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AXIALLY SYMMETRIC DARK ENERGY COSMOLOGICAL MODEL IN SCALAR TENSOR THEORY OF GRAVITATION WITH CONSTANT DECELERATION PARAMETER

V P Kadam
Dept of Math,
G S Gawande College,
Umarkhed- 445206, India

Abstract:-In this paper, we have studied an Axially Symmetric dark energy model with constant deceleration parameter, in the scalar tensor theory of Gravitation proposed by Saez and Ballester (1986). We obtain exact solutions to the field equations using special law of variation for Hubble's parameter presented by Bermann (1983) and condition that expansion scalar is proportional to shear scalar. Some physical aspects of the dark energy models are investigated.

Keywords:- Cosmological model, Dark Energy, Scalar-Tensor theory, Deceleration parameter.

1. Introduction:-

Recent observations of type Ia supernovae (SN Ia) (Permutter et al.[1-3], Riess et al. [4-5]), Dunlop et al. [6], Spinard et al. [7], galaxy redshift survays (Fedeli et al. [8]), cosmic microwave background radiation (CMBR) data (Caldwell and Doran [9] Huang et al [10]) and large scale structure (Daniel et al. [11] strongly suggest that the universe is accelerating. As universe is expanding , the paramount characteristic of dark energy is a constant or slightly changing energy density, but nature of dark energy is still not known [12-20].

In recent years there has been a considerable interest in scalar-tensor theories of gravitation proposed by Lyra [21], Brans and Dicke [22], Nordtvedt [23], Sen [24], Sen and Dunn [25], Barber [26], Saez and Ballester [27]. Brans-Dicke has developed a new scalar tensor theory of gravitation which includes a long range scalar field interacting equally with all forms of matter, where as Saez-Ballester has developed a new scalar tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner.

The field equations of Saez-Ballester scalar tensor theory for the combined scalar and tensor fields are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -kT_{ij} \tag{1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, R the scalar curvature, ω and n are constants, T_{ij} is the stress tensor of matter and comma and semicolon denote partial and covariant differentiation respectively.

Also we have energy conservation equation,

$$T^{ij}_{;i} = 0 \tag{3}$$

which is the consequences of the field equation (1) and (2).

For last some few decades, the study of cosmological models in the frame work of scalar – tensor theories has been the active area of research. Some important features of cosmological models in the frame work of Lyra geometry , are studied by some authors like Soleng,[28], Reddy, et al. [29], Rahaman, F.et al. [30-34].

Also, Singh et al.[35], Shri Ram, et al.[36,37], Reddy, et al. [38], Rao et al. [39,40] are investigated the cosmological models in scalar tensor theory proposed by Saez-Ballester. Mukhopadhyay,U., et al. [41,42], Usmani, et al. [43] have been investigated various models for various form of time dependent w. Recently, dark energy models with variable EoS parameter in different contexts have obtained by Mukhopadhyay, U. et al. [44], Ray, S.et al. [45], Aksaru, O. et al. [46], Yadav, et al. [47,48], Pradhan, et al. [49-51].

With the motivation of the above cosmologist, in present paper we discussed an Axially Symmetric dark energy model with constant deceleration parameter in Saez -Ballester scalar tensor theory of Gravitation. Some other properties are also studied.

2. Field Equations:-

We consider axially symmetric space-time given by

$$ds^2 = dt^2 - A^2\left[d\chi^2 + f^2 d\psi^2\right] - B^2 dz^2$$

(4)

With convention $x^1 = \chi$, $x^2 = \psi$, $x^3 = z, x^4 = t$ and A, B are the functions of the proper time t alone while f is a function of co-ordinate χ alone.

