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SOME FIXED POINT THEOREMS IN GENERALIZED METRIC SPACE USING TRIANGULAR FUZZY NUMBERS.

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ABSTRACT

In this Paper, a fixed point theory concept of triangular fuzzy numbers is considered to study fuzzy metric space and established some properties of triangular metric space along with contraction mapping are revised in terms of triangular fuzzy numbers and also $\bar{\bar{X}}$ *be a contraction on* $[(\underline{x}_1, \underline{x}_2, \underline{x}_3), d]$ *then* $\bar{\bar{T}}$ *has unique fixed points theorem proved. In this method it gives more accurate and approximate solution of real life situation and numerical illustrations are given*

KEYWORDS:

Fuzzy Topology,
Fuzzy metric space,
Fuzzy contraction,
Fuzzy set,
topological space,
triangular fuzzy number, etc.

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1. INTRODUCTION

The Fuzzy concept was first introduced in 1965, Zadeh [2], the concept of fuzzy set theory and there after it has been developed by several authors through the contribution of the different articles on this concept and applied on different branches of pure and applied

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mathematics. The concept of fuzzy theory was introduced by Katsaras [23] in 1984 and in 1992. Felbin [20] introduced the idea of fuzzy normed linear space. Cheng - Moderson [1] introduced , Later on Bag and Samanta [17] modified the definition of fuzzy norm of Cheng – Moderson and the basic fuzzy idea have applied in literature that includesthe applying of fuzzy sets to pattern recognition, judgment issues, perform approximation, system theory, logical system, fuzzy algorithms, fuzzy automata, fuzzy grammars, fuzzy language, fuzzy mathematics, fuzzy topology, etc. during this note , our interests are in the study of certain concepts in triangular fuzzy metric Space.

A fixed point of a function $f: s \rightarrow s$ is a point x in s such that$f(x) = x$. Fixed point theory has a beautiful mixture of analysis, topology and geometry. Since from 1922 the theory of fixed points has been revealed as a very powerful and important technique for solving a variety of applied problems in mathematical sciences and engineering. In particular fixed point techniques have been applied in such diverse fields as biology, chemistry, economics, game theory and physics i.e quantum mechanics. Also in the method of successive approximation for proving existence and uniqueness of solutions of differential and integral equations.

In this paper, the concept of fuzzy metric space is discussed in the knowledge of triangular fuzzy system. Fuzzy contraction mapping is applied to study the fixed point theorem.

2. PRELIMINARIES

The preliminary segments are the best definition of local minimum, local maximum, global optima, and convex, concave and general nonlinear problem.

2.1 Fuzzy number: [5]

The Fuzzy numbers are the great consequence of fuzzy systems. Regularly used the fuzzy numbers in applications of the triangular and the trapezoidal shaped. [7].

Definition 1: [5]

A fuzzy number is a fuzzy set like $\bar{\bar{u}}: \bar{\bar{R}} \rightarrow \bar{\bar{I}} = [0, 1]$ which satisfies [2-21],

1. $\bar{\bar{u}}$is upper semi continuous function.
2. $\bar{\bar{u}}(x) = 0$ is outside of the interval [c, d].
3. There exists a real numbers a, b such that $c \le a \le b \le d$ and

   3.1 $\bar{\bar{u}}(x)$is the monotonically increasing function on [c, a].

   3.2 $\bar{\bar{u}}(x)$is the monotonically decreasing function on [b, d].

3.3 $\bar{\bar{u}}(x) = 1,\ x \in [a,b]$.

It is denoted by $F(\bar{\bar{R}})$. This is also specified in [13]. An alternative description or parametric form of a fuzzy number which yields the same $F(\bar{\bar{R}})$ is given by kelva [16].

Arithmetic operations among two triangular fuzzy numbers defined on universal set of real numbers $F(\bar{\bar{R}})$ are reviewed [4].

2.2 Fuzzy Number:[5]

A fuzzy set $\bar{\bar{u}}$ on $\bar{\bar{R}}$must possess at least the following three properties to qualify as a fuzzy number.

i$\bar{\bar{u}}$is a normal fuzzy set.

ii The closed interval $^{\alpha}\bar{\bar{u}}$, for every $\alpha \in [0,1]$

iii $\bar{\bar{u}}$, $^{0+}\bar{\bar{u}}$ must be bounded.

