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SUPRO IDEMPOTENT STRUCTURES OF S-NORM NEAR-RINGS

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Abstract: In this paper, we introduce the notion of new kind of subgroup called S-fuzzy right R-subgroup using S-norm, and investigate some related properties. Finally, suproidempotent property of S-norm over near-ring is also discussed.

Key words: S-norm, near-ring, idempotent, fuzzy set, S-fuzzy right R-subgroup, level set, min-operation.
AMS Subject Classification: , 03G25, 06F35

Section-1 Introduction:Basic concept of fuzzy sets and its operation is first defined by Zadeh [7]. S.Abou-zoid [4] introduced the concept of a fuzzy sub near-ring and explained fuzzy left (resp., right) ideals of a near-ring. K.H.Kim [5] discussed the properties of .fuzzy R-subgroups in near-rings. M.T.Abu Osman [3] investigated on some product of fuzzy subgroups.Also, S.Abou-zoid[4]introduced the concept of fuzzy ideals of a ring, and many authors are discussed in extension of the near-rings.Generalised product of subgroups and t- level subgroups discussed by [1].Various kind of invariant fuzzy subgroups and ideals investigated by Liu [6].In this paper, we introduce the notion of new kind of subgroup called S-fuzzy right R-subgroup using S-norm, and investigate some related properties. Finally, suproidempotent property of S-norm over near-ring is also discussed.

Section-2 Preliminaries

In this section, we include some elementary aspects that are necessary for this paper.

By a near-ring ,we mean a non-empty set R with two binary operations '*Addition*' and *'Multiplication'* satisfying the following conditions;

(NR-1) : (R, +) is a group.

(NR-2) (R , •) is a semi group.

(NR-3) a • (b + c) = a • b + a • c (left distributive) and

(a + b) • c = a • c + b • c (right distributive) for all a,b,c ε R.

Precisely speaking, it is a right near-ring because it satisfies the right distributive law.we will use the word "near-ring" instead of "right near-ring". We denote xy instead of x • y. Note that 0x =0 and (-y)x = -(xy) but in general 0x ≠ 0 for some x ε R.

A two sided R-subgroup of a near-ring R is a subset H of R such that

(i) (H , +) is a subgroup of (R, +).

(ii) RH is a subset of H.

(iii) HR is a subset of H.

If H satisfies (i) and (iii) , then it is called a right R-subgroup of R.

Definition 2.1: A fuzzy set A in a set R is a function A : R→ [0,1].

Example 2.2: Let X ={a,b,c} be a non-empty set. A fuzzy set A is defined by

1.1 X	**1.2 a**	**1.3 b**	**1.4 c**
1.5 Membership value	**1.6 0.9**	**1.7 0.3**	**1.8 0.1**

Definition 2.3: Let (R, +, •) be a near-ring. A fuzzy set A in R is called a fuzzy right R-subgroup of R if

(i) A is a fuzzy subgroup of (R, +).

(ii) $A(xr)^m \geq A(x^m)$ for all x,r m ε R.

Definition 2.4: By a s-norm S, we mean a function S : [0,1] × [0,1] → [0,1] satisfying the following conditions;

(S1) S(x, y) = S(y, x)

(S2) S(x,z) < S(y ,z) , if x < z

(S3) S(x, S(y, z)) = S(S (x,y), z)

(S4)S(0 ,x) = x for all x ∊ [0,1].

For a s-norm S on [0,1], denoted by Δs the set of all element α ε [0,1] such that S(α , α) = α. That is Δs = { α ε [0,1] / S(α , α) = α }.

Proposition 2.5: Every s-norm S have a useful property; S(α , β) ≥ max { α , β } for all α , β ε [0,1].

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Throughout this paper, all standard proofs are going to proceed the only right cases, because the right cases are obtained from similar rule.

In what follows, the term "fuzzy R-subgroup" means "fuzzy right R-subgroup" ("S-fuzzy R-subgroup") respectively.

Definition 2.6: A function $A : R \to [0,1]$ is called a S-fuzzy right R-subgroup of R with respect to s-norm S (briefly, a S-fuzzy right R-subgroup of R) if

(i) $A(x\text{-}y)^m \leq S\,(A(x^m), A(y^m))$

(ii) $A(xr)^m \leq A(x^m)$ for all x,r,m ε R.

It is easy to show that every fuzzy right R-subgroup is a S-fuzzy right R-subgroup of R with $S(\alpha, \beta) = \alpha \vee \beta$ for each $\alpha, \beta \in [0,1]$.

