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# UNSTEADY MHD FREE CONVECTION FLOW IN A CASSON NANO FLUID THROUGH POROUS MEDIUM WITH SUCTION AND HEAT SOURCE

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## ABSTRACT

*The unsteady MHD flow of a casson nano fluid past a moving vertical permeable semi infinite flat plate with constant heat source through porous medium is studied theoretically. The temperature is assumed to be fluctuating harmonically with time from a constant mean at the plate. Similarity transformations are used to convert partial differential equations to ordinary differential equation and the solved by using perturbation technique to obtain expressions for velocity field, temperature field, skin friction and rate of heat transfer. The results are discussed in detailed by using graphs and observed the effect of different parameters.*

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## KEYWORDS:

*Casson nanofluid;*

*MHD;*

*Free convection;*

*Porous medium;*

*Suction ;*

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## 1. INTRODUCTION

Due to the vast, variety and complexity applications , Nanofluids play a vital role in the past decade research in convective heat transfer using suspensions of nanometer-sized solid particles is attracted by many researchers. The study of heat and mass transfer of nanofluid flow with effects of chemical reaction has a wide range of applications in chemical industries, petroleum industries, food processing, nuclear reactors and in cooling and drying processes. The term nanofluid carries the metallic particle the heat transfer and

diffusion together is an interesting phenomenon with wide application in the field of base fluid with suspension of metallic nanosized particles. Many authors Buongiorno J , M. A.A. Hamad, I. Pop , Khan, W.A., A. Aziz ,Kuznetsov AV, and Nield DA studied the nanofluid flow and heat transfer effects in various geometries and various effects. In Maxwell or Hamilton and Crosser model models there is no description about the particle diameter or layer around the particle. Many of the researches carried out so far ignored the natural phenomenon of formation of liquid like layer around nano particle because of the chemical reaction with the solvent. The formation of liquid like layer limits the contact of the metallic nano particle with the boundary. This reduces the effective heat transfer. Enhancing thermal conductivity of fluids with nano particles is given by Cho[1]. Tran and Soong[2] explained the preparation of nano fluid using laser ablation in liquid technique. Dae-Hwang Yo and others[3] studied Cheng and e thermal conductivity of nanofluids for the application of heat transfer fluids. Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike was discussed by Cheng and Minkowycz[4]. As a particular case of study on enhanced thermal conductivity of TiO<sub>2</sub>-water based nanofluids studied by Murshed and others[5] The idea of using small particles to collect solar energy was first investigated by Hunt [6] in the 1970s. Masuda and others[7] given the alteration of thermal conductivity and viscosity by dispersion ultra fine particles in the liquid. Choi and Eastman [8] were introduced the concept of nano particles and they have found the way to control heat transfer by using nano particles. Buongiorno [9] developed the non homogeneous equilibrium mathematical model for convective transport of nanofluids. In his concluding remarks Brownian motion and Thermophoretic diffusion of nano particles are the important mechanism for abnormal convective heat transfer enhancement. The related procedure described by Batchelar[10], Walker and others[11] and Pratsinis and Kim[12], Investigations in the nanofluid flows have received remarkable popularity in research community in last couple of decades primarily due to their variety of applications in generation of power , also in transportation where nanofluid can be used in vehicles as coolant, shock absorber, fuel additives etc., in heating and cooling problems which can be involve the usage of nanofluids for cooling of microchips in computer processors , in improving the performance of efficiency of air-conditioners and refrigerant etc. and in Biomedical applications in magnetic nano particles which can be used medicine, cancer therapy and tumor analysis. Otanicar and Golden[13] studied the comparative environmental and economic analysis of conventional and nano fluid solar hot water technologies. Tyagi and others[14] predicted the efficiency of a low

temperature nanofluid based direct absorption solar collector. Hu and Buong[15] identified heat transfer enhancement in nano fluids for nuclear reactors. Humnic G and Humnic A [16] finds different applications in heat exchangers. The effect the effect of Chemical reaction and radiation was studied by Kameswaran et al. (18) examined the Chemical reaction and Soret effect of convective nanofluid flow through a stretching or shrinking sheet in the presence of magnetic field and they concluded that heat and mass transfer rates is inversely proportional to magnetic parameter. Hence it has been widely investigated by many researchers like Mohan Krishna and others [19], Sandeep [20], Haddy [21] T. Hayath etc all[22], Ghaly[23] Saidhur[24] and Mohamed[25].

