

For all x, y ∈ G. Therefore, $A_1 \cap A_2$ is an m-dimensional normal flexible fuzzy soft subgroup of G.

Theorem 4.4: Let $A_1, A_2, A_3 \in F_m(G)$ be such that A_1 and A_2 are m-dimensional normal flexible fuzzy soft subgroups of A_3. Then $A_1 \cap A_2$ is an m-dimensional normal flexible fuzzy soft subgroup of A_3.

Proof: Since A_1, $A_2 \in F_m(G)$, it follows that $A_1 \cap A_2 \in F_m(G)$ and $A_1 \cap A_2$ is subset of A_3. Now

$(A_1 \cap A_2)(xyx^{-1}) = \inf (A_1(xyx^{-1}), A_2(xyx^{-1})\}$

$\geq \inf \{\inf (A_1(y), A_3(x)),\ \inf (A_2(y), A_3(x))\}$

$\geq \inf \{(A_1 \cap A_2)(y), A_3(x)\}$.

Hence $A_1 \cap A_2$ is an m-dimensional normal flexible fuzzy soft subgroup of A_3.

Theorem 4.5: If A_1' is an m-dimensional normal flexible fuzzy soft subgroup of A_1 and A_1, $A_2 \in F_m(G)$, then $A_1' \cap A_2$ is an m-dimensional normal flexible fuzzy soft subgroup of $A_1 \cap A_2$.

Proof: Clearly, $A_1' \cap A_2 \in F_m(G)$ and $A_1' \cap A_2$ is subset of $A_1 \cap A_2$. By definition- 4.2,

$(A_1' \cap A_2)(xyx^{-1}) = \min \{ \inf (A_1'(y)), \inf A_2(xyx^{-1})\}$

$\geq \min \{ \inf A_1'(y)\ , \inf (A_2(x), A_2(y), A_2(x^{-1}))\}$

$= \min \{ \inf (A_1' \cap A_2)(y), \inf (A_1 \cap A_2(x)\}$

For all x, y $\in$ G. Therefore, $A_1' \cap A_2$ is an m-dimensional normal flexible fuzzy soft subgroup of $A_1 \cap A_2$.

Now we state the following theorem without its proof

Theorem 4.6: Let $A \in N_m(G)$. Then $A_{\#} = \{ x/ x \in G, A(x) = A(e)\}$ and $A^{\#} = \{ x/ x \in G , A(x) \geq (0, 0, \ldots\ldots\ldots, 0)\}$ are normal flexible fuzzy soft subgroups of G.

Theorem 4.7: Let A be an m-dimensional flexible fuzzy soft subgroup of a group G. Then A is an m-dimensional normal flexible fuzzy soft subgroup of G if and only if

$A([x, y]) \geq A(x)$ for all x, y $\in$ G.

Proof: Suppose that A is an m-dimensional normal flexible fuzzy soft subgroup of G. Then $A(x^{-1}y^{-1}xy) \geq \inf (A(y^{-1}xy), A(x^{-1}))$

$= \inf (A(x), A(x)) = A(x)$.

Now suppose that A satisfies the relation $A([x,y]) \geq A(x)$ for all x, y $\in$ G. Then for x, z $\in$ G, we have $A(x^{-1}zx) = A(zz^{-1}x^{-1}zx)$

$\geq \inf (A(z), A([z,x])) = A(z)$.

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Thus

$A(x^{-1}zx) \geq A(z)$ for all $z,x \in G$. -- (1)

Again, we get

$A(z) = A(x.x^{-1}zxx^{-1}) \geq \inf (A(x), A(x^{-1}zx))$. ------------------------------------- (2)

Now if $\inf (A(x), A(x^{-1}zx)) = A(x)$, then we get that $A(z) \geq A(x)$ for all $x, z \in G$.Then A is a constant function, and there is nothing to prove. So we assume that $\inf (A(x), A(x^{-1}zx)) = A(x^{-1}zx)$. Then (2) gives that $A(z) \geq A(x^{-1}zx)$ for all $x,z \in G$. By this inequality with (1),we have $A(x^{-1}zx) = A(z)$ for all $x,z \in G$. Hence A is constant on the conjugate classes of G.

Example 4.8: Let D_8 be a dihedral group of order 8 given by $D_8 = \{ e,a,a^2,a^3,b,ab,a^2b,a^3b \}$. where we have $a^4 = b^2 = e$ and $a^3b = ba$. Define $A: D_8 \to P^*[0.1]^m$ by setting

$A(e) = A(a^3b) = 1$,

$A(a) = A(a^2) = A(a^3) = A(b) = A(ab) = A(a^2b) = 0.3$.

It is easy to see that $A \in F_m(D_8)$ and A does not belong to $N_m(D_8)$ because $A(a^3.b) \neq A(b.a^3)$.

Now the cossets of A in D_8 is given by

$Ae = \{ 1, 0.3, 0.3. 0.3, 0.3, 0.3, 0.3, 1\}$

$Aa = \{ 0.3, 1, 0.2,0.3, 0.3, 0.3, 1, 0.3\}$

$Aa^2= \{0.3.0.3, 1, 0.3, 0.3, 0.3, 0.3, 1\}$

$Aa^3= \{ 0.3,0.3,0.3,1, 0.3, 0.3, 0.3,1\}$

$Ab = \{1, 0.3,0.3,0.3, 0.3, 0.3, 1, 0.3\}$

$Aab =\{0.3,0.3,0.3,0.3,0.3, 1, 0.3, 1\}$

$Aa^2b = \{0.3,0.3, 0.3, 0.3, 1,0.3,1,0.3\}$

$Aa^3b = \{1,0.3, 0.3,0.3,0.3,0.3,0.3,1\}$.

Here $Ab = Aa^3$, $Aa^2 = Aab$. But $Ab \text{ o } Aa^2 = Aa^2b \neq Aa^3 \text{ o } Aab = Ab$.

Hence A is m-dimensional normal flexible fuzzy soft subgroup.

Conclusion: Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as computer science, information science, topological spaces and so on. This provides sufficient motivation to researchers to review various concepts and results on algebraic structures and in the broader framework of soft set setting. .These include smoothness of functions, game theory, operations research, Reimann and Perron integrations, probability theory and measure theory. In this article, we discuss the concept of m-dimensional structures on flexible fuzzy soft subgroup, and investigate some of its properties. The characterisations of various structures of normal flexible fuzzy soft subgroups related to cossets with illustrative examples.

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