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EMERGING APPLICATIONS ON DOUBLE N-FUZZY SOFT MARKOVIAN IDEALS STRUCTURES

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ABSTRACT: *In this paper, we define a new type of double Markovian group (M(G)-group) action, called double N-fuzzy M(G)-group soft intersection (DNFGSI) action and double N-fuzzy M(G)-ideal soft intersection(DNFGSI) action on a soft set. This new concept illustrates how a soft set effects on a double N-fuzzy M(G)-group in the mean of union and inclusion of sets and its function as bridge among soft set theory, set theory and double N-fuzzy M(G)-group theory. We also obtain some analog of classical double N-fuzzy M(G)-group theoretic concepts for double fuzzy M(G)-group SI-action. Finally, we give the application of SU-actions on double N-fuzzy M(G)-group to M(G)-group theory.*

KEYWORDS: *Soft set, M(G)-group, double N-fuzzy M(G)-group SI-action, double N-fuzzy M(G)--ideal SI-action, soft pre-image, soft anti-image, α-inclusion.*

AMS Mathematics Subject Classification: 03E70, 08E40.

1. INTRODUCTION

Soft set theory was introduced in 1999 by Molodtsov [22] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 25, 28]. Moreover, Atagun and Sezgin [4] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Maji et al. [19] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [3] introduced several operations of soft sets and Sezgin and Atagun [26] studied on soft set operations as well. Furthermore, soft set relations and functions [5] and soft mappings [21] with many related concepts were discussed. The theory of soft set has also wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 23, 29].

Sezgin et.al [25] introduced a new concept to the literature of N-group called N-group soft intersection action. In this paper, we define a new type of double N-fuzzy M(G)-group action on a soft set, which we call double N-fuzzy M(G)-group soft intersection action and abbreviate as "*double N-fuzzy M(G)-group SI action*" which is based on the inclusion relation and union of sets. Since double N-fuzzy M(G)-group SU-action gathers soft set theory and set theory and double N-fuzzy M(G)–group theory, it is useful in improving the soft set theory with respect to double fuzzy M(G)-group structures. Based on this new notion, we then introduce the concepts of double N-fuzzy M(G)-ideal SU-action and show that if double fuzzy M(G)-group SU-action over U. Moreover, we investigate these notions with respect to soft image, soft pre-image and give their applications to double N-fuzzy M(G)-group theory.

2. PRELIMINARIES

In this section, we recall some basic notions relevant to near-ring modules and fuzzy soft sets.

By a near-ring, we shall mean an algebraic system (M(G),+,●),

where (N_1) (M(G), +) forms a group (not necessarily abelian)

(N_2) (M(G), ●) forms a semi group and

(N_3) (x + y)z = xz + yz, for all x,y,z ∈ M(G).

Throughout this paper, M(G) will always denote group. A normal subgroup H of M(G) is called a left ideal of M(G) if g(s+h) – gs ∈ H for all g ∈ M(G) and h ∈ I and denoted by $H \lhd_{\ell} M(G)$.For a group M(G), the zero-symmetric part of M(G) denoted by $M(G)_0$ is defined by $M(G)_0$= {g∈ M(G) / g0 = 0}.

Let (S,+) be a group and A: M(G)×S →S, (g,s)→s.

(S,A) is called M(G)-group if for all x,y ∈ S,

(i) x(ys) = (xy)s

(ii) (x+y)s = xs+ys.

It is denoted by N^S. Clearly M(G) itself is an double fuzzy M(G)–group by natural operations. A subgroup T of $M(G)^S$ with M(G)T⊆T is said to be M(G)–sub group of S and denoted by $T \le N^S$. A normal subgroup T of S is called an M(G)–ideal of $M(G)^S$ and denoted by a group. S and χ two M(G)–groups. Then h: S→χ is called a double fuzzy M(G)–group homomorphism if s,$\delta \in$ S, for all g∈ M(G),

(i) h((s+δ)) = h(s)+h(δ) and

(ii) h(gs) = g(h(s)).

For all undefined concepts and notions we refer to (24). From now on, U refers to on initial universe, E is a set of parameters P(U) is the power set of U and A,B,C⊆ E.

Definition 2.1: [22] A pair (F,A) is called a soft set over U, where F is a mapping given by F : A→P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U. Note that a soft set (F,A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A respectively. For more details, we refer to [11, 17, 18, 26, 29, 7].

Definition 2.2: [6] The relative complement of the soft set F_A over U is denoted by F^r_A, where F^r_A: A → P(U) is a mapping given as $F^r_A(a)$ =U \$F_A(a)$, for all a ∈A.

Definition 2.3: [6] Let F_A and G_B be two soft sets over U such that A∩B ≠ ∅,. The restricted intersection of F_A and G_B is denoted by $F_A \Cup G_B$, and is defined as $F_A \Cup G_B$ = (H,C), where C = A∩B and for all c ∈C, H(c) = F(c)∩G(c).