2.3 Triangular Fuzzy number: [4][5][8]

The Triangular fuzzy number represents the three points as:$\bar{\bar{u}} = (\underline{u}_1, \underline{u}_2, \underline{u}_3)$

The membership functions of triangular fuzzy numbers satisfies the following conditions

(i) $\underline{u}_1$to $\underline{u}_2$is increasing function

(ii) $\underline{u}_2 to\ \underline{u}_3$ is decreasing function

(iii) $\underline{u}_1 \le \underline{u}_2 \le \underline{u}_3$

$$\mu_A(x) = \begin{cases} 0, & for \quad x < \underline{u}_1 \\ \dfrac{x - \underline{u}_1}{\underline{u}_2 - \underline{u}_1}, & for\ \underline{u}_1 \le x \le \underline{u}_2 \\ \dfrac{\underline{u}_3 - x}{\underline{u}_3 - \underline{u}_1}, & for\ \underline{u}_2 \le x \le \underline{u}_3 \\ 0, & for \quad x > \underline{u}_3 \end{cases}$$

2.4 Positive triangular fuzzy number: [4] [5]

A Triangular fuzzy number $\bar{\bar{u}} = (\underline{u}_1, \underline{u}_2, \underline{u}_3)$ is positive, if $\forall\ \underline{u}_i$'s > 0 $\forall$ i = 1, 2, 3.

2.5 Negative triangular fuzzy number: [4] 5]

A Triangular fuzzy number$\bar{\bar{u}} = (\underline{u}_1, \underline{u}_2, \underline{u}_3)$ is negative, if $\forall\ \underline{u}_i$'s < 0 $\forall$ i = 1, 2, 3.

Note: The Negative multiplication of negative fuzzy number is positive triangular fuzzy number.

Example: when $\bar{\bar{A}} = (-3, -2, -1)$ is a negative fuzzy number, this can be written as $\bar{\bar{A}} = -(1,2,3)$

2.6 Equal Triangular fuzzy number: [4], [5]

If $\bar{\bar{u}}$ and $\bar{\bar{v}}$ are identically equal, then $\bar{\bar{u}} = \bar{\bar{v}}$, if $\underline{u}_1 = \underline{v}_1$, $\underline{u}_2 = \underline{v}_2$ and $\underline{u}_3 = \underline{v}_3$,

3.PROPERTIES OF METRIC SPACE

Some elementary properties of linear space are discussed here.

3.1.Properties:

Let $\bar{\bar{X}} = [0, \infty]$ and $\{\bar{\bar{A}}, \bar{\bar{B}}, \bar{\bar{C}}\} \subset \bar{\bar{X}}$ are $\bar{\bar{A}} = (\underline{a}_1, \underline{a}_2, \underline{a}_3), \bar{\bar{B}} = (\underline{b}_1, \underline{b}_2, \underline{b}_3) \& \bar{\bar{C}} = (\underline{c}_1, \underline{c}_2, \underline{c}_3)$ such that $\underline{a}_1 \le \underline{b}_1 \le \underline{c}_1, \underline{a}_2 \le \underline{b}_2 \le \underline{c}_2$ and $\underline{a}_3 \le \underline{b}_3 \le \underline{c}_3$ then

i). $d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{b}_1, \underline{b}_2, \underline{b}_3)\right) \ge 0$

ii). $d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{b}_1, \underline{b}_2, \underline{b}_3)\right) = 0 \quad if\ (\underline{a}_1, \underline{a}_2, \underline{a}_3) = (\underline{b}_1, \underline{b}_2, \underline{b}_3)$

iii). $d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{b}_1, \underline{b}_2, \underline{b}_3)\right) = d\left((\underline{b}_1, \underline{b}_2, \underline{b}_3), (\underline{a}_1, \underline{a}_2, \underline{a}_3)\right)$

iv).