Example 2.7: Let R = {1,2,3,4} be a set with addition and multiplication as follows

1.9 +	**1.10 1**	**1.11 2**	**1.12 3**	**1.13 4**
1.14 1	**1.15 1**	**1.16 2**	**1.17 3**	**1.18 4**
1.19 2	**1.20 2**	**1.21 1**	**1.22 4**	**1.23 3**
1.24 3	**1.25 3**	**1.26 4**	**1.27 2**	**1.28 1**
1.29 4	**1.30 4**	**1.31 3**	**1.32 1**	**1.33 2**

1.34 •	**1.35 1**	**1.36 2**	**1.37 3**	**1.38 4**
1.39 1	**1.40 1**	**1.41 1**	**1.42 1**	**1.43 1**
1.44 2	**1.45 1**	**1.46 1**	**1.47 1**	**1.48 1**
1.49 3	**1.50 1**	**1.51 1**	**1.52 1**	**1.53 2**

We define fuzzy subset A: R → [0,1] by A(3) = A(4) > A(2) > A(1). Then A is called S-fuzzy right R-subgroup of a near-ring R.

Definition 2.8: Let S be a s-norm. A fuzzy set A in R is said to fulfil supro idempotent property if Im (A) ⊇Δs.

Section -3: STRUCTURES OF S-FUZZY RIGHT R-SUBGROUP OF NEAR-RING

Proposition-3.1: Let S be a s-norm on [0,1]. If A is idempotent S-fuzzy right R-subgroup of R, then we have $A(0^m) \leq A(x^m)$ for all x ∊ R.

Proof: For every x ∊ R, we have

$A(0^m) = A(x\text{-}x)^m \leq S\ (A(x^m), A(x^m)) = A(x^m)$. This completes the proof.

Proposition-3.2: Let S be a s-norm on [0,1]. If A is an idempotent S-fuzzy right R-subgroup of R, then the set A = { x∊ R / $A(x^m) \leq A(\Omega^m)$ is an R-subgroup of a near-ring R.

Proof: Let x,y∊A^Ω.Then $A(x^m) \leq A(\Omega^m)$ and $A(y^m) \leq A(\Omega^m)$. Since A is an idempotent S-fuzzy right R-subgroup of R, it follows that

$A(x\text{-}x)^m \leq S\ (A(x^m), A(y^m)) \leq S(A(x^m)\ , A(\Omega^m)) \leq S(\ A(\Omega^m)\ A(\Omega^m)) = A(\Omega^m)$.

Now let r ∊ R, x ∊ AΩ.Then $A(xr)^m \leq A(x^m) \leq A(\Omega^m)$ and $A(xr)^m \leq A(\Omega^m)$, that is x-y ∊A^Ω and xr∊ A^Ω. The proof is completed.

Corrollary-3.3: Let S be a s-norm. If A is an idempotent S-fuzzy right R-subgroup of R, then the set A_R = {x ∊ R / $A(x^m) = A(0^m)$} is an R-subgroup of a near-ring R.

Proof: From the proposition-1, A_R = {x ∊ R / $A(x^m) = A(0^m)$} = {x ∊ R / $A(x^m) = A(0^m)$}, hence A_R is an R-subgroup of a near-ring R from proposition-2.

Let Φ_I denoted the characteristic function of a non-empty subset I of a near-ring R.

Theorem-3.4: Let $R \subseteq I$. Then I is an R-subgroup of a near-ring if and only if Φ_I is a S-fuzzy right R-subgroup of a near-ring R.

Proof: Let I be an R-subgroup of R. Then it is easy to show that Φ_I is an S-fuzzy right R-subgroup of R.

In fact, let $x,y \in I$ and $r \in R$.

Then $x-y \in I$ and $xr \in I$.Hence

$\Phi_I(x-y)^m = 1 = S(\Phi_I(x^m), \Phi_I(y^m))$ and

$\Phi_I(xr)^m \leq \Phi_I(x^m) = 1$.

If $x \in I$, $y \notin I$, then we have $\Phi_I(x^m) = 1$ or $\Phi_I(y^m)$. This means that

$\Phi_I(x-y)^m \leq S(\Phi_I(x^m), \Phi_I(y^m)) = 0$ and $\Phi_I(xr)^m \leq \Phi_I(x^m) = 0$.

Conversly, suppose that Φ_I is a S-fuzzy right R-subgroup of R.

Now let $x,y \in I$.