Definition 2.4: [6] Let F_A and G_B be two soft sets over U such that A∩B ≠ ∅,. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B$ = (H,C), where C = A∩B and for all c ∈C, H(c) = F(c)∪G(c).

Definition 2.5: [12] Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi(F_A)$ over U, where ψ (F_A) : B→P(U) is a set valued function defined by ψ $(F_A)(b)$ = ∪{F(a) | a∈A and ψ (a) = b},

if $\psi^{-1}(b) \neq \emptyset$,

= 0 otherwise for all b ∈B.

Here, ψ (F_A) is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B)$: A → P(U) is a set-valued function defined by

$\psi^{-1}(G_B)(a) = G(\psi(a))$ for all $a \in A$. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

Definition 2.6: [13] Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^\star(F_A)$ over U, where $\psi^\star(F_A)$:B→P(U) is a set-valued function defined by $\psi^\star(F_A)(b) = \cap\{F(a) \mid a\in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \emptyset$,

= 0, otherwise, for all b∈B.

Here, $\psi^\star(F_A)$ is called the soft anti image of F_A under ψ.

Definition 2.7: [8] Let f_A be a soft set over U and α be a subset of U. Then, upper α-inclusion of a soft set f_A, denoted by $f^{\alpha}{}_A$, is defined as $f^{\alpha}{}_A = \{x \in A : f_A(x) \supseteq \alpha\}$

Definition 2.8: Let X be a non-empty set. A mapping $f_s : X \to [-1,0]$ is called Negative fuzzy set [N-fuzzy set] in X.

3. DOUBLE N-FUZZY M(G) –GROUP SI-ACTION

In this section, we first define soft intersection action, abbreviated as SI-action on double N-fuzzy M(G)–group and double N-fuzzy M(G)-ideal structures with illustrative examples. We then study their basic results with respect to soft set operation.

Definition 3.1: Let S be a N-fuzzy M(G)–group and f_s be a soft set over U. Then f_s is called SI-action on double N-fuzzy M(G)–group over U if it satisfies the following conditions;

(DNFSG-1) $f_s((x+y)) \supseteq f_s(x) \cap f_s(y)$

(DNFSG-2) $f_s(-x) \supseteq f_s(x)$

(DNFSG-3) $f_s(gx) \supseteq f_s(x)$, for all x,y∈S and g∈M(G)..

Example 3.2: Consider the module M(G) = {0,x,y,z}, be the near-ring under the operation defined by the following table:

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

•	0	x	y	z
0	0	0	0	0
x	x	x	x	x
y	0	0	0	0
z	x	x	x	x

Let S = M(G) and S be the set of parameters

and U= $\left\{\begin{bmatrix} -a & -a \\ 0 & -a \end{bmatrix} / a, b \in -Z_6\right\}$, 2×2 matrices with $-Z_6$ terms, is the universal set. We construct a fuzzy soft set.

$f_s(0)$ = $\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} -3 & -3 \\ 0 & -3 \end{bmatrix}\right\}$, f_s (x)= $\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 3 & -3 \\ 0 & -3 \end{bmatrix}\right\}$,

f_s (y) = $\left\{\begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix}\right\}$, and f_s (z) = $\left\{\begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix}\right\}$

Then one can easily show that the soft set f_s is a SI-action on double N-fuzzy M(G)–group

Proposition 3.3: Let f_s be a SI-action on double N-fuzzy M(G)–group over U. Then, $f_s(0) \supseteq f_s(x)$, for all x∈ S.

Proof: Assume that f_s is SI-action over U. Then, for all x∈S,

$f_s(0) = f_s((x\text{-}x)) \supseteq f_s(x) \cap f_s(-x) = f_s(x) \cap f_s(x) = f_s(x)$.

Theorem 3.4: Let S be a SI-action on double N-fuzzy M(G)–group and f_s be a soft set over U.

Then f_s is SI-action of double N-fuzzy M(G)–group over U if and only if

(i) $f_s((x\text{-}y)) \supseteq f_s(x) \cap f_s(y)$

(ii) $f_s(gx) \supseteq f_s(x)$, for all x,y ∈S and g∈M(G).

Proof: Suppose f_s is a fuzzy SI-action on double N-fuzzy M(G)–group over U. Then, by definition-3.1,

$f_s(xy) \supseteq f_s(y)$ and $f_s(m(x\text{-}y)) \supseteq f_s(x) \cap f_s(-y) = f_s(x) \cap f_s(y)$ for all x,y ∈ S

Conversely, assume that $f_s(xy) \supseteq f_s(y)$ and $f_s(m(x\text{-}y)) \supseteq f_s(x) \cap f_s(y)$, for all x,y ∈S.

If we choose x=0, then $f_s(0\text{-}y) = f_s(-y) \supseteq f_s(0) \cap f_s(y) = f_s(y)$ by proposition-3.1.