In recent times, due to increasing industrial applications the flows involving casson nano fluids grab significant attention of modern day researchers. Many materials in real field, like, melts, muds, condensed milk, glues, printing ink, emulsions, soaps, sugar solution, paints, shampoos, tomato paste etc. show properties which differs from those of Newtonian fluids. But, the main difficulty is to construct a single constitutive equation which follows all properties of such casson nano fluids. It also plays a vital role in Nuclear Physics and in Geographical flows. Many researchers have identified different effects on casson nano fluids. The boundary layer flow of casson nano fluid over vertical exponentially stretching cylinder is analysed by Malik and others [28]. Sarojamma and Vendabai [29] studied the boundary layer flow of a casson nanofluid over a vertical exponentially stretching cylinder. The flow of casson nano fluid with viscous dissipation and convective conditions was given by Hussain and others[30]. Sathies kumar and Gangadhar[31] identified the effect of chemical reaction on slip flow of MHD Casson fluid over stretching sheet with heat and mass transfer. A model for flow of casson nanofluid past a nonlinearly stretching sheet considering magnetic field effects was established by Mustafa and Junaid [32]. In an Unsteady casson nanofluid flow over stretching sheet with thermal radiation, convective and slip boundary conditions are imposed by Ibukun Sarah Oyelakin [33] J V Ramana Reddy and others[34] are studied the effect of cross diffusion on non-newtonian fluid flow past a stretching sheet with non uniform heat source/sink. MHD stagnation point flow of casson nanofluid over stretching sheet with effect of viscous dissipation was studied by Srinivasulu and others [35]. Nabil and others[36] explained about the unsteady MHD Non-darcian flow of casson nano fluid between two parallel plates with heat and mass transfer and soret and doufour effects with convective boundary condition given by Imran Ullah[37].

Hence, the object of the present paper is to analyse the unsteady MHD flow of a Casson nano fluid past a moving vertical permeable semi infinite flat plate with constant heat source through porous medium. However, the interactions of the unsteady two-dimensional flow of a non-Newtonian fluid over a stretching surface having a prescribed surface temperature in the presence of radiation and suction/injection is considered. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically using perturbation technique. The influence of velocity, temperature, concentration profile on different governing parameter are analysed graphically.

## 2. MATHEMATICAL FORMULATION

Consider the unsteady free convection flow of a Casson nanofluid past a vertical permeable semi-infinite plate in the presence of an applied magnetic field with suction, variable free stream and heat source. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is [26]

$$\tau = \tau_0 + \mu\sigma$$

(or)

$$\tau = \begin{cases} 2 \left( \mu_B + \frac{p_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\ 2 \left( \mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) e_{ij}, & \pi_c < \pi \end{cases} \quad (1)$$

where  $\tau$  is shear stress,  $\tau_0$  Casson fluid shear stress,  $\mu$  is the dynamic viscosity,  $\sigma$  is the shear rate;  $\pi = e_{ij}e_{ij}$  in which  $e_{ij}$  is the product of the component of deformation rate with itself;  $\pi_c$  is critical value of this product based on non-Newtonian model.  $\mu_B$  is plastic dynamic viscosity of the non-Newtonian fluid,  $p_y$  is the yield stress of fluid. The velocity and temperature are functions of  $y, t$  only. So, if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move.

The  $x'$ -axis is taken along surface in flow direction and  $y'$ -axis is normal to be acting along the  $y$ -axis. the surface. A uniform external magnetic field  $B_0$  is taken to be acting along the  $y$ -axis. We consider that initially at  $t' = 0$  the fluid as well as plate is at rest and temperature  $T'_w$  but for  $t' \geq 0$  the plate has an move at a constant velocity  $U_0$ , and temperature at the plate fluctuates with time harmonically from a constant mean. It is further assumed that the regular fluid and the suspended nano particles are in thermal

equilibrium and no slip occurs between them. Following the Casson nano fluid model proposed by Tiwari and Das [17] along with the Boussinesq and boundary layer approximations, the boundary layer equations governing the flow and temperature are,

$$\frac{\partial v'}{\partial y'} = 0 \quad (2)$$

$$\rho_{nf} \left( \frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = \left( 1 + \frac{1}{\gamma} \right) \mu_{nf} \frac{\partial^2 u'}{\partial y'^2} + (\rho\beta)_{nf} g(T' - T'_{\infty}) - \sigma B_0^2 u' - \frac{u'}{K'} \quad (3)$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \alpha_{nf} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q'}{(\rho c_p)_{nf}} (T' - T'_{\infty}) \quad (4)$$