$$d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{c}_1, \underline{c}_2, \underline{c}_3)\right) = d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{b}_1, \underline{b}_2, \underline{b}_3)\right) + d\left((\underline{b}_1, \underline{b}_2, \underline{b}_3), (\underline{c}_1, \underline{c}_2, \underline{c}_3)\right)$$

v). $d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3) + k, (\underline{b}_1, \underline{b}_2, \underline{b}_3) + k\right) = d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{b}_1, \underline{b}_2, \underline{b}_3)\right)$

vi). $d\left(\lambda(\underline{a}_1, \underline{a}_2, \underline{a}_3), \lambda(\underline{b}_1, \underline{b}_2, \underline{b}_3)\right) = \lambda\, d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{b}_1, \underline{b}_2, \underline{b}_3)\right)$

Proof:

Let $\bar{\bar{X}} = [0, \infty]$ and $\{\bar{\bar{A}}, \bar{\bar{B}}, \bar{\bar{C}}\} \subset \bar{\bar{X}}$

Here $\bar{\bar{A}} = (\underline{a}_1, \underline{a}_2, \underline{a}_3), \bar{\bar{B}} = (\underline{b}_1, \underline{b}_2, \underline{b}_3)$

i). By $d\left((\underline{a}_1,\underline{a}_2,\underline{a}_3),(\underline{b}_1,\underline{b}_2,\underline{b}_3)\right) = |\underline{a}_1-\underline{b}_1|,|\underline{a}_2-\underline{b}_2|,|\underline{a}_3-\underline{b}_3|$

Where $\underline{a}_1 \le \underline{b}_1$,$\underline{a}_2 \le \underline{b}_2$ and $\underline{a}_3 \le \underline{b}_3$ then

$$\Rightarrow |\underline{a}_1-\underline{b}_1|,|\underline{a}_2-\underline{b}_2|,|\underline{a}_3-\underline{b}_3| \ne 0 \quad and \; |\underline{a}_1-\underline{b}_1|,|\underline{a}_2-\underline{b}_2|,|\underline{a}_3-\underline{b}_3| > 0$$

$$d\left((\underline{a}_1,\underline{a}_2,\underline{a}_3),(\underline{b}_1,\underline{b}_2,\underline{b}_3)\right) \ge 0$$

ii) Let By $d\left((\underline{a}_1,\underline{a}_2,\underline{a}_3),(\underline{b}_1,\underline{b}_2,\underline{b}_3)\right) = 0$

Where$\underline{a}_1 \le \underline{b}_1$,$\underline{a}_2 \le \underline{b}_2$ and $\underline{a}_3 \le \underline{b}_3$ then

$$\Rightarrow |\underline{a}_1-\underline{b}_1|,|\underline{a}_2-\underline{b}_2|,|\underline{a}_3-\underline{b}_3| = 0$$

$$\Rightarrow |\underline{a}_1-\underline{b}_1| = 0, |\underline{a}_2-\underline{b}_2| = 0, |\underline{a}_3-\underline{b}_3| = 0$$

Shall only occur when $|\underline{a}_1-\underline{b}_1| = 0, |\underline{a}_2-\underline{b}_2| = 0 \; and \; |\underline{a}_3-\underline{b}_3| = 0$

$$\Rightarrow \underline{a}_1-\underline{b}_1 = 0,\;\; \underline{a}_2-\underline{b}_2 = 0, and \; \underline{a}_3-\underline{b}_3 = 0$$

$$\Rightarrow \underline{a}_1 = \underline{b}_1, \qquad \underline{a}_2 = \underline{b}_2, \qquad and \; \underline{a}_3 = \underline{b}_3$$

$$\therefore d\left((\underline{a}_1,\underline{a}_2,\underline{a}_3),(\underline{b}_1,\underline{b}_2,\underline{b}_3)\right) = 0$$

Again, Let $(\underline{a}_1,\underline{a}_2,\underline{a}_3) = (\underline{b}_1,\underline{b}_2,\underline{b}_3)$

$$\Rightarrow \underline{a}_1 = \underline{b}_1, \qquad \underline{a}_2 = \underline{b}_2, \qquad and \; \underline{a}_3 = \underline{b}_3$$

$$\Rightarrow |\underline{a}_1-\underline{b}_1| = 0, |\underline{a}_2-\underline{b}_2| = 0 \; and \; |\underline{a}_3-\underline{b}_3| = 0$$