Similarly $f_s(gy) = f_s(-(-y)) \supseteq f_s(-y)$, thus $f_s(-y) = f_s(y)$ for all y ∈S.

Also, by assumption $f_s((x\text{-}y)) \supseteq f_s(x) \cap f_s(-y) = f_s(x) \cap f_s(y)$.

This completes the proof.

Theorem 3.5: Let f_s be a SI-action on double N-fuzzy M(G)–group over U.

(i) If $f_s((x\text{-}y)) = f_s(0)$ for any x,y ∈S, then $f_s(x) = f_s(y)$.

(ii) $f_s((x\text{-}y)) = f_s(0)$ for any x,y ∈S, then $f_s(x) = f_s(y)$.

Proof: Assume that $f_s(x\text{-}y) = f_s(0)$ for any x,y ∈S, then

$f_s(x) = f_s((x\text{-}y\text{+}y)) \supseteq f_s(x\text{-}y) \cap f_s(y)$

$= f_s(0) \cap f_s(y) = f_s(y)$

and similarly,

$f_s(gy) = f_s(n(y\text{-}x)+x)) \supseteq f_s(y\text{-}x) \cap f_s(x)$

$= f_s(-(y\text{-}x)) \cap f_s(x)$

$= f_s(0) \cap f_s(x) = f_s(x)$

Thus, $f_s(x) = f_s(y)$ which completes the proof.

Similarly, we can show the result (ii).

It is known that if S is double N-fuzzy M(G)–group, then (S,+)is a group but not necessarily abelian. That is, for any x,y∈S, x+y needs not be equal to y+x. However, we have the following:

Theorem 3.6: Let f_s be SI-action on double N-fuzzy M(G)–group over U and x∈S. Then, $f_s(x) = f_s(0) \Leftrightarrow f_s(x+y) = f_s(y+x) = f_s(y)$ for all y ∈S .

Proof: Suppose that $f_s(x+y) = f_s(y+x) = f_s(y)$ for all y ∈S. Then, by choosing y=0, we obtain that $f_s(x) = f_s(0)$.

Conversely, assume that $f_s(x) = f_s(0)$.

Then by proposition-3.1, we have

$f_s(0) = f_s(x) \supseteq f_s(y), \forall\, y \in S$…………… (1)

Since f_s SI-action on double N-fuzzy M(G)–group over U, then

$f_s((x+y)) \supseteq f_s(x) \cap f_s(y) = f_s(y), \forall\, y \in S$. Moreover, for all y ∈S

$f_s(gy) = f_s(g(\text{-}x)+x)+y)) = f_s(g(\text{-}x+(x+y))) \supseteq f_s(\text{-}x) \cap f_s(x+y)$

$= f_s(x) \cap f_s(x+y) = f_s(x+y)$

Since by equation (1), $f_s(x) \supseteq f_s(y)$ for all y ∈S and x,y ∈S, implies that x+y ∈S. Thus, it follows that $f_s(x) \supseteq f_s(x+y)$. So $f_s(x+y) = f_s(y)$ for all y ∈S.

Now, let x ∈S. Then, for all x, y ∈S

$f_s((y+x)) = f_s((y+x+(y\text{-}y)))$

$= f_s((y+(x+y)\text{-}y))$

$\supseteq f_s(y) \cap f_s(x+y) \cap f_s(y)$

$= f_s(y) \cap f_s(x+y) = f_s(y)$

Since $f_s(x+y) = f_s(y)$. Furthermore, for all y ∈S

$f_s(gy) = f_s(g(y+(x\text{-}x)))$

= f_s((y+x)-x)

⊇ f_s(y+x)∩ f_s(x)

= f_s(y+x) by equation (1).

It follows that f_s(y+x) = f_s(y) and sof_s(x+y)= f_s(y+x)= f_s(y), for all y ∈S, which completes the proof.

Theorem 3.7: Let S be a near-field and f_s be a soft set over U. If f_s(0)⊇ f_s(1) =f_s(x), ∀ 0≠ x∈S, then it is SI-action on double N-fuzzy M(G)–group over U.

Proof: Suppose that f_s(0)⊇ f_s(1) = f_s(x) for all 0≠ x ∈S. In order to prove that it is SI-action on double fuzzy M(G)–group over U, it is enough to prove that f_s((x-y))⊇ f_s(x)∩ f_s(y) and f_s(gx)⊇ f_s(x).

Let x, y ∈S. Then we have the following cases:

Case-1: Suppose that x ≠ 0 and y= 0 or x= 0 and y ≠ 0. Since S is a near-field, so it follows that nx = 0 and f_s(gx) = f_s(0). Since f_s(0)⊇ f_s(x), for all x ∈S, sof_s(gx) = f_s(0) ⊇ f_s(x), and f_s(gx) = f_s(0) ⊇ f_s(y). This implies, f_s(gx) ⊇ f_s(x).