The energy momentum tensor of fluid is taken as

$$T_j^i = diag\left[T_0^0, T_1^1, T_2^2, T_3^3\right]$$

(5)

One can parameterize as follows

$$T_j^i = diag\left[\rho, -p_x, -p_y, -p_z\right]$$

$$T_j^i = diag\left[1, -w_x, -w_y, -w_z\right]\rho$$

$$T_j^i = diag[1, -w, -(w+\delta), -(w+\eta)]\rho$$

(6)

where ρ is the energy density of the fluid, p_x, p_y, p_z are pressures and w_x, w_y, w_z are the directional EoS parameters along the x, y and z axis respectively. $w(t) = \frac{p}{\rho}$ is the free EoS parameter of the fluid, we have parameterize the deviation from isotropy by setting $w_x = w$ and then introducing skewness parameters δ, η are the deviation from w along y and z axes.

In commoving co-ordinate system Saez-Ballester field equations (1)-(3) for the metric (4) with the help of (5) and (6) take the form

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -w\rho$$

(7)

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -(w+\delta)\rho$$

(8)

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 - \frac{f_{11}}{A^2 f} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -(w+\eta)\rho$$

(9)

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4}{A}\frac{B_4}{B} - \frac{f_{11}}{A^2 f} + \frac{\omega}{2}\phi^n {\phi_4}^2 = \rho$$

(10)

$$\phi_{44} + \left(2\frac{A_4}{A} + \frac{B_4}{B}\right)\phi_4 + \frac{n}{2}\frac{{\phi_4}^2}{\phi} = 0$$

(11)

$$\rho_4 + (\omega + \delta)\left(2\frac{A_4}{A} + \frac{B_4}{B}\right)\rho = 0$$

(12)

where the suffixes 1 and 4 after an unknown functions denote partial differentiation with respect to χ and t respectively.

The functional dependence of the metric together with (9) and (10) imply that

$$\frac{f_{11}}{f} = k^2, \qquad k^2 = \text{constant}$$

(13)

If $k = 0$ the $f(\chi) = constant, 0, \chi < \alpha$. This constant can be made equal to 1 by suitably choosing units for ψ. Thus we shall have $f(\chi) = \chi$ resulting in the flat model of the universe (Hawking and Ellis 1976 [52]).

Using equations (7) and (8), we have

$$\delta = 0$$

(14)

Now the field equations (7) – (12) gives us the following independent equations

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -w\rho$$

(15)

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 - \frac{\omega}{2}\phi^n {\phi_4}^2 = -(w+\eta)\rho$$

(16)

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4}{A}\frac{B_4}{B} + \frac{\omega}{2}\phi^n {\phi_4}^2 = \rho$$

(17)

$$\phi_{44} + \left(2\frac{A_4}{A} + \frac{B_4}{B}\right)\phi_4 + \frac{n}{2}\frac{\phi_4^{\ 2}}{\phi} = 0$$

(18)

$$\rho_4 + \omega\rho\left(2\frac{A_4}{A} + \frac{B_4}{B}\right) = 0$$

(19)

The equations (15) to (18) are four linear equations with six unknowns A, B, ϕ, w, ρ and η. Two additional constraints relating these parameters are required to obtain explicit solutions of the system.

i)The law of variation of Hubble's parameter that yields a constant value of deceleration parameter. Such types of relation have been already considered by Berman [53] for solving FRW models. Later on many authors (Singh et al. [54, 55] and references their in) have studied flat FRW and Bianchi type models by using the special law of Hubble's parameter.

ii) We assume that expansion scalar (θ) in the model is proportional to shear scalar (σ). This condition leads to

$$A = B^m$$

(20)

where m is proportionality constant. The motive behind assuming condition (ii) is explained with reference to Throne [56], the observation of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropy today within ≈ 30 percent (Kantowski and Sachs [57]; Kristian and Sachs [58]) To put more precisely, red-shift studies place the limit $\frac{\sigma}{H} \le 0.3$ on the ratio of shear to Hubble's parameter in the neighborhood of our galaxy today. Collin et al. [59] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfied that the condition $\frac{\sigma}{\theta}$ is constant.

The average scale factor of axially symmetric metric is given by

$$R = (A^2 B)^{\frac{1}{3}}$$

(21)

We define, the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_1 + H_2 + H_3)$$

(22)

where $H_1 = \frac{A_4}{A} = H_2$, $H_3 = \frac{B_4}{B}$ are the directional Hubble's parameter in the direction of x, y, and z resp.

