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MAGIC LABLING ON HUMAN CHAIN GRAPH

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Abstract: In this paper, we investigate Z_3- vertex magic total labeling, Z_3- edge magic total labeling, Z_4-bi magic labeling, total magic cordial labeling and n-edge magic labeling for human chain graph.

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Key words: Human chain, Magic, Total magic, Z_3- vertex, Z_4-bi magic

1. Introduction

Let G= $HC_{n,m}(p,q)$, n∈N, m≥3 be a Human chain graph and it is a simple, finite and undirected graph with p= 2mn+n+1 vertices and q=2mn+2n edges. For a summary on various labeling see the Dynamic survey of graph labeling by Gallian [1]. Magic labeling was introduced by Sedlacek in 1963 [3,4]. The original concept of total edge magic graph is due to Kotzig and Rosa [2].We have referred Z_3- vertex magic total labeling and Z_3- edge magic total labeling which has been extracted from various articles [5,7]. The concept of Human chain graph was introduced by K.Anitha and B.Selvam[6]. In this paper, we investigate Z_3- vertex magic total, Z_3- edge magic total, Z_4-bi magic, total magic cordial labeling and n-edge magic labeling of Human chain graph.

2. Preliminaries

In this section, we provide some basic definitions which needed to this paper.

Definition 2.1 Z_3-Vertex magic total labeling : A graph G(V,E) is said to admit Z_3-vertex magic total labeling if f: V∪E→A^* where $A^*=Z_3-[0]$ such that the induced map f* on V defined by $f^*(v_i)=\{f(v_i)+\sum f(e)\}(\text{mod}3) = k$, a constant where e is the edge incident at v_i. A graph which admits Z_3- vertex magic total labeling is called Z_3- vertex magic total graph.

Definition 2.2 Z_3-edge magic total labeling : A graph G(V,E) is said to admit Z_3- edge magic total labeling if f: V∪E→A^* where $A^*=Z_3-[0]$ such that the induced map f* on E defined by $f^*(v_iv_j)=\{f(v_i)+f(v_j)+f(v_i v_j)\}(\text{mod}3) = k$, a constant for all edges $v_i v_j \in E$. A graph which admits Z_3- edge magic total labeling is called Z_3- edge magic total graph.

Definition 2.3 Z_4-bi magic labeling : A graph G(V,E) is said to admit Z_4- bi magic labeling if there exists a function f: E→ $\{1,2,3\}$ such that the induced map f* on V defined by $f^*(v_i)=\sum f(e)(\text{mod } 4) = k_1$ or k_2 , a constant $e=v_iv_j \in E$.

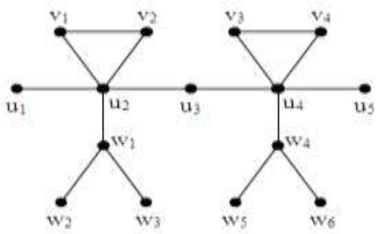
Definition 2.4 Total magic cordial labeling : A graph G(V,E) is said to admit total magic cordial labeling if f:V∪E→ $\{0,1\}$ such that (i) { f(x)+f(y)+f(xy)}(mod 2) is constant for all edges xy∈E. (ii) for all i, j∈{0,1}, $\left|\{m_i(f)+n_i(f)\}-\{m_j(f)+n_j(f)\}\right| \leq 1, (i \neq j)$, where $m_i(f)$= {e∈ E/f(e)=i} and $n_i(f)$= {v∈ V/f(v)=i}. A graph which admits total magic cordial labeling is called total magic cordial.

Definition 2.5 n-edge magic labeling : Let G(V,E) be a graph. Let f: V→ $\{-1, n+1\}$ and f^*:E→{n} such that for all uv ∈ E , f*(uv)= f(u)+f(v)=n, then the labeling is called n-edge magic labeling.

Definition 2.6 Human chain graph:

A human chain graph $HC_{n,m}(p,q)$ is obtained a path $u_1, u_2, \ldots, u_{2n+1}$, $n \in N$ by joining a cycle of length m (C_m) and Y-tree Y_{m+1}, $m \geq 3$ to each u_{2i} for $1 \leq i \leq n$. The vertices of C_m and Y-tree Y_{m+1} are $v_1, v_2, \ldots, v_{(m-1)n}$ and $w_1, w_2, \ldots, w_{mn}$ respectively.