Case-2: Suppose that x≠ 0 and y≠0. It follows that nx≠0. Then, f_s(gx) = f_s(1) = f_s(x) and f_s(gx) = f_s(1) = f_s(y), which implies that f_s(gx)⊇ f_s(x).

Case-3: Suppose that x=0 and y=0, then clearly f_s(gx)⊇ f_s(x). Hence f_s(gx)⊇ f_s(x), ∀ x,y∈S.

Now, let x,y∈S. Then x-y=0 or x+y≠ 0. If x+y=0, then either x=y=0 or x≠ 0, y≠ 0 and x=y. But, since f_s(x+y) = f_s(0) ⊇ f_s(x), for all x∈S it follows that f_s((x+y)) = f_s(0) ⊇ f_s(x)∩ f_s(y). If x+y≠0, then either x≠ 0, y≠ 0 and x≠y or x≠ 0 and y=0 or x=0 and y≠ 0.

Assume that x≠ 0, y≠ 0 and x≠y. This follows that f_s((x-y)) = f_s(1) = f_s(x)⊇ f_s(x)∩ f_s(y). Now, let x≠ 0 and y=0. Then f_s((x+y)) ⊇ f_s(x)∩ f_s(y). Finally, let x = 0 and y ≠ 0. Then, f_s((x+y)) ⊇ f_s(x)∩ f_s(y). Hence f_s(x-y) ⊇ f_s(x)∩ f_s(y), for all x, y ∈S.

Thus, f_s is SI-action on double N-fuzzy M(G)–group over U.

Theorem 3.8: Let f_s and f_T be two SI-action on double N-fuzzy M(G)–group over U. Then $f_s \wedge f_T$ is soft SI-action on double N-fuzzy M(G)–group over U.

Proof: Let (x_1, y_1), (x_2, y_2) ∈ S×T. Then

$f_{S\wedge T}\big(((x_1,y_1) - (x_2,y_2))\big)$ = $f_{S\wedge T}((x_1 - x_2, y_1 - y_2))$

$$= f_S\,(x_{1+}x_2\,) \cap f_T\,(y_{1+}y_2\,)$$
$$\supseteq (f_S\,(x_1\,) \cap f_S\,(x_2\,)) \cap (f_T\,(y_1\,) \cap f_T\,(y_2\,))$$
$$= (f_S\,(x_1\,) \cap f_T\,(y_1\,)) \cap (f_S\,(x_2\,) \cap f_T\,(y_2\,))$$
$$= f_{S\wedge T}(x_{1,}y_1) \cap f_{S\wedge T}(x_{2,}y_2)$$

And, $f_{S\wedge T}\left((g_{1,}g_2),(x_{2,}y_2)\right) = f_{S\wedge T}(g_1x_{2,}g_2y_2)$
$$= f_S(g_1x_2) \cap f_T\,(g_2y_2\,)$$
$$\supseteq f_S(x_2) \cap f_T(y_2)$$
$$= f_{S\wedge T}(x_{2,}y_2)$$

Thus $f_s \wedge f_T$ is SI-action on double N-fuzzy M(G)–group over U.

Note that $f_s \vee f_T$ is not SI-action on double N-fuzzy M(G)–group over U.

Example 3.9: Assume U = p_3 is the universal set. Let S = Z_3 and H = $\left\{\begin{bmatrix} a & a \\ b & b \end{bmatrix} / a, b \in Z_3\right\}$ 2× 2 matrices with Z_3 terms are set of parameters. We define SI-action on double N-fuzzy M(G)–groupf_S over U = p_3 by

$f_S(0) = p_3$

$f_S(1) = \{(1), (1\ 2), (1\ 3\ 2)\}$

$f_S(2) = \{(1), (1\ 2), (1\ 2\ 3), (1\ 3\ 2)\}$

We define SI-action on M-N-module $f_H$over U = p_3 by

$f_H\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right\}$ =p_3

$f_H\left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\right\}$ =$\{(1), (1\ 2), (1\ 3\ 2)\}$

Then $f_s \vee f_T$ is not SI-action on double N-fuzzy M(G)–group over U.

Definition 3.10: Let $f_{S,} g_T$ be SI-action on double N-fuzzy M(G)–group over U. Then product of fuzzy SI-action on double N-fuzzy M(G)–group f_S and g_T is defined as $f_S \times g_T = h_{S\times T}$, where $h_{S\times T}(x, y) = f_S(x) \times g_T$(y) for all (x,y) ∈ S × T.

Theorem 3.11: If f_S *and* g_T are SI-action on double N-fuzzy M(G)–group over U. Then so is $f_S \times g_T$ over U×U.

Proof: By definition-3.2, let $f_S \times g_T = h_{S\times T}$, where $h_{S\times T}(x, y) = f_S(x) \times g_T$(y) for all (x,y) ∈ S × T.

Then for all $(x_1, y_1), (x_2, y_2) \in$ S × T and $(g_{1,}g_2)$ = M(G)×M(G).