Therefore equation (22), may be reduces to

$$H = \frac{R_4}{R} = \frac{1}{3}\left(2\frac{A_4}{A} + \frac{B_4}{B}\right)$$

(23)

Since the line element (1) is completely characterized by Hubble's parameter H Let us consider that mean Hubble parameter H is related to average scale factor R by following relation

$$H = k_1 R^{-s}$$

(24)

Where $k_1 > 0$ and $s \geq 0$ are constant.

The deceleration parameter is defined by as

$$q = -\frac{RR_{44}}{R_4^{\ 2}}$$

(25)

From equations (23) and (24), we get

$$R_4 = k_1 R^{-s+1}$$

(26)

$$R_{44} = -k_1^{\ 2}(s-1)R^{-2s+1}$$

(27)

Using equations (25), (26) and (27), we get

$$q = s - 1$$

(28)

The signs of q indicates whether the model inflates or not. The positive signs of q corresponds to standard decelerating model where as the negative signs of indicates inflation.

From equation (25), we obtain the law of average scale factor R as

$$R = \begin{cases} (Dt + c_1)^{\frac{1}{3}}, s \neq 0 \\ c_2 e^{k_1 t}, s = 0 \end{cases}$$

(29)

Where c_1 and c_2 are the constant of integration.

From equation (28), for $s \neq 0$,it is clear that the condition for expansion of universe is $s > 0$ i.e. $q + 1 > 0$. Therefore for expansion model of universe the deceleration parameter q should be greater than -1.

3. Solution Of The Field Equations:-

3.1 Case (i): when $s \neq 0$

Equations leads to

$$A = l(Dt + c_1)^{\frac{1}{r}}$$

(30)

Where c_0 is constant of integration & $c_0^{\frac{m}{2m+1}} = l$ and $r = \frac{(2m+1)s}{3m}$.

Equations (20) and (30) leads to

$$B = l_1(Dt + c_1)^{\frac{1}{mr}}$$

(31)

where $l_1 = l^{\frac{1}{m}}$.

From equation (18), we get

$$\phi = k_2(Dt + c_1)^{2\left[\frac{(r-2)m-1}{mr(n+2)}\right]}$$

(32)

where $$k_2 = \left[\frac{(n+2)\,m\,r}{2l^2 l_1[(r-2)m-1}\right]^{\frac{2}{n+2}}$$

(33)

The dark energy model corresponding to equation (30) and (31) can be written as

$$ds^2 = dt^2 - l^2(Dt + c_1)^{\frac{2}{r}}\left[d\chi^2 + f^2 d\psi^2\right] - l_1^2(Dt + c_1)^{\frac{2}{mr}} dz^2$$

(34)

Thus Hubble's parameter H, scalar of expansion θ, anisotropy parameter A_m and shear scalar σ are given by

$$H = \frac{k_1}{(Dt + c_1)}$$

(35)

$$\theta = \frac{3k_1}{(Dt + c_1)}$$

(36)

$$A_m = \frac{1}{3}\left[\frac{2m^2(s-r)^2 + (s-mr)^2}{m^2 r^2}\right]$$

(37)

$$\sigma^2 = \frac{7}{2}\frac{k_1{}^2}{(Dt + c_1)^2}$$

(38)

From equations (17), the energy density of fluid is obtained as

$$\rho = \frac{(m+2)D^2}{mr^2(Dt+c_1)^2} + 2\omega k_2^{n+2}\left[\frac{(r-2)m-1}{mr(n+2)}\right]D^2(Dt+c_1)^{\frac{2(n+2)[(r-2)m-1]}{mr(n+2)}-2}$$

(39)