Example:1 ($HC_{2,3}$)



Structural properties of $HC_{n,m}$

1. The vertex set of $HC_{n,m} = \{u_i, v_j, w_k / 1 \leq i \leq 2n+1, 1 \leq j \leq (m-1)n, 1 \leq k \leq mn\}$.

2. The total number of vertices of $HC_{n,m} = |V| = 2mn+n+1$.

3.The edge of set of $HC_{n,m} = |E|$ $\{u_i u_{i+1} / 1 \leq i \leq 2n\} \cup$

$\{u_{2i} w_{m(i-1)+1}; u_{2i} v_{(m-1)i}; u_{2i} v_{(m-1)(i-1)+1}; w_{mi} w_{mi-2} / 1 \leq i \leq n\} \cup \{w_{mi+j} w_{mi+j+1};$

$v_{(m-1)i+j} v_{(m-i)+j+1} / 0 \leq i \leq n-1, 1 \leq j \leq m-2\}$.

4.The total number of edges of $HC_{n,m} = |E| = 2mn+2n$.

5. The maximum degree of $HC_{n,m} = \Delta = 5$.

6. The minimum degree of $HC_{n,m} = \delta = 1$.

3. MAIN RESULTS

Algorithm 3.1

Procedure: (Z_3-Vertex magic total labeling of $HC_{n,m}$)

Input: $V \leftarrow \{u_1, u_2, \ldots, u_{2n+1}, v_1, v_2, \ldots, v_{(m-1)n}, w_1, w_2, \ldots, w_{mn}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{2mn+2n}\}$

if n≥1

```
            u1, u2n+1 ← 2
        for i= 1 to(m-1)n do
            vi ← 1
    end for
        for i= 1 to mn do
            wi ← 1
    end for
        for i= 1 to (2n-1) do
           ui+1 ← 1
    end for
        for i= 1 to 2n do
```

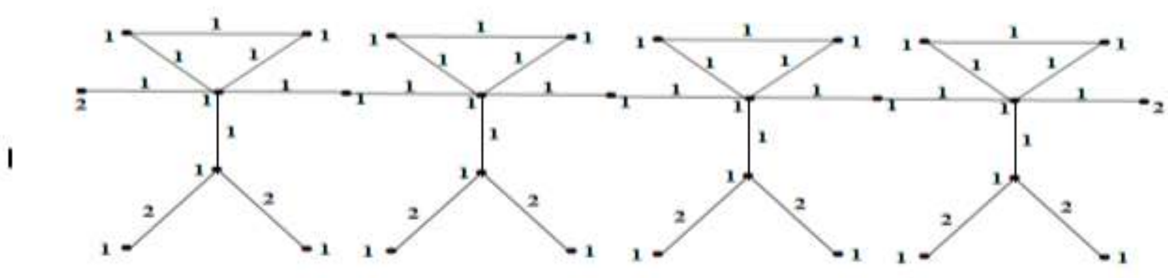
$u_i u_{i+1} \leftarrow 1$
end for
for i= 1 to n do
$u_{2i}\ v_{(m-1)i} \leftarrow 1$
$u_{2i}\ v_{(m-1)(i-1)+1} \leftarrow 1$
$u_{2i}\ w_{m(i-1)+1} \leftarrow 1$
$w_{mi}\ w_{mi-2} \leftarrow 2$
end for
for i= 0 to (n-1) do
j= 1 to (m-2) do
$v_{(m-1)i+j}\ v_{(m-1)i+j+1} \leftarrow 1$
$w_{(mi+j)}\ w_{(mi+j+1)} \leftarrow 2$
end for
end if
end procedure

Theorem 3.1: For m≥3 and n≥1, the human chain graph admits Z_3 – vertex magic total labeling.