So, $\Rightarrow |\underline{a}_1-\underline{b}_1|,|\underline{a}_2-\underline{b}_2|,|\underline{a}_3-\underline{b}_3| = 0$

$$d\left((\underline{a}_1,\underline{a}_2,\underline{a}_3),(\underline{b}_1,\underline{b}_2,\underline{b}_3)\right) = 0$$

iii). Let $\underline{a}_1 \le \underline{b}_1$,$\underline{a}_2 \le \underline{b}_2$ and $\underline{a}_3 \le \underline{b}_3$

By $d\left((\underline{a}_1,\underline{a}_2,\underline{a}_3),(\underline{b}_1,\underline{b}_2,\underline{b}_3)\right) = |\underline{a}_1-\underline{b}_1|,|\underline{a}_2-\underline{b}_2|,|\underline{a}_3-\underline{b}_3|$

$$=|\underline{b}_1-\underline{a}_1|,|\underline{b}_2-\underline{a}_2|,|\underline{a}_3-\underline{b}_3|$$

$$= d\left((\underline{b}_1,\underline{b}_2,\underline{b}_3),(\underline{a}_1,\underline{a}_2,\underline{a}_3)\right)$$

iv). we have$\bar{\bar{A}} = (\underline{a}_1,\underline{a}_2,\underline{a}_3), \bar{\bar{B}} = (\underline{b}_1,\underline{b}_2,\underline{b}_3) \& \bar{\bar{C}} = (\underline{c}_1,\underline{c}_2,\underline{c}_3)$

such that $\underline{a}_1 \leq \underline{b}_1 \leq \underline{c}_1, \underline{a}_2 \leq \underline{b}_2 \leq \underline{c}_2$ and $\underline{a}_3 \leq \underline{b}_3 \leq \underline{c}_3$,

So, $d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{c}_1, \underline{c}_2, \underline{c}_3)\right) = \left(|\underline{a}_1 - \underline{c}_1|, |\underline{a}_2 - \underline{c}_2|, |\underline{a}_3 - \underline{c}_3|\right)$

$\leq \left(|\underline{a}_1 - \underline{b}_1|, |\underline{a}_2 - \underline{b}_2|, |\underline{a}_3 - \underline{b}_3|\right) + \left(|\underline{b}_1 - \underline{c}_1|, |\underline{b}_2 - \underline{c}_2|, |\underline{b}_3 - \underline{c}_3|\right)$

$= d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{b}_1, \underline{b}_2, \underline{b}_3)\right) + d\left((\underline{b}_1, \underline{b}_2, \underline{b}_3), (\underline{c}_1, \underline{c}_2, \underline{c}_3)\right)$

v). clearly $[(\underline{a}_1, \underline{a}_2, \underline{a}_3) + k, (\underline{b}_1, \underline{b}_2, \underline{b}_3) + k]$

$= [(\underline{a}_1 + k, \underline{a}_2 + k, \underline{a}_3 + k), (\underline{b}_1 + k, \underline{b}_2 + k, \underline{b}_3 + +k)]$

$= d[(\underline{a}_1 + k, \underline{a}_2 + k, \underline{a}_3 + k), (\underline{b}_1 + k, \underline{b}_2 + k, \underline{b}_3 + +k)]$

$= \left(|\underline{a}_1 + k - \underline{b}_1 - k|, |\underline{a}_2 + k - \underline{b}_2 - k|, |\underline{a}_3 + k - \underline{b}_3 - k|\right)$

$= \left(|\underline{a}_1 - \underline{b}_1|, |\underline{a}_2 - \underline{b}_2|, |\underline{a}_3 - \underline{b}_3|\right)$

$= d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{b}_1, \underline{b}_2, \underline{b}_3)\right)$

vi). Clearly, for $\lambda > 0$, $[\lambda(\underline{a}_1, \underline{a}_2, \underline{a}_3), \lambda(\underline{b}_1, \underline{b}_2, \underline{b}_3)] = [(\lambda\underline{a}_1, \lambda\underline{a}_2, \lambda\underline{a}_3), (\lambda\underline{b}_1, \lambda\underline{b}_2, \lambda\underline{b}_3)]$