Then $\Phi_I(x-y)^m \leq S(\Phi_I(x^m), \Phi I(y^m)) = 1$ and $\Phi_I(xr)^m \leq \Phi_I(x^m) =$, that is $x-y \in I$.

Let $r \in R$, $x \in I$. Then $\Phi_I(xr)^m \leq \Phi_I(x^m) = 1$, and $xr \in I$. The proof is completed.

Lemma 3.5: Let S be a s-norm. Then $S(S(p,q), S(\alpha, \beta)) = S(S(p,\alpha), S(q,\beta))$, for all $p,q,\alpha,\beta \in [0,1]$.

Proposition 3.6: If $A: R \rightarrow [0,1]$ and $B:R \rightarrow [0,1]$ are S-fuzzy right R-subgroups of a near-ring R.Then $A \cap B : R \rightarrow [0,1]$ defined by $(A \cap B)(x) = S(A(x), A(y))$ for all $x \in R$ is an S-fuzzy right R-subgroup of a near-ring R.

Proof: Let $x,y \in R$ and $r \in R$.Then we have

$(A\cap B)(x-y)^m = S(A(x-y)^m, B(x-y)^m)$

$\leq S(S(A(x^m), A(y^m)), B(x^m), B(y^m)))$

$= S(S(A(x^m), B(x^m)), S(A(y^m), B(y^m)))$

$= S((A\cap B)(x^m), (A\cap B)(y^m))$

And

$(A\cap B)(xr)^m = S(A(xr)^m, B(xr)^m)$

$\leq S(A(x^m), B(x^m))$

$= (A\cap B)(x^m)$.

This completes the proof.

Definition 3.7: A fuzzy right R-subgroup A of a near-ring is said to be normal if A(0) = 1.

Definition 3.8: Let A be a fuzzy subset of a set R, S a s-norm and $\alpha \in [0,1]$. Then we define a S-level subset of a fuzzy subset A as

$A^S_\alpha = \{x \in R \,/\, S(A(x^m), \alpha) \leq \alpha\}$.

Theorem 3.9: Let R be a near-ring and Aa fuzzy right R-subgroup of R.Then S-level subset A^S_α is an R-subgroup of R where $S(A(0), \alpha) \leq \alpha$ for $\alpha \in [0,1]$.

Proof: $A^S_\alpha = \{x \in R \,/\, S(A(x^m), \alpha) \leq \alpha\}$ is clearly non-empty.

Let $x,y \in A^S_\alpha$

Then we have

$S(A(x^m), \alpha) \leq \alpha$ and $S(A(y^m), \alpha) \leq \alpha$. Since A is a S-fuzzy right R-subgroup of R,

$S(A(x-y)^m, \alpha) \leq S(S(A(x^m), A(y^m)), \alpha)$

$= S(A(x^m), S(A(y^m), \alpha))$

$\leq S(A(x^m), \alpha) \leq \alpha$.

Hence $x-y \in A^S_\alpha$.

Now let $r \in R$ and $x \in A^S_\alpha$. Then we have

$S(A(x^m), \alpha) \leq \alpha$. Since A is a S-fuzzy right R-subgroup of R, we have

$A(xr)^m \leq A(x^m)$. And so

$S(A(xr)^m, \alpha) \leq S(A(x^m), \alpha) \leq \alpha$. This means that $xr \in A^S_\alpha$.

Therefore A^S_α is a R-subgroup of R.

Conclusion: Based on the definition of S-fuzzy right R-subgroup of R, we can generate this idea with minimum operations in a near-rings. This will very applicable in the field of computer design and automation. This concept will be very useful for further research work.

References

1.H.Aktas and N.Cagman, Generalised product of subgroups and t- level subgroups, Mathematical communications 11 (2006), 121-128.

2.Atagun A.O.,Sezgin A., Soft substructures of rings, fields and modules, Comput. Math. Appl., 61(3)(2011), 592-601.

3. M.T.AbuOsman, On some product of fuzzy subgroups, Fuzzy sets and system 24(1987), 79-86.

4. Abou-zoid, On fuzzy subnear-rings and ideals, Fuzzy sets and system 44 (1991), 139-146.

5. K.H.Kim, T-fuzzy R-subgroups of near-rings, Pan American Mathematical Journal 12 (2002), 21-29.

6.W.Liu, Fuzzy invariant subgroups and fuzzy ideals, Fuzzy sets and systems 8 (1982), 133-139.

7.L.A.Zadeh, Fuzzy sets, Inform. And Control, 8 (1965), 338-353.