The initial and boundary conditions are as follows

$$t' < 0, \quad u'(y', t') = 0, \quad T' = T'_{\infty} \text{ at } y' = 0$$

$$t' \geq 0, \quad u'(y', t') = U_0, \quad T' = T'_w + (T'_w - T'_{\infty}) e^{iw't'}$$

at  $y = 0$  (5)

$$u'(y', t') = 0, \quad T' = T'_{\infty} \quad \text{at } y' = \infty$$

where  $T'$  is the local temperature of the nanofluid and  $Q'$  is the additional heat source.  $B_f$  and  $\beta_c$  are the coefficients of thermal expansion of the fluid and of the solid,  $\rho_f$  and  $\rho_c$  are the densities of the fluid and of the solid fraction,  $\gamma$  is Casson parameter, and  $K'$  is the porous parameter.  $\rho_{nf}$ ,  $\mu_{nf}$ ,  $\alpha_{nf}$  and  $(\rho c_p)_{nf}$ , are the density, viscosity, thermal diffusivity, heat capacitance of the nanofluid and these are given by [Abu-Nada et al (27)]

$$\begin{aligned} \rho_{nf} &= (1 - \phi)\rho_f + \rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \\ \alpha_{nf} &= \frac{K_{nf}}{(\rho c_p)_{nf}}, \quad K_{nf} = \frac{K_f + 2K_s - 2\phi(K_f - K_s)}{K_s + 2K_f + 2\phi(K_f - K_s)} \\ (\rho c_p)_{nf} &= (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s, \\ (\rho\beta)_{nf} &= (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \end{aligned} \quad (6)$$

Where  $\phi$  is volume solid fraction of the nanoparticles and  $K_{nf}$  is thermal conductivity of the nanofluid,  $K_f$  and  $K_s$  are the thermal conductivities of the base fluid and the solid.  $\sigma$  is electrical conductivity of the fluid and  $B_0$  is the strength of magnetic field applied in the  $y$  direction.  $c_p$  is the specific heat,

Solving equation (1) we may get,

$$v' = -v_0 \quad (7)$$

where  $v_0$  is the normal velocity at the plate, which is positive for suction ( $v_0 > 0$ ), and negative for blowing or injection ( $v_0 < 0$ ).

Now by introducing non dimensional variables as follows.

$$u = \frac{u'}{U_0}, y = \frac{U_0 y'}{v_f}, t = \frac{U_0^2 t'}{a}, \omega = \frac{v_f \omega'}{U_0^2}, \theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty}, M = \frac{\sigma B_0^2}{\rho_f U_0^2}, S_0 = \frac{V_0}{U_0},$$

$$K = \frac{\rho_f U_0^2 k'}{v_f}, Q = \frac{Q' v_f^2}{k_f U_0^2}, P_r = \frac{v_f}{\alpha_f}, G_r = \frac{(\rho\beta)_{nf} g v_f}{\rho_f U_0^2} (T'_\omega - T'_\infty) \quad (8)$$

In view of equation (8), equations (3) and (4) reduces , the dimensionless governing equations together with the appropriate boundary conditions can be written as

$$A \left( \frac{\partial u}{\partial t} - S \frac{\partial u}{\partial y} \right) = D \frac{\partial^2 u}{\partial y^2} - \left( M + \frac{1}{K} \right) u + B G_r \theta \quad (9)$$

$$C \left( \frac{\partial \theta}{\partial t} - S \frac{\partial \theta}{\partial y} \right) = \frac{1}{P_r} \left( E \frac{\partial^2 \theta}{\partial y^2} - Q \theta \right) \quad (10)$$

the dimensionless boundary conditions are

$$t' < 0, \quad u = 0, \quad \theta = 0 \quad \text{at } y = 0,$$

$$t' \geq 0, \quad u = 1, \theta = 1 + \epsilon e^{i\omega t} \quad \text{at } y = 0, \quad u = 0, \theta = 0 \quad \text{at } y = \infty \quad (11)$$

### 3. MATHEMATICAL FORMULATION

Equations (12) to (14) are solved by series method by considering the following solution

$$u(y, t) = u_0(y) + \epsilon u_1(y) e^{i\omega t}$$

$$\theta(y, t) = \theta_0(y) + \epsilon \theta_1(y) e^{i\omega t} \quad (12)$$

Substituting (16) into the equations (12) to (14), equating the harmonic and non-harmonic terms, we obtain

$$D \frac{\partial^2 u_1}{\partial y^2} + A S \frac{\partial u_1}{\partial y} - \left( M + \frac{1}{K} + i A \omega \right) u_1 = -B G_r \theta_1 \quad (13)$$

$$D \frac{\partial^2 u_0}{\partial y^2} + A S \frac{\partial u_0}{\partial y} - \left( M + \frac{1}{K} \right) u_0 = -B G_r \theta_0 \quad (14)$$

$$E \frac{\partial^2 \theta_1}{\partial y^2} + C S P_r \frac{\partial \theta_1}{\partial y} - (Q + i c P_r \omega) \theta_1 = 0 \quad (15)$$

$$E \frac{\partial^2 \theta_0}{\partial y^2} + C S P_r \frac{\partial \theta_0}{\partial y} - Q \theta_0 = 0 \quad (16)$$