Proof: Let $HC_{n,m}(p,q)$ be a human chain graph with p= 2mn+n+1 vertices and q= 2mn+2n edges. Using algorithm 3.1, the 2mn+n+1 vertices and 2mn+2n edges are labeled by defining a function $f:V\cup E \rightarrow \{1,2\}$. The induced function is defined by $f^*:V\rightarrow N\cup\{0\}$, such that $f^*(v_i)= \{f(v_i)+\sum f(e)\}(mod3) = k$, a constant for all edges $v_iv_j\in E$. The total weight of each vertex is $f^*(v)= \{f(v)+\sum f(uv)\}(mod3) = 3$ or $6(mod\ 3)= 0$, a constant for all edges $uv\in E$. Thus the induced function yields the weight '0' to all the vertices. Therefore, for m≥3 and n≥1, the human chain graph admits Z_3 – vertex magic total labeling.

Example 2 : Z_3 – vertex magic total labeling for $HC_{4,3}$

Algorithm 3.2



Procedure: (Z_3-edge magic total labeling of $HC_{n,m}$)

Input: $V\leftarrow\{u_1,u_2,\ldots,u_{2n+1}, v_1,v_2,\ldots,v_{(m-1)n}, w_1,w_2,\ldots,w_{mn}\}$
$E\leftarrow\{ e_1,e_2,\ldots,e_{2mn+2n}\}$

if n≥1, m≥3

```
        for i= 1 to (2n+1) do
                u_i ← 1
    end for
        for i= 1 to(m-1)n do
                v_i ← 1
    end for
        for i= 1 to mn do
                w_i ← 1
    end for
        for i= 1 to n do
              u_2i v_(m-1)i ← 2
              u_2i v_(m-1)(i-1)+1 ← 2
              u_2i w_m(i-1)+1 ← 2
              w_mi w_mi-2 ← 2
        end for
        for i= 1 to 2n do
              u_i u_i+1 ← 2
        end for
        for i= 0 to (n-1) do
                j= 1 to (m-2) do
              v_(m-1)i+j v_(m-1)i+j+1 ← 2
              w_(mi+j) w_(mi+j+1) ← 2
       end for
```

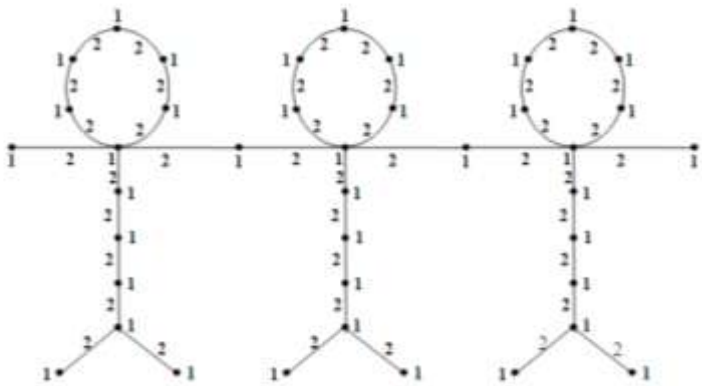
end if

end procedure

Theorem 3.2: For $m\geq3$ and $n\geq1$, the human chain graph admits Z_3 – edge magic total labeling.

Proof: Let $HC_{n,m}(p,q)$ be a human chain graph with p= 2mn+n+1 vertices and q= 2mn+2n edges. Using algorithm 3.2, the 2mn+n+1 vertices and 2mn+2n edges are labeled by defining a function $f:V\cup E \rightarrow \{1,2\}$. The induced function is defined by $f^*:E\rightarrow N\cup$**{0},** such that $f^*(uv)= \{f(u)+f(v)+f(uv)\}(mod3) = k$. The induced function yields the labels for as follows. $f^*(uv)= \{f(u)+f(v)+f(uv)\}= 1+1+2=4(mod3)=1$. Therefore, for $m\geq3$ and $n\geq1$, the human chain graph admits Z_3 – edge magic total labeling.