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$$h_{S\times T}(((x_1,y_1)-(x_2,y_2))) = h_{S\times T}((x_{1+}x_{2,}y_{1+}y_2))$$
$$= f_S((x_{1+}x_{2,} \times g_T((y_{1+}y_2)))$$
$$\supseteq (f_S(x_1) \cap f_S(x_2)) \times (g_T(y_1) \cap$$
□□ (□2)
$$= (f_S(x_1) \times g_T(y_1)) \cap (f_S(x_2) \times$$
□□ (□2)
$$= h_{S\times T}(x_1,y_1) \cap h_{S\times T}(x_2,y_2)$$
$$h_{S\times T}((g_1,g_2)(x_2,y_2)) = h_{S\times T}(g_1x_2,g_2y_2)$$
$$= f_S(g_1x_2) \times g_T(g_2y_2)$$
$$\supseteq f_S(x_2) \times g_T(y_2)$$
$$= h_{S\times T}(x_2,y_2)$$

Hence $f_S \times g_T = h_{S\times T}$ is SI-action on double N-fuzzy M(G)–group over U.

Theorem 3.12: If f_S and h_S are SI-action on double N-fuzzy M(G)–group over U, then so is $f_S \tilde{\cap} h_S$ over U.

Proof: Let $x, y \in$ s and g$\in$ M(G) then

$$(f_S \tilde{\cap} h_S)((x+y)) = f_S((x+y)) \cap h_S((x+y))$$
$$\supseteq (f_S(x) \cap f_S(y)) \cap (h_S(x) \cap h_S(y))$$
$$= (f_S(x) \cap h_S(x)) \cup (f_S(y) \cap h_S(y))$$
$$= (f_S \tilde{\cap} h_S)(x) \cap (f_S \tilde{\cap} h_S)(y)$$
$$(f_S \tilde{\cap} h_S)(gx) = f_S(gx) \cap h_S(gx)$$
$$\supseteq f_S(x) \cap h_S(x)$$
$$= (f_S \tilde{\cap} h_S)(x).$$

Therefore, $(f_S \tilde{\cap} h_S)$ is SI-action on double N-fuzzy M(G)–group over U.

4. SI-ACTION ON DOUBLE N-FUZZY M(G) –IDEAL STRUCTURES

Definition 4.1: Let S be a double N-fuzzy M(G)–group and f_S be a soft set over U. Then f_S is called SI-action on double N-fuzzy M(G)–ideal of S over U if the following conditions are satisfied:

(i) $f_s((x+y)) \supseteq f_s(x) \cap f_s(y)$

(ii) $f_s(-x) = f_s(x)$

(iii) $f_s(x + y - x) \supseteq f_s(y)$

(iv) $f_s(g(x + y) - gx) \supseteq f_s(y)$ for all $x, y \in$ S and $g \in$ M(G).

Here, note that

$f_s(x + y) \supseteq f_s(x) \cap f_s(y)$ and $f_s(-x) = f_s(x)$ imply $f_s(x - y) \supseteq f_s(x) \cap f_s(y)$

Example 4.2: Consider M(G) ={0, x, y, z} with the following tables

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

•	0	x	y	z
0	0	0	0	0
x	0	0	0	x
y	0	x	y	y
z	0	x	y	z

Let S = M(G) be the parameters and U = D_2, dihedral group, be the universal set. We define a N-fuzzy soft set f_s over U by $f_s(0) = D_2$, $f_s(x) = \{e, b, ba\}$, $f_s(y) = \{a, b\}$, $f_s(z) = \{b\}$.

Then, one can show that f_s is SI-action on double N-fuzzy M(G)–ideal of S over U.

Example 4.3: Consider M(G) = {0,1,2,3} with the following tables

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

•	0	x	y	z
0	0	0	0	0
x	0	1	0	1
y	0	3	0	3
z	0	2	0	2

Let S=M(G) be the set of parameters and U= Z^+ be the universal set. We define a N-fuzzy soft set f_s over U by

$$f_s(0) = \{1,2,3,5,6,7,9,10,11,17\}$$

$$f_s(1) = f_s(3) = \{1,3,5,7,9,11\}$$

$$f_s(2) = \{1,5,7,9,11\}$$

Since, $f_S(2.(3+1)-2.3) = f_S(2.1-2.3) = f_S(3-3) = f_S(0) \nsubseteq f_S(1)$

Therefore, f_S is not SI-action on double N-fuzzy M(G) –ideal over U.

It is known that if M(G) is a zero-symmetric near-ring, then every double N-fuzzy M(G)-ideal of S is also double N-fuzzy M(G)–group of S. Here, we have an analog for this case.

Theorem 4.4: Let M(G) be a zero- symmetric near-ring. Then, every SI-action on double N-fuzzy M(G) –ideal is SI-action on double N-fuzzy M(G)–group over U.