$d[\lambda(\underline{a}_1, \underline{a}_2, \underline{a}_3), \lambda(\underline{b}_1, \underline{b}_2, \underline{b}_3)] = d[(\lambda\underline{a}_1, \lambda\underline{a}_2, \lambda\underline{a}_3), (\lambda\underline{b}_1, \lambda\underline{b}_2, \lambda\underline{b}_3)]$

$= |\lambda\underline{a}_1 - \lambda\underline{b}_1|, |\lambda\underline{a}_2 - \lambda\underline{b}_2|, |\lambda\underline{a}_3 - \lambda\underline{b}_3|$

$= \lambda\left(|\underline{a}_1 - \underline{b}_1|, |\underline{a}_2 - \underline{b}_2|, |\underline{a}_3 - \underline{b}_3|\right)$

$= \lambda\, d\left((\underline{a}_1, \underline{a}_2, \underline{a}_3), (\underline{b}_1, \underline{b}_2, \underline{b}_3)\right)$

4. FIXED POINT OF TRIANGULAR FUZZY METRIC SPACE.

In classical Topology there are notations of fixed point contraction mapping. In this section we shall present the notions in fuzzy context.

4.1. Definition of fixed point

Suppose $\bar{\bar{X}}$ is any set and $\bar{\bar{T}}: \bar{\bar{X}} \longrightarrow \bar{\bar{X}}$ is a mapping, then $\bar{\bar{x}} \in \bar{\bar{X}}$ is called fixed point of $\bar{\bar{T}}$ if $\bar{\bar{T}}(\bar{\bar{x}}) = \bar{\bar{x}}$

4.2. Definition of triangular fuzzy fixed point.

Let n $\bar{\bar{X}} \in \mathbb{I}^R$ be a triangular fuzzy number and $(\underline{a}_1, \underline{a}_2, \underline{a}_3) \subset [0, \ \infty]$ be the set of $\bar{\bar{X}}$.

Suppose $\bar{\bar{T}} : (\underline{a}_1, \underline{a}_2, \underline{a}_3) \rightarrow (\underline{a}_1, \underline{a}_2, \underline{a}_3)$ is the mapping. Then $(\underline{a}_1, \underline{a}_2, \underline{a}_3) \in \bar{\bar{X}}$ is called a fixed point of $\bar{\bar{T}}$, If $\bar{\bar{T}}\,(\underline{a}_1, \underline{a}_2, \underline{a}_3) = (\underline{a}_1, \underline{a}_2, \underline{a}_3)$

4.3. Definition of triangular fuzzy contraction mapping.

Let $(\bar{\bar{X}}, \ d)$ be the metric space. A mapping $\bar{\bar{T}}: \bar{\bar{X}} \longrightarrow \bar{\bar{X}}$ is called contraction on $\bar{\bar{X}}$, if there is a positive real number $k < 1$ such that for all $\bar{\bar{x}}, \bar{\bar{y}} \in \bar{\bar{X}}$, $d(\bar{\bar{T}}\bar{\bar{x}}, \bar{\bar{T}}\bar{\bar{y}}) = k\, d(\bar{\bar{x}}, \bar{\bar{y}})$

4.5. Theorem

Let $\bar{\bar{X}}$ be a contraction on $[(\underline{x}_1, \underline{x}_2, \underline{x}_3), d]$. Then $\bar{\bar{T}}$ has a unique fixed point

Proof

Let for an arbitrary $i \in N$, $\bar{\bar{x}}_i \in \bar{\bar{X}}$, here $\bar{\bar{x}}_i = \bar{\bar{x}}_1, \bar{\bar{x}}_2, \bar{\bar{x}}_3, \ldots..$ and $\bar{\bar{x}}_1 = (\underline{a}_1, \underline{a}_2, \underline{a}_3)$