Example 3 : Z_3 – edge magic total labeling for $HC_{3,6}$



Algorithm 3.3

Procedure: (Z_4-bi magic lebeling of $HC_{n,m}$)

Input: $V \leftarrow \{u_1, u_2, \ldots, u_{2n+1}, v_1, v_2, \ldots, v_{(m-1)n}, w_1, w_2, \ldots, w_{mn}\}$

$E \leftarrow \{ e_1, e_2, \ldots, e_{2mn+2n}\}$

if n≥1, m≥3

$u_1 u_2 \leftarrow 3$

$u_2 v_1 \leftarrow 2$

$u_2 v_{m-1} \leftarrow 2$

for i= 1 to n do

$w_{mi} w_{mi-2} \leftarrow 3$

$w_{mi-2} w_{mi-1} \leftarrow 3$

$u_{2i} u_{2i+1} \leftarrow 3$

$u_{2i} w_{m(i-1)+1} \leftarrow 2$

end for

for i= 0 to (n-1) do

j=1 to m-2 do

$v_{(m-1)i+j} v_{(m-i)+j+1} \leftarrow 2$

end for

end if

if n>1, m≥3

for i= 2 to n do

$u_{2i} v_{(m-1)i} \leftarrow 1$

$u_{2i} v_{(m-1)(i-1)+1} \leftarrow 1$

end for

for i= 1 to (n-1) do

$u_{2i+1} u_{2i+2} \leftarrow 1$

end for

end if

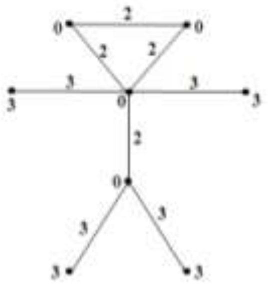
if n>1, m>3

for i= 0 to (n-1)

j= 1 to $\left\lfloor \frac{m-2}{2} \right\rfloor$ do
$w_{mi+2j}\, w_{mi+2j-1} \leftarrow 1$
end for
end if
if n>1, **m>4**
for i= 0 to (n-1)
j= 1 to $\left\lfloor \frac{m-3}{2} \right\rfloor$ do
$w_{mi+2j+1}\, w_{mi+2j} \leftarrow 2$
end for
end if
if n=1, **m>3**
for i= 1 to $\left\lfloor \frac{m-2}{2} \right\rfloor$ do
$w_{2i-1}\, w_{2i} \leftarrow 1$
end for
end if
if n=1, **m>4**
for i= 1 to $\left\lfloor \frac{m}{3} \right\rfloor$ do
$w_{2i+1}\, w_{2i} \leftarrow 2$
end for
end if
end procedure

Theorem 3.3: For m≥3 and n≥1, the human chain graph admits Z_4 – bi magic labeling.
Proof: Let $HC_{n,m}(p,q)$ be a human chain graph with p= 2mn+n+1 vertices and q= 2mn+2n edges. Using algorithm 3.3, the 2mn+2n edges are labeled by defining a function f: $E \rightarrow \{1,2,3\}$ such that the induced function is defined by $f^*:V\rightarrow\{0,1,2,3\}$defined by $f^*(v)=\{\sum f(uv)(mod\ 4)/u \in N(v)\}$)=$k_1$ or k_2, constants.Thus all the weight of the vertices are either 0 or 3. Therefore, for m≥3 and n≥1, the human chain graph admits Z_4 – bi magic labeling.

Example 4 : Z_4 – bi magic labeling for $HC_{1,3}$



Algorithm 3.4

Procedure: (Total Magic Cordial labeling of $HC_{n,m}$)

Input: $V \leftarrow \{u_1, u_2, \ldots, u_{2n+1}, v_1, v_2, \ldots, v_{(m-1)n}, w_1, w_2, \ldots, w_{mn}\}$

