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A NOTE ON ALMOST ALPHA-COSYMPLECTİCMANIFOLDS

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Abstract

In this paper, we are especially interested in almost alpha-cosymplecticmanifold whose structure (φ,ξ,η,g) satisfies the certain nullity condition. Also, we obtain some results using D-homothetic deformation for almost alpha-cosymplecticmanifolds. Finally, we give an illustrative example with dimension3.

Keywords:

Kenmotsu manifold; Almost,Kenmotsu manifold; D-homothetic deformation; (k,m,v)-space;

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1. Introduction

An extensive research about D-homothetic deformation on contact geometry is carried out in recent years. A serious study in the literature was introduced by Tanno in 1968 (see [14]). Under a D-homothetic deformation we mean a change of structure tensors of the form

$$\eta' = b\eta, \xi' = (1/b)\xi, \varphi' = \varphi, g' = bg + b(b-1)\eta \otimes \eta, \quad (1.1)$$

where$b$ is a positiveconstant (see[14]). Inparticular, someauthorsstudied$D$-homothetic deformations of structures (see[3]and[5]).

Using a D-homothetic deformation to an almost cosymplectic structures (φ,ξ,η,g), we have an almost contact metric manifold satisfying the following special condition

$$R(X,Y)\xi = \eta(Y)(\kappa I + \mu h + \nu\varphi h)X - \eta(X)(\kappa I + \mu h + \nu\varphi h)Y \ , (1.2)$$

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for$\kappa, \mu, \nu \in R_\eta(M^{2n+1})$, where $R_\eta(M^{2n+1})$ be the subring of the ring of smooth functions f on M^{2n+1} such that $df \wedge \eta = 0$ (see [4]). Such manifolds are called almost cosymplectic(κ, μ, ν)-spaces. The condition (1.2) is invariant with respect to the D-homothetic deformations of these structures.

This paper is devoted to study almost almost alpha-cosymplectic manifolds whose almost alpha-cosymplectic structure (φ, ξ, η, g) satisfies the condition (1.2)for $\kappa, \mu, \nu \in R_\eta(M^{2n+1})$. We give some basic concepts of almost alpha-cosymplectic manifolds and D-homothetic deformation on almost alpha-cosymplectic manifolds where alpha is a smooth function such that $d\alpha \wedge \eta = 0$.Next, we obtain some results on such manifolds. We conclude the paper with an illustrative example.

2. Research Method

Almost contact manifolds have odd-dimension. Let us denote the manifold by $^{2n+1}$. Then it carries two fields and a 1-form. These fields are denoted by $\varphi$and ξ. The field φ represents the endomorphisms of the tangent spaces. The field ξ is called characteristic vector field. Also, η is an 1-form given by

$$\varphi^2 = -I + \eta \otimes \xi, \eta(\xi) = 1,$$

such that $I: TM^{2n+1} \to TM^{2n+1}$ is the identity transformation. In light of the above information, it follows that

$$\varphi\xi = 0, \eta \circ \varphi = 0,$$

and the (1,1)-tensor field φ is of constant rank $2n$ (see [16]). Let $(M^{2n+1}, \varphi, \xi, \eta)$ be an almost contact manifold. This manifold called normal if the following tensor field N

$$N = [\varphi, \varphi] + 2d\eta \otimes \xi,$$

vanishes identically. Furthermore, $[\varphi, \varphi]$ represents the Nijenhuis tensor of the tensor field φ.It is well known that $(M^{2n+1}, \varphi, \xi, \eta)$ inducts the following Riemannian metric g

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \quad (2.1)$$

for arbitrary vector fields X, Y on M^{2n+1}. The above metric g is said to be a compatible metric. Thus the structure given with this quadruplecalledalmostcontactmetricstructure. Suchmanifoldsaresaidto be thesame name. According to (2.1), we have $\eta = g(., \xi)$. Moreover, The Φ represents the 2-form of the manifold that is given by

$$\Phi(X, Y) = g(\varphi X, Y),$$

Then it is called the fundamental 2-form of M^{2n+1}.For an almost contact metric manifold, if both η and Φ are closed, then it is said to be an almost cosymplectic manifold. In addition, if an almost contact metric manifold holds the following equations

$$d\eta = 0, d\Phi = 2\eta \wedge \Phi.$$

Then it is called an almost Kenmotsu manifold. These manifolds are studied in (see [7], [8] and [15]) .