Proof: Let f_S be an SI-action on double N-fuzzy M(G)–ideal on S over U. Since f_S(g(x+y)-gx) ⊇ f_S(y), for all x,y ∈ S, and g∈ M(G) in particular for x=0, it follows that f_S(g(0+y)-g.0) = f_S(gy-0)=f_S(y)⊇f_S(y).

Since the other condition is satisfied by definition-4.1, f_S is SI-action on double N-fuzzy M(G)–ideal of S over U.

Theorem 4.5: Let f_S be SI-action on double N-fuzzy M(G)–ideal of S and f_T be SI-action on double N-fuzzy M(G)–ideal of T over U. Then $f_S \wedge f_T$ is SI-action on double N-fuzzy M(G)–ideal of S×T over U.

Theorem 4.6: If f_S is SI-action on double N-fuzzy M(G)–ideal of S and f_T be SI-action on double N-fuzzy M(G)–ideal of T over U, then $f_S \times f_T$ is SI-action on double N-fuzzy M(G)–ideal over U×U.

Theorem 4.7: If f_S and h_S are two SI-action on double N-fuzzy M(G)–group of S over U, then $f_S \tilde{\cap} h_S$ is SI-action on double N-fuzzy M(G)–ideal over U.

5. APPLICATIONS OF DOUBLE N-FUZZY SI-ACTION ON M(G) –IDEAL

In this section, we give the applications of soft image, soft pre-image, lower α-inclusion of soft sets and double N-fuzzy M(G)-group homomorphism with respect to SI-action on double N-fuzzy M(G) –group and double N-fuzzy M(G)–ideal.

Theorem 5.1: If f_S is SI-action on double N-fuzzy M(G)–ideal of S over U, then S^f={x ∈S / f_S(x) =f_S(0)} is a double N-fuzzy M(G)–ideal of S.

Proof: It is obvious that 0 ∈ S^f we need to show that

(i) x-y ∈ S^f,

(ii) s+x-s ∈ S^f and

(iii) g(s+x)-gs ∈ S^f for all x,y ∈ S^f and g∈M(G) and s∈S.

If x,y ∈ S^f, then f_S(x)=f_S(y) = f_S(0).

By proposition-3.1,

$f_S(0) \supseteq f_S(x\text{-}y)$, $f_S(0) \supseteq f_S(s+x-s)$, and $f_S(0) \supseteq f_S(g(s+x)\text{-}gs)$ for all x,y ∈ S^f and g ∈M(G) and s∈S.

Since f_S is fuzzy SI-action on M(G) –ideal of S over U, then for all x,y ∈ S^f and g∈M(G) and s∈S.

(i) $f_S((x\text{-}y)) \supseteq f_S(x) \cap f_S(y) = f_S(0)$.

(ii) $f_S(s+x-s) \supseteq f_S(x) = f_S(0)$.

(iii) $f_S(g(s+x)\text{-}gs) \supseteq f_S(x) = f_S(0)$.

Hence $f_S(x\text{-}y) = f_S(0)$, $f_S(s+x-s) = f_S(0)$ and $f_S(g(s+x)\text{-}gs) = f_S(0)$, for all x,y ∈ S^f and g∈M(G) and s∈S.

Therefore S^f isdouble N-fuzzy M(G)–ideal of S.

Theorem 5.2: Let f_S be soft set over U and α be a subset of U such that $\emptyset \supseteq \alpha \supseteq f_S(0)$. If f_S is SI-action on double N-fuzzy M(G)–ideal over U, then $f_S^{\supseteq\alpha}$ is a double fuzzy M(G)–ideal of S.

Proof: Since $f_S(0) \supseteq \alpha$, then $0 \in f_S^{\supseteq\alpha}$ and $\emptyset \neq f_S^{\subseteq\alpha} \supseteq$ S. Let x,y ∈ $f_S^{\supseteq\alpha}$, then $f_S(x) \supseteq \alpha$ and $f_S(y) \supseteq \alpha$. We need to show that

(i) x-y ∈ $f_S^{\supseteq\alpha}$

(ii) s+x-s ∈ $f_S^{\supseteq\alpha}$

(iii) g(s+x)-gs ∈ $f_S^{\supseteq\alpha}$ for all x,y ∈ $f_S^{\supseteq\alpha}$ and n∈N and s∈S.

Since f_S is SI-action on M(G) –ideal over U, it follows that

(i) $f_S((x\text{-}y)) \supseteq f_S(x) \cap f_S(y) \supseteq \alpha \cap \alpha = \alpha$,

(ii) $f_S(s+x-s) \supseteq f_S(x) \supseteq \alpha$ and

(iii) $f_S(g(s+x)\text{-}gs) \supseteq f_S(x) \supseteq \alpha$. Thus, the proof is completed.

Theorem 5.3: Let f_S and f_T be soft sets over U and χ be an M(G)-isomorphism from S to T. If f_S is SI-action on double N-fuzzy M(G)–ideal of S over U, then $\chi(f_S)$ is SI-action on double N-fuzzy M(G)–ideal of T over U.