$E \leftarrow \{ e_1, e_2, \ldots, e_{2mn+2n}\}$

if n≥1, m≥3

for i= 1 to $\left\lfloor \frac{n+2}{2} \right\rfloor$ do

$u_{4i-3} \leftarrow 1$

end for

for i= 1 to $\left\lfloor \frac{n+1}{2} \right\rfloor$ do

$u_{4i-1} \leftarrow 0$

$u_{4i-2}\, u_{4i-1} \leftarrow 1$

$u_{4i-3}\, u_{4i-2} \leftarrow 1$

end for

for i= 1 to n do

$u_{2i} \leftarrow 0$

$w_{mi} \leftarrow 0$

$w_{mi-1} \leftarrow 1$

$u_{2i}\, w_{m(i-1)+1} \leftarrow 0$

$u_{2i}\, v_{(m-1)(i-1)+1} \leftarrow 1$

end for

end if

if m is odd

for i= 1 to n do

$w_{mi}\, w_{mi-2} \leftarrow 0$

$w_{mi-1}\, w_{mi-2} \leftarrow 1$

end for

end if

if m is even

for i= 1 to n do

$w_{mi}\, w_{mi-2} \leftarrow 1$

$w_{mi-1}\, w_{mi-2} \leftarrow 0$

end for

for i= 1 to $\left\lfloor \frac{n}{2} \right\rfloor$ do

$u_{4i-1}\, u_{4i} \leftarrow 0$

$u_{4i}\, u_{4i+1} \leftarrow 1$

end for

for i= 1 to n do

j= 1 to $\left\lfloor \frac{m-1}{2} \right\rfloor$ do
$w_{mi-m+2j-1} \leftarrow 0$
$w_{mi+2j-m} \leftarrow 1$
end for
for i= 0 to (n-1) do
j= 1 to $\left\lfloor \frac{m-1}{2} \right\rfloor$ do
$v_{(m-1)i+j}\ v_{(m-1)i+j+1} \leftarrow 0$
end for
for i= 1 to n do
j= 1 to $\left\lfloor \frac{m+1}{2} \right\rfloor$ do
$v_{(m-1)i+j-m+1} \leftarrow 1$
end for
end if
if m>3
for i= 1 to n do
j= 1 to $\left\lfloor \frac{m-2}{2} \right\rfloor$ do
$v_{(m-1)i-j+1} \leftarrow 0$
end for
for i= 0 to (n-1) do

j= 1 to (m-3) do
$w_{mi+j}\ w_{mi+j=1} \leftarrow 1$
end for
for i= 1 to n do
$u_{2i}\ v_{(m-1)i} \leftarrow 0$
$v_{(m-1)i-\left\lfloor \frac{m}{2} \right\rfloor+1} v_{(m-1)i-\left\lfloor \frac{m}{2} \right\rfloor+2} \leftarrow 1$
end for
end if
if m= 3
for i= 1 to n do
$u_{2i}\ v_{(m-1)i} \leftarrow 1$
end for
end if
if m>5
for i= 1 to n do
j= 1 to $\left\lfloor \frac{m-4}{2} \right\rfloor$ do
$v_{(m-1)i+j+1}\ v_{(m-1)i+j+2} \leftarrow 0$

end for
end if
end procedure

Theorem 3.4 : For m≥3 and n≥1, the human chain graph admits total magic cordial labeling.

Proof: Let $HC_{n,m}(p,q)$ be a human chain graph with p= 2mn+n+1 vertices and q= 2mn+2n edges. Using algorithm 3.4, the 2mn+n+1 vertices and 2mn+2n edges are labeled by defining a function f:V∪ E →{0,1}

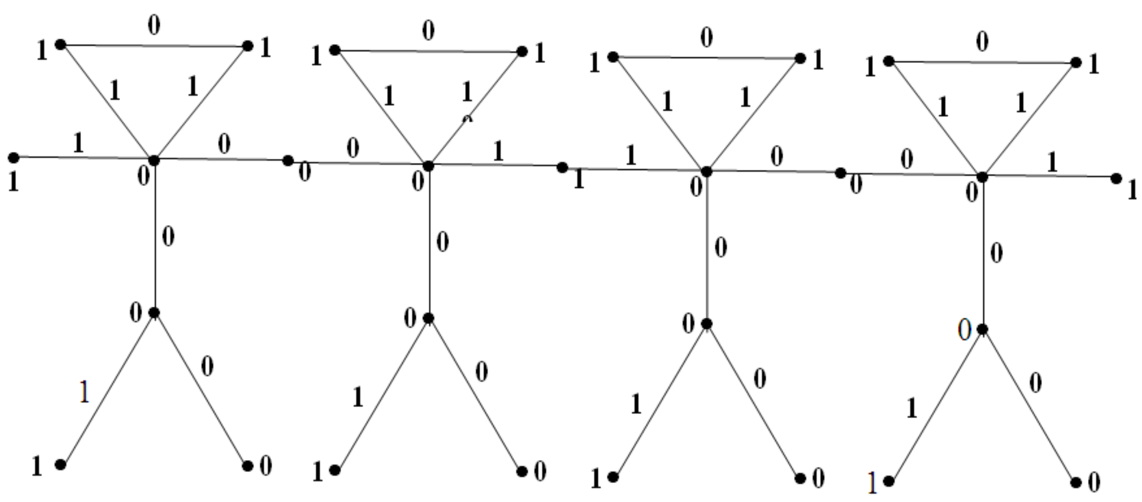
Case (i) If n is odd, the number of vertices labeled with '0' and '1' is mn+(n+1)/2 respectively and the number of edges labeled with '0' and '1' is mn+n respectively. From this we conclude that, the number of vertices and edges labeled with '0' and with '1' is mn+(n+1)/2+mn+n=2mn+(3n/2)+(1/2) which differ by at most one.