Considering the below deformation

$$\eta^* = \left(\frac{1}{\alpha}\right)\eta, \xi^* = \alpha\xi, (2.2)$$

$$\varphi^* = \varphi, g^* = \left(\frac{1}{\alpha^2}\right)g,$$

where alpha is a real constant with $\alpha \neq 0$. Thus we have an almost alpha-Kenmotsu structure $(\varphi^*, \xi^*, \eta^*, g^*)$. In general, this deformation is said to be a homothetic deformation (see [4]). The almost alpha-Kenmotsu structure is connected with some local conformal deformations of almost cosymplectic structures (see [15]).
The notion defined by $d\eta = 0$ and $d\Phi = 2\alpha\eta \wedge \Phi$ called almost alpha-cosymplectic manifold for arbitrary real number alpha (see [8]).

We define $A = -\nabla\xi$ and $h = (1/2)L_\xi\varphi$ for all vector fields where alpha is a smooth function such that $d\alpha \wedge \eta = 0$ and recall that $A(\xi) = 0$ and $h(\xi) = 0$. Then we have

$$\nabla_X\xi = -\alpha\varphi^2X - \varphi hX, (2.3)$$

$$(\varphi h)X + (h\varphi)X = 0, (2.4)$$

$$(\varphi A)X + (A\varphi)X = -2\alpha\varphi, \qquad (2.5)$$

$$tr(h) = 0, \qquad (2.6)$$

for arbitrary vector fields X, Y on M^{2n+1}, (see [8]).

Also, we have the following curvature relations on$(M^{2n+1}, \varphi, \xi, \eta, g)$ almost alpha-cosymplectic manifold.Here alpha is a smooth function where $d\alpha \wedge \eta = 0$, and $l = R(.,\xi)\xi$ is the Jacobi operator (see [11] and [12]):

$$R(X,Y)\xi = (\nabla_Y\varphi h)X - (\nabla_X\varphi h)Y - \alpha[\eta(X)\varphi hY - \eta(Y)\varphi hX]$$
$$+[\alpha^2 + \xi(\alpha)][\eta(X)Y - \eta(Y)X], (2.7)$$

$$lX = [\alpha^2 + \xi(\alpha)]\varphi^2X + 2\alpha\varphi hX - h^2X + \varphi(\nabla_\xi h)X, (2.8)$$

$$(\nabla_\xi h)X = -\varphi lX - [\alpha^2 + \xi(\alpha)]\varphi X - 2\alpha hX - \varphi h^2X, (2.9)$$

$$S(X,\xi) = -2n[\alpha^2 + \xi(\alpha)]\eta(X) - \big(div(\varphi h)\big)X, (2.10)$$

$$S(\xi,\xi) = -\left[2n\left(\alpha^2+\xi(\alpha)\right)+tr(h^2)\right]. \qquad (2.11)$$

3. Results and Analysis

In this section, we are especially interested in almost alpha-cosymplectic manifolds whose almost alpha-cosymplectic structure (φ,ξ,η,g) satisfies the condition (1.2)for $\kappa,\mu,\nu \in R_\eta(M^{2n+1})$. Such manifolds are called almost alpha-cosymplectic (κ,μ,ν)-spaces and (φ,ξ,η,g) be called almost alpha-cosymplectic (κ,μ,ν)-structure. We notice that the functions κ,μ,ν don't have to be constant functions on M^{2n+1} such that $df\wedge\eta = 0$.

Definition 3.1. The structure of (φ,ξ,η,g) on almost cosymplectic manifold by the help of D-homothetic deformation is defined as

$$\varphi^* = \varphi, \xi^* = (1/\zeta)\xi,$$
$$\eta^* = \zeta\eta, \qquad g^* = \gamma g + (\zeta^2-\gamma)\eta\otimes\eta, \qquad (3.1)$$

where$R_\eta(M^{2n+1})$ be the subring of the smooth functions f such that $f: M\to R$ satisfying $df\wedge\eta = 0$ on M^{2n+1}. Here γ is a positive constant and $\zeta\in R_\eta((M^{2n+1}), \zeta\neq 0$ at any point of M^{2n+1} (see [4]).

Firstly, wegivesomecertainresultsthatproved in [11] forthe D-homothetic deformations of almost alpha-cosymplectic manifolds. We will use these results in later usage.