We define iterative sequence $\bar{\bar{x}}_n \in \bar{\bar{X}}$, by $\bar{\bar{x}}_0$,

$$\bar{\bar{x}}_1 = \bar{\bar{T}}(\bar{\bar{x}}_0), \ \bar{\bar{x}}_2 = \bar{\bar{T}}(\bar{\bar{x}}_1), \bar{\bar{x}}_3 = \bar{\bar{T}}(\bar{\bar{x}}_2), \ldots, \bar{\bar{x}}_n = \bar{\bar{T}}(\bar{\bar{x}}_{n-1})$$

$$\text{Then } \bar{\bar{x}}_2 = \bar{\bar{T}}\left(\bar{\bar{T}}(\bar{\bar{x}}_0)\right) = \bar{\bar{T}}^2(\bar{\bar{x}}_0)$$

$$\bar{\bar{x}}_3 = \bar{\bar{T}}\left(\bar{\bar{T}}^2(\bar{\bar{x}}_0)\right) = \bar{\bar{T}}^3(\bar{\bar{x}}_0)$$

$$\bar{\bar{x}}_4 = \bar{\bar{T}}\left(\bar{\bar{T}}^3(\bar{\bar{x}}_0)\right) = \bar{\bar{T}}^4(\bar{\bar{x}}_0)$$

.

.

.

$$\bar{\bar{x}}_n = \bar{\bar{T}}\left(\bar{\bar{T}}^{n-1}(\bar{\bar{x}}_0)\right) = \bar{\bar{T}}^n(\bar{\bar{x}}_0)$$

We shall show that the sequence $\bar{\bar{x}}_n$ is Cauchy sequence

If $n > m$, then $d(\bar{\bar{x}}_{m+1}, \bar{\bar{x}}_m) = d\left(\bar{\bar{T}}(\bar{\bar{x}}_m), \bar{\bar{T}}(\bar{\bar{x}}_{m-1})\right)$

$$\leq k\, d\big((\bar{\bar{x}}_m), (\bar{\bar{x}}_{m-1})\big)$$

$$\leq k^2\, d\big((\bar{\bar{x}}_{m-1}), (\bar{\bar{x}}_{m-2})\big)$$
$$\leq k^3\, d\big((\bar{\bar{x}}_{m-2}), (\bar{\bar{x}}_{m-3})\big)$$

.

$$d(\bar{\bar{x}}_{m+1}, \bar{\bar{x}}_m) \leq k^m\, d\big((\bar{\bar{x}}_1), (\bar{\bar{x}}_0)\big)$$

By Triangle inequality, we obtain for $n > m$

$$d(\bar{\bar{x}}_m, \bar{\bar{x}}_n) \leq d(\bar{\bar{x}}_m, \bar{\bar{x}}_{m+1}) + d(\bar{\bar{x}}_{m+1}, \bar{\bar{x}}_{m+2}) + d(\bar{\bar{x}}_{m+2}, \bar{\bar{x}}_{m+3}) + d(\bar{\bar{x}}_{m+3}, \bar{\bar{x}}_{m+4}) + \cdots + d(\bar{\bar{x}}_0, \bar{\bar{x}}_1)$$
$$\leq k^m d(\bar{\bar{x}}_0, \bar{\bar{x}}_1) + k^{m+1} d(\bar{\bar{x}}_0, \bar{\bar{x}}_1) + k^{m+2} d(\bar{\bar{x}}_0, \bar{\bar{x}}_1) + \cdots + k^{n-1} d(\bar{\bar{x}}_0, \bar{\bar{x}}_1)$$
$$\leq k^m (1 + k + k^2 + k^3 + \cdots + k^{n-1-m}) d(\bar{\bar{x}}_0, \bar{\bar{x}}_1)$$
$$\leq k^m \frac{1 - k^{n-m}}{1 - k} d(\bar{\bar{x}}_0, \bar{\bar{x}}_1)$$

Since $0 < K < 1$, so the number $1 - k^{n-m} < 1$

Therefore

$$d(\bar{\bar{x}}_m, \bar{\bar{x}}_n) \leq \frac{1 - k^{n-m}}{1 - k} d(\bar{\bar{x}}_0, \bar{\bar{x}}_1)$$

Since the space $\bar{\bar{X}}$ is complete, there exists a $\bar{\bar{x}}_0 \in \bar{\bar{X}}$ such that $\bar{\bar{x}}_n \to \bar{\bar{x}}_0$

Now we show that this $\bar{\bar{x}}_0 \in \bar{\bar{X}}$ is fixed under the mapping $\bar{\bar{T}}$.