Case (ii) If n is even , the number of vertices labeled with '0' is mn+(n/2)+1 and labeled with '1' is mn+(n/2) and the number of edges labeled with '0' and '1' is mn+n respectively. From this we conclude that, the number of vertices and edges labeled with '0' is mn+(n/2)+1+mn+n=2mn+(3n/2)+1 and with '1' is mn+(n/2)+mn+n=2mn+(3n/2) which differ by at most one.

Thus the 2mn+n+1 vertices and 2mn+2n edges are labeled such that the number of vertices labeled with '0' and '1' differ by atmost one. The induced function is defined by f*:E→N∪**{0},** such that $f^*(uv)$= {f(u)+f(v)+f(uv)}(mod2) = k.Thus we have $f^*(uv)$= {f(u)+f(v)+f(uv)}(mod2)= 0+0+0 (or)1+1+0(mod2)=0, which is a constant.

Hence for m≥3 and n≥1, the human chain graph admits total magic cordial labeling.

Example 5: Total magic cordial labeling for $HC_{4,3}$



Algorithm 3.5

Procedure: (n-edge magic labeling of $HC_{n,m}$, m =2n+2, n≥1)

Input: $V \leftarrow \{u_1, u_2, \ldots, u_{2n+1}, v_1, v_2, \ldots, v_{(m-1)n}, w_1, w_2, \ldots, w_{mn}\}$

$E \leftarrow \{ e_1, e_2, \ldots, e_{2mn+2n}\}$

if n≥1, m≥4

```
    for i= 1 to (n+1) do
      u_{2i-1} ← -1
  end for
    for i= 1 to n do
      u_{2i} ← n + 1
  end for
    for i= 0 to (n-1) do
          j= 1 to (m/2) do
          w_{mi+2j-1} ← -1
  end for
    for i= 0 to (n-1) do
          j= 1 to (m-2)/2 do
          w_{mi+2j} ← n + 1
  end for
    for i= 1 to n
      w_{mi} ← -1
    end for
    for i= 1 to n do
          j= 1 to (m/2) do
          v_{(m-1)i-m+2j} ← -1
    end for
    for i= 1 to n do
          j= 1 to (m-2)/2 do
          v_{(m-1)i-m+2j+1} ← n + 1
    end for
```

end if

end procedure

Theorem 3.5 : For m≥3 and n≥1, the human chain graph admits n– edge magic labeling.

Proof: Let $HC_{n,m}(p,q)$ be a human chain graph with p= 2mn+n+1 vertices and q= 2mn+2n edges. Using algorithm 3.5, the 2mn+n+1 vertices are labeled by defining a function f: V→{-1, n+1/n∈N}and 2mn+2n edges are labeled by defining a function f*:E→N,such that $f^*(uv)= \{f(u)+f(v)\}$ =-1+n-1= n, a constant for all uv∈E. Therefore, for m≥3 and n≥1, the human chain graph admits n - edge magic labeling.

4. Conclusion: In this paper, we have constructed algorithms for labeling the vertices and edges and also proved the existence of Z_3 – vertex magic total, Z_3-edge magic total, Z_4-bi magic, total magic cordial and n- edge magic labeling for human chain graph.

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