Proof: Let δ_1, δ_2 and n∈ N. Since χ is surjective, there exists s_1, s_2 ∈ S such that $\chi(s_1) = \delta_1$ and $\chi(s_2) = \delta_2$. Then

$$(\chi f_S)((\delta_1 - \delta_2)) = \cup \{ f_S(s) / s \in S, \chi(s) = \delta_1 - \delta_2 \}$$

$= \cup \{ f_s(s) / s \in S , s = \chi^{-1}(\delta_1 - \delta_2) \}$

$= \cup \{ f_s(s) / s \in S , s = \chi^{-1}(\chi(s_1 - s_2)) = s_1 - s_2 \}$

$= \cup \{ f_s(s_1 - s_2) / s_i \in S , \chi(s_i) = \delta_i , i = 1,2,\ldots \}$

$\supseteq \cup \{ f_s(s_1) \cap f_s(s_2)) / s_i \in S , \chi(s_i) = \delta_i , i = 1,2,\ldots \}$

$= (\{\cup \{f_s(s_1)/s_1 \in S , \chi(s_1) = \delta_1\}\} \cap \{\cup \{f_s(s_2)/s_2 \in$ S , χ(□2)=□2

$= (\chi(f_s))(\delta_1) \cap (\chi(f_s))(\delta_2)$

Also, $(\chi f_s)(\delta_1 + \delta_2 - \delta_1) = \cup \{ f_s(s) / s \in S , \chi(s) = \delta_1 + \delta_2 - \delta_1 \}$

$= \cup \{ f_s(s) / s \in S , s = \chi^{-1}(\delta_1 + \delta_2 - \delta_1) \}$

$= \cup \{ f_s(s) / s \in S , s = \chi^{-1}(\chi(s_1 + s_2 - s_1)) = s_1 + s_2 - s_1 \}$

$= \cup \{ f_s(s_1 + s_2 - s_1) / s_i \in S , \chi(s_i) = \delta_i , i = 1,2,\ldots \}$

$\supseteq \cup \{f_s(s_2)/s_2 \in S , \chi(s_2) = \delta_2\}$

$= (\chi(f_s))(\delta_2)$

Furthermore,

$(\chi f_s)(g(\delta_1 + \delta_2) - g\delta_1) = \cup \{ f_s(s) / s \in S , \chi(s) = g(\delta_1 + \delta_2) - g\delta_1 \}$

$= \cup \{ f_s(s) / s \in S , s = \chi^{-1}(g(\delta_1 + \delta_2) - g\delta_1) \}$

$= \cup \{ f_s(s) / s \in S , s = g(s_1 + s_2) - gs_1 \}$

$= \cup \{ f_s(g(s_1 + s_2) - gs_1) / s_i \in S , \chi(s_i) = \delta_i , i = 1,2,\ldots \}$

$\supseteq \cup \left\{\frac{f_s(s_2)}{s_2} \in S,\ \chi(s_2) = \delta_2\right\}$

$= (\chi(f_s))(\delta_2)$.

Hence, $\chi(f_s)$ is SI-action on double N-fuzzy M(G)–ideal of T over U.

Theorem 5.4: Let f_S and f_T be soft sets over U and χ be an M-N-isomorphism from S to T. If f_T is SI-action on double N-fuzzy M(G)–ideal of T over U, then $\chi^{-1}(f_T)$ is SI-action on double N-fuzzy M(G)–ideal of S over U.

Proof: Let $s_1, s_2 \in S$ and $n \in N$. Then

$(\chi^{-1}(f_T))((s_1 - s_2)) = f_T(\chi(s_1 - s_2))$

$= f_T(\chi(s_1) - \chi(s_2))$

$\supseteq f_T(\chi(s_1)) \cap f_T(\chi(s_2))$

$$= (\chi^{-1}(f_T))(s_1) \cup (\chi^{-1}(f_T))(s_2).$$

Also, $(\chi^{-1}(f_T))\,(s_1+s_2 - s_1) = f_T(\chi\,(s_1+s_2 - s_1))$

$$= f_T(\chi\,(s_1)+\chi(s_2) - \chi(s_1))$$

$$\supseteq f_T(\chi\,(s_2)) = (\chi^{-1}(f_T))(s_2)$$

Furthermore, $(\chi^{-1}(f_T))\,(g(s_1+s_2) - ng) = f_T(\chi\,(g(s_1+s_2) - ng))$

$$= f_T(g(\chi\,(s_1)+\chi(s_2)) - g\chi(s_1))$$

$$\supseteq f_T(\chi\,(s_2)) = (\chi^{-1}(f_T))(s_2)$$

Hence, $(\chi^{-1}(f_T))$ is SI-action on double N-fuzzy M(G)–ideal of S over U.