Theorem 3.1. Let $(M^{2n+1},\varphi,\xi,\eta,g)$ be an almost alpha-cosymplectic manifold. The manifold is transformed into a new almost ζ^*-cosymplectic manifold where alpha is parallel along ξ.

Theorem 3.2. Let $(M^{2n+1},\varphi,\xi,\eta,g)$ be an almost alpha-cosymplectic manifolds. For a D-homothetic deformation of almost alpha-cosymplectic structure, the Levi-Civita connections $\nabla^*$and ∇ are can be written as follows:

$$\nabla_X Y = \nabla^*{}_X Y + \frac{(\zeta^2-\gamma)}{\zeta^2} g(AX,Y)\xi - \frac{d\zeta(\xi)}{\zeta}\eta(X)\eta(Y)\xi,$$

whereα is parallel along ξ.

Theorem 3.3. For a D-homothetic deformation of almost alpha-cosymplectic structure, the following equations are held

$$A^*X = (1/\zeta)AX, h^*X = (1/\zeta)hX, \qquad (3.1)$$

$$R^*(X,Y)\xi^* = (1/\zeta)R(X,Y)\xi + (1/\zeta^2)d\zeta(\xi)[\eta(X)AY - \eta(Y)AX], \qquad (3.2)$$

for any vector fields X,Y,Z and $\xi(\alpha) = 0$.

In the light of these relations, we can state the following results:

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Theorem 3.4. Let (φ,ξ,η,g) be an almost alpha-cosymplectic(κ,μ,ν)-structure. Then there exists an almost ζ^*-cosymplectic structure $(\varphi^*,\xi^*,\eta^*,g^*)$with $\kappa^*,\mu^*,\nu^* \in R_{\eta^*}(M^{2n+1})$ satisfying the following relations

$$\kappa^* = \left(\frac{\kappa}{\zeta^2}\right),\ \ \mu^* = \left(\frac{\mu}{\zeta}\right),\ \nu^* = \frac{\zeta\nu - d\zeta(\xi)}{\zeta^2}, \tag{3.3}$$

and

$$R^*(X,Y)\xi^* = \zeta\kappa^*[\eta(Y)X - \eta(X)Y] + \mu^*[\eta(Y)hX - \eta(X)hY] \\ +\nu^*[\eta(Y)\varphi hX - \eta(X)\varphi hY], \tag{3.4}$$

where alpha is parallel along ξ.

Proof. Let (φ,ξ,η,g) be an almost alpha-cosymplectic(κ,μ,ν)-structure. With the help of Definition 3.1, we have

$$\Phi^* = \gamma\Phi, \qquad d\eta^* = (d\zeta \wedge \eta) + \zeta d\eta. \tag{3.5}$$

Then applying (1.2), (3.1) and (3.5) into (3.2) we can easily obtain (3.4). By using simple computations, we also have,

$$[\eta(Y)X - \eta(X)Y](\zeta\kappa^*) + [\eta(Y)hX - \eta(X)hY](\mu^*) \\ +[\eta(Y)\varphi hX - \eta(X)\varphi hY](\nu^*) = [\eta(Y)X - \eta(X)Y]\left(\left(\frac{\kappa}{\zeta}\right)\right) + [\eta(Y)hX - \eta(X)hY]\left(\left(\frac{\mu}{\zeta}\right)\right) \\ +[\eta(Y)\varphi hX - \eta(X)\varphi hY](\nu/\zeta - d\zeta(\xi)/\zeta^2). \tag{3.6}$$

It follows from (3.6) we get (3.3). Thus the proof is completed.

Theorem 3.5. An almost alpha-cosymplectic(κ,μ,ν)-structure can be D-homothetically transformed to an almost ζ^*-cosymplectic $(-1-(3\alpha^2+\alpha\nu)/\zeta),\mu/\zeta,2\alpha/\zeta)$-structure with $\zeta^2 = -(\kappa+\alpha^2)$ for $\kappa < -\alpha^2$ and $\xi(\alpha) = 0$.

Proof. Let (φ,ξ,η,g) be an almost alpha-cosymplectic(κ,μ,ν)-structure.For $\kappa < -\alpha^2$and $\zeta^2 = -(\kappa+\alpha^2)$, we examine a D-homothetic deformation on almost alpha-cosymplectic(κ,μ,ν)-structure where alpha is parallel along ξ.