By definition and triangle inequality we have $d\big(\bar{\bar{x}}_0, \bar{\bar{T}}(\bar{\bar{x}}_0)\big) \leq d(\bar{\bar{x}}_0, \bar{\bar{x}}_n) + d\big(\bar{\bar{x}}_n, \bar{\bar{T}}(\bar{\bar{x}}_0)\big)$

$$\Rightarrow d\big(\bar{\bar{x}}_0, \bar{\bar{T}}(\bar{\bar{x}}_0)\big) \leq d(\bar{\bar{x}}_0, \bar{\bar{x}}_n) + k\, d(\bar{\bar{x}}_{n-1}, \bar{\bar{x}}_0)$$

We know that $d(\bar{\bar{x}}, \bar{\bar{y}}) = 0\ if\ \bar{\bar{x}} = \bar{\bar{y}})$

Since $\bar{\bar{x}}_n \to \bar{\bar{x}}_0$, so $d(\bar{\bar{x}}_0, \bar{\bar{x}}_n) \to 0$ and $d(\bar{\bar{x}}_{n-1}, \bar{\bar{x}}_0) \to 0$

Which implies that $d\big(\bar{\bar{x}}_0, \bar{\bar{T}}(\bar{\bar{x}}_0)\big) = 0$ and hence $\bar{\bar{T}}(\bar{\bar{x}}_0) = \bar{\bar{x}}_0$. This shows that $\bar{\bar{x}}_0$ fixed of $\bar{\bar{T}}$.

Now we shall show that $\bar{\bar{x}}_0$ is unique of $\bar{\bar{T}}$

Suppose that $\bar{\bar{x}}_1$ is another fixed of $\bar{\bar{T}}$

Then $\bar{\bar{T}}(\bar{\bar{x}}_1) = \bar{\bar{x}}_1$

$$d(\bar{\bar{x}}_0, \bar{\bar{x}}_1) \leq d\big(\bar{\bar{T}}(\bar{\bar{x}}_0), \bar{\bar{T}}(\bar{\bar{x}}_1)\big) \leq k\, d(\bar{\bar{x}}_0, \bar{\bar{x}}_1)$$

Since $K < 1$, this implies that $\therefore d(\bar{\bar{x}}_0, \bar{\bar{x}}_1)$

Hence $\bar{\bar{x}}_0 = \bar{\bar{x}}_1$, thus the $\bar{\bar{x}}_0$ is unique of $\bar{\bar{T}}$

4. Numerical Examples:

Consider the two triangular fuzzy numbers as follows: $\bar{\bar{A}} = (0.1, 0.2, 0.3)$ and $\bar{\bar{B}} = (0.2, 0.3, 0.4)$ as in [23].

$$d(\bar{\bar{A}}, \bar{\bar{B}}) = (|0.1 - 0.2|, |0.2 - 0.3|, |0.3 - 0.4|)$$
$$=(|-0.1|, |-0.1|, |-0.1|)$$
$$= (0.1, 0.1, 0.1)$$

The comparison of the proposed methods with the other existing methods

The distance measure between $\bar{\bar{A}}$ and $\bar{\bar{B}}$ by chen[24] is $d(A, B) = 0.9$, S.J. chen and S.M. chen[25] is obtained: $d(A, B) = 0.81$, Guha and Chakraborty [26] is $d(A, B) = (0.8, 0.9, 1)$, Sadi-Nezhad's methods [27] is DAB = (0,0,0.33). Also Mostafa Ali Beigi ,TayabehHajjari and ErfanGhasemkhani [28] is DistAB= (0.1,0.1,0.3). By Proposed Method we get $d(\bar{\bar{A}}, \bar{\bar{B}}) = (0.1, 0.1, 0.1)$ which the result is reasonable.

5. CONCLUSION

Fuzzy distance measure play an essential part in image processing under impression, as well as it can be commonly used in data mining and pattern recognition. In this paper, we review on some fuzzy distance methods, then we discussed about a distance for two triangular fuzzy numbers that unlike to existing methods. We constructed new approaches to solve basic properties of metric space using triangular fuzzy numbers and the contraction of triangular fuzzy metric space is defined, and further the existence and uniqueness theorem are proved.

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