6. CONCLUSION

In this paper, we have defined a new type of double N-fuzzy M(G)–group action on a soft set, called SI-action on double N-fuzzy M(G)–group by using the soft sets. This new concept picks up the soft set theory and double N-fuzzy M(G)–group theory together and therefore, it is very functional for obtaining results in the mean of M(G)–group structure. Based on this definition, we have introduced the concept of SI-action on double N-fuzzy M(G)–ideal. We have investigated these notions with respect to soft image, soft pre-image and upper α-inclusion of soft sets. Finally, we give some application of SI-action on M-N-ideal to double N-fuzzy M(G)–group theory.

Future Work: To extend this study, one can further study the other algebraic structures such as different algebra in view of their SI-actions.

REFERENCES

[1] Acar U., Koyuncu F., Tanay B., Soft sets and soft rings, Comput. Math. Appl., 59(2010), 3458-3463.

[2] Aktas. H., C. agman N., Soft sets and soft groups, Inform. Sci., 177(2007), 2726-2735.

[3] Ali M.I.,Feng F., Liu X., Min W.K., Shabir M., On some new operations in soft set theory, Comput. Math. Appl., 57(2009), 1547-1553.

[4] Atagun A.O., Sezgin A., Soft substructures of rings, fields and modules, Comput. Math. Appl., 61(3)(2011), 592-601.

[5] Babitha K.V., Sunil J.J., Soft set relations and functions, Comput. Math. Appl., 60(7)(2010), 1840-1849.

[6] Cagman N., Enginoglu S.,Soft matrix theory and its decision making, Comput. Math. Appl., 59(2010), 3308-3314.

[7] Cagman N, Enginoglu S., Soft set theory and uni-int decision making, Eur. J. Oper. Res., 207(2010), 848-855.

[8] Cagman N., C¸ ıtak F., Aktas¸ H., Soft int-groups and its applications to group theory, Neural Comput. Appl., DOI: 10.1007/s00521-011-0752-x.

[9] Cagman N., Sezgin A., Atagun A.O., Soft uni-groups and its applications to group theory, (submitted).

[10] Cagman N., Sezgin A., Atagun A.O., α-inclusions and their applications to group theory, (submitted).

[11] Feng F., Jun Y.B., Zhao X., Soft semirings, Comput. Math. Appl., 56(2008), 2621–2628.

[12] Feng F., Liu X.Y., Leoreanu-Fotea V., Jun Y.B., Soft sets and soft rough sets, Inform. Sci., 181(6)(2011), 1125-1137.

[13] Feng F., Li C., Davvaz B., Ali M.I., Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Comput., 14(6)(2010), 899-911.

[14] Jun Y.B., Soft BCK/BCI-algebras, Comput. Math. Appl., 56(2008), 1408 -1413.

[15] Jun Y.B., Park C.H., Applications of soft sets in ideal theory of BCK/ BCI-algebras, Inform. Sci., 178(2008), 2466-2475.

[16] Jun Y.B., Lee K.J., Zhan J., Soft p-ideals of soft BCI-algebras, Comput. Math. Appl., 58(2009), 2060-2068.

[17] Jun Y.B., Lee K.J., Park C.H., Soft set theory applied to ideals in d-algebras, Comput. Math. Appl., 57(3)(2009), 367-378.

[18] Kazancı O., Yılmaz S¸., Yamak S., Soft sets and soft BCH-algebras, Hacet. J. Math. Stat., 39(2)(2010), 205-217.

[19] Maji P.K., Biswas R., Roy A.R., Soft set theory, Comput. Math. Appl., 45(2003), 555-562. [20] Maji P.K., Roy A.R., Biswas R., An application of soft sets in a decision making problem, Comput. Math. Appl., 44(2002), 1077-1083.

[21] Majumdar P., Samanta S.K., On soft mappings, Comput. Math. Appl., 60 (9)(2010), 2666-2672.

[22] Molodtsov D., Soft set theory-first results, Comput. Math. Appl., 37(1999), 19-31.

[23] Molodtsov D.A., Leonov V.Yu., Kovkov D.V., Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya, 1(1)(2006), 8-39.

[24] Pilz G., Near-rings, North Holland Publishing Company, Amsterdam-New York-Oxford, 1983.

[25] Sezgin A., Atagun A.O., Aygun E., A note on soft near-rings and idealistic soft near-rings, Filomat., 25(1)(2011), 53-68.

[26] Sezgin A., Atagun A.O., On operations of soft sets, Comput. Math. Appl., 61(5)(2011), 1457-1467.

[27] Wendt G., On Zero Divisors in Near-Rings, Int. J. Algebra, 3(1)(2009), 21-32.

[28] Zhan J., Jun Y.B., Soft BL-algebras based on fuzzy sets, Comput. Math. Appl., 59(6)(2010), 2037-2046.

[29] Zou Y., Xiao Z., Data analysis approaches of soft sets under incomplete information, Knowl-Based Syst., 21(2008), 941-945.