In this case, by using the following relations

$$\kappa^* = \left(\frac{\kappa}{\zeta^2} + \frac{\xi(\zeta)}{\zeta^3}\right),$$

$$\xi(\zeta) = -\left(\frac{\xi(\kappa)}{2\zeta}\right),$$

$$\xi(\kappa) = 2(\nu - 2\alpha)(\kappa+\alpha^2),$$

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weobtain

$$\kappa^* = \left(\frac{\kappa - 2\alpha^2 + \alpha\nu}{\zeta^2}\right),$$

$$\mu^* = \left(\frac{\mu}{\sqrt{-(\kappa + \alpha^2)}}\right).$$

Then follows from (3.3) we have $\nu^* = \left(\frac{2\alpha}{\zeta}\right)$. Thus the (κ, μ, ν)-structure can be transformed into $(\kappa - 2\alpha^2 + \alpha\nu/\zeta^2), \mu/\zeta, 2\alpha/\zeta)$-structure after applying$D$-homothetic deformation with$\zeta^2 = -(\kappa + \alpha^2)$and $\xi(\alpha) = 0$. Thus it completes the proof.

Example 3.1. We assume that 3-dimensional manifold is defined as

$$M^3 = \{(x, y, z) \in R^3, \qquad z \neq 0\},$$

where(x, y, z) are the cartesian coordinates in R^3. We define three vector fields on M^3 as

$$e = \left(\frac{\partial}{\partial x}\right), \varphi e = \left(\frac{\partial}{\partial y}\right), \xi$$
$$= \left[\alpha x - y\left(e^{\{-2\alpha z\}} + z\right)\right]\left(\frac{\partial}{\partial x}\right) + \left[x\left(z - e^{\{-2\alpha z\}}\right) + \alpha y\right]\left(\frac{\partial}{\partial y}\right) + \left(\frac{\partial}{\partial z}\right).$$

Furthermore, the matrice form of the metric tensor g, the tensor fields ϕ and h are given by

$$g = \begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & -k \\ -d & -k & 1 + d^2 + k^2 \end{pmatrix}, \quad \varphi = \begin{pmatrix} 0 & -d & k \\ 1 & 0 & -d \\ 0 & 0 & 0 \end{pmatrix},$$
$$h = \begin{pmatrix} e^{-2z} & 0 & k - de^{-2z} \\ 0 & -e^{-2z} & ke^{-2z} \\ 0 & 0 & 0 \end{pmatrix},$$

where $d = \alpha x - y(e^{-2\alpha z} + z), k = x(z - e^{-2\alpha z} + \alpha y$.

Let η be the 1-form defined by $\eta = k_1 dx + k_2 dy + k_3 dz$ for all vector fields on M^3. Since $\eta(X) = g(X, \xi)$, we can easily obtain that $\eta(e) = 0, \eta(\varphi e) = 0$and $\eta(\xi) = 1$. By using these equations, we get $\eta = dz$ for all vector fields. Since $d\eta = d(dz) = d^2 z$, we obtain $d\eta = 0$. Using Koszul's formula, we have seen that $d\Phi = 2\alpha\eta \wedge \Phi$. Hence, it has been showed that M^3 is an almost alpha-cosymplectic manifold. Thus we obtain

$$R(X, Y)\xi = -(e^{-4\alpha z} + \alpha^2)[\eta(Y)X - \eta(X)Y] + 2z[\eta(Y)hX - \eta(X)hY],$$

where$\kappa = -(e^{-4\alpha z} + \alpha^2) and\ \mu = 2z$.Thusaccording to Theorem 7.3.1 in [10], this example is provided for $\xi(\alpha) = 0$.

4. Conclusion

In this paper, westudy almost alpha-cosymplectic manifolds in the light of (1.1). Some certain results are obtained related to D-homothetic deformation on almost alpha-cosymplectic manifolds where alpha is a smooth function such that $d\alpha \wedge \eta = 0$.We obtain some results using a D-homothetic deformation on almost alpha-cosymplectic manifolds. Our forthcoming papers are devoted to investigate almost alpha-cosymplectic(κ, μ, ν)-spaces in terms of a certain D-homothetic deformation given in this paper. Open problems are so interesting for these spaces where the smooth functions κ, μ, ν are not constants. When we use different tensor fields on such manifolds, we can obtain some different results.

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