From equation (15), EoS parameter w is given by

$$w = \frac{-\left\{\frac{[(1-r)m^2 + (1-mr)]D^2}{m^2 r^2 (Dt+c_1)^2} - 2\omega k_2^{n+2} D^2\left[\frac{(r-2)m-1}{mr(n+2)}\right]^2 (Dt+c_1)^{\frac{2(n+2)[(r-2)m-1]}{mr(n+2)}-2}\right\}}{\frac{(m+2)D^2}{mr^2(Dt+c_1)^2} + 2\omega k_2^{n+2}\left[\frac{(r-2)m-1}{mr(n+2)}\right]D^2(Dt+c_1)^{\frac{2(n+2)[(r-2)m-1]}{mr(n+2)}-2}}$$

(40)

From equation (16), we get

$$\eta = \frac{-\left\{\frac{[(2-r)m^2-(1-mr)]D^2}{m^2r^2(Dt+c_1)^2}\right\}}{\left\{\frac{(m+2)D^2}{mr^2(Dt+c_1)^2}+2\omega k_2^{n+2}\left[\frac{(r-2)m-1}{mr(n+2)}\right]D^2(Dt+c_1)^{\frac{2(n+2)[(r-2)m-1]}{mr(n+2)}-2}\right\}}$$

(41)

3.2 Case (ii): when $s=0$.

Equations leads to

$$A = I_0 e^{k_3 t}$$

(42)

Where I_0 is constant of integration & $k_3 = \frac{3mk_1}{2m+1}$.

Equations (20) and (30) leads to

$$B = I_1 e^{k_3 mt}$$

(43)

Where $I_1 = I_0{}^m$.

From equation (18), we get

$$\phi = k_4 e^{\frac{-2(2+m)k_3 t}{(n+2)}}$$

(44)

Where $k_4 = \left[\frac{-(n+2)c}{2(2+m)I_o^2 I_1 k_3}\right]^{\frac{2}{n+2}}$

(45)

The dark energy model corresponding to equation (42) and (43) can be written as

$$ds^2 = dt^2 - I_0{}^2 e^{2k_3 t}\left[d\chi^2 + f^2 d\psi^2\right] - I_1^2 e^{2mk_3 t} dz^2$$

(46)

Thus Hubble's parameter H, scalar of expansion θ, anisotropy parameter A_m and shear scalar σ are given by

$$H = \frac{1}{3}(2+m)k_3$$

(47)

$$\theta = (2+m)k_3$$

(48)

$$A_m = \frac{1}{3}(2+m)k_3$$

(49)

$$\sigma^2 = \frac{7}{18}(2+m)^2 k_3^2$$

(50)

From equations (17), the energy density of fluid is obtained as

$$\rho = (1+2m)k_3^2 + 2\omega k_3^2 k_4^{n+2}\left(\frac{m+2}{n+2}\right)^2 e^{-2(m+2)k_3 t}$$

(51)

From equation (15), EoS parameter w is given by

$$w = \frac{-\left\{(1+m+m^2)k_3^2 - 2\omega k_4^{n+2}\left(\frac{m+2}{n+2}\right)^2 e^{-2(m+2)k_3 t}\right\}}{\left\{(1+2m)k_3^2 + 2\omega k_4^{n+2}\left(\frac{m+2}{n+2}\right)^2 e^{-2(m+2)k_3 t}\right\}}$$

(52)

From equation (16), we get

$$\eta = \frac{-\{m^2 + m - 2\}}{\left\{(1+2m)k_3^2 + 2\omega k_4^{n+2}\left(\frac{m+2}{n+2}\right)^2 e^{-2(m+2)k_3 t}\right\}}$$

(53)

4. Concluding Remarks:

In this paper, we have investigated an anisotropic axially symmetric dark energy model with variable EoS parameter w, considering two cases for $s \neq 0$ and $s = 0$ in Saez Ballester scalar tensor theory of gravitation. Also, it is observed that in both cases EoS parameter is variable function of time. It is observed that when $s \neq 0$ then Hubble's parameter H, scalar of expansion θ and shear scalar σ are decreases with time while in second case i.e. when $s = 0$ then Hubble's parameter H, scalar of expansion θ and shear scalar σ are not depends on time. Also it is observed that in both cases the energy density of fluid ρ, EoS parameter w and η are time dependent.

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