With boundary conditions

$$t' < 0, \quad u_0 = 1, u_1 = 0, \quad \theta_0 = 1, \theta_1 = 1 \quad \text{at } y = 0$$

$$t' \geq 0, \quad u_0 = 0, u_1 = 0, \quad \theta_0 = 0, \theta_1 = 0 \quad \text{at } y = \infty \quad (17)$$

Therefore solution of Eq (13) to (16) with the boundary conditions (17) is

$$u(y, t) = A_2 e^{-m_3 y} + A_1 e^{-m_1 y} + \epsilon (A_4 e^{-m_4 y} + A_3 e^{-m_2 y}) e^{i\omega t}$$

$$\theta(y, t) = e^{-m_1 y} + \epsilon e^{i\omega t} e^{-m_2 y}$$

The skin friction coefficient( $\tau$ ) at the plate is given by

$$\tau = (\partial u / \partial y)_{y=0} = (-m_3 A_2 - m_1 A_1) + \epsilon (-m_4 A_4 - m_2 A_3) e^{i\omega t}$$

The Local Nusselt number (Nu) at  $y=0$  is given by

$$Nu = -(\partial \theta / \partial y)_{y=0} = m_1 + m_2 \epsilon e^{i\omega t}$$

Where  $A = 1 - \phi + \phi \frac{(\rho)_s}{(\rho)_f}$ ,  $B = 1 - \phi + \phi \frac{(\rho\beta)_s}{(\rho\beta)_f}$ ,  $C = 1 - \phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}$ ,

$$D = \left(1 + \frac{1}{\gamma}\right) \frac{1}{(1 - \phi)^{2s}}, \quad E = 1 - \phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}$$

$$m_1 = \frac{scp_r + \sqrt{(scp_r)^2 + 4EQ}}{2E}$$

$$m_2 = \frac{scp_r + \sqrt{(scp_r)^2 + 4(icwp_r + Q)}}{2E}$$

$$m_3 = \frac{As + \sqrt{A^2 s^2 + 4D \left(M + \frac{1}{K}\right)}}{2D}$$

$$m_4 = \frac{As + \sqrt{A^2 s^2 + 4D \left(iAw + M + \frac{1}{K}\right)}}{2D}$$

$$A_1 = \frac{G_r B}{Dm_1^2 - Asm_1 - \left(M + \frac{1}{K}\right)}, \quad A_2 = 1 - A_1$$

$$A_3 = \frac{G_r B}{Dm_2^2 - Asm_2 - \left(iAw + M + \frac{1}{K}\right)}, \quad A_4 = -A_3$$

### 3. RESULTS AND ANALYSIS

Fig 1 and 2 shows the velocity and temperature profiles for different values of Casson parameter. For large amount of Casson parameter, yield stress decreases so that the casson fluid behave like a Newtonian fluid and hence velocity profile decreases and temperature profile increases. We have identified this on both the nano fluid copper mixed water and aluminium oxide mixed water. Fig 3 and 4 shows the velocity and temperature profiles for different values of magnetic parameter. As magnetic parameter increases, the Lorentz force will also increases and it suppress the flow of the fluid, hence the velocity profile decreases. But this is reversible incase of temperature profile. As magnetic parameter increases, the Lorentz force will also increases and it convert some thermal energy to heat energy and hence the velocity profile increases. At the sheet it is

considerable but away from the sheet is negligible Fig 5 shows the velocity profiles for different values of Grashof number. As Grashof number increases the velocity profile increases. At the sheet it is considerable but away from the sheet is negligible. Fig 6 shows the velocity profiles for different values of Heat source parameter. As heat source parameter increases the velocity profile decreases. Fig 7 shows the velocity profiles for different values of permeability parameter. As permeability parameter increases the velocity profile increases. Fig 8 shows the velocity profiles for different values of suction parameter. As suction parameter increases the velocity profile increases. Fig 9 shows the velocity profiles for different values of solid volume fraction parameter. As solid volume fraction parameter increases the velocity profile increases.

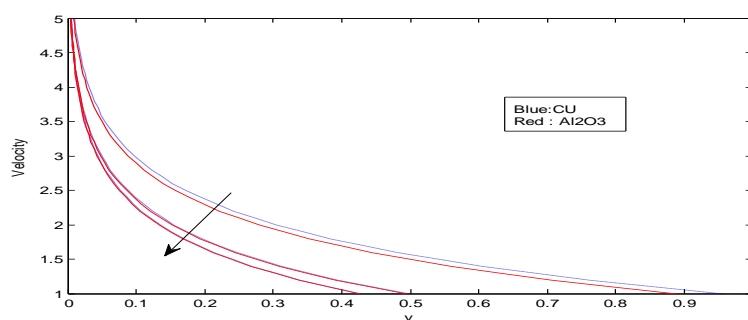


Figure 1 Velocity profile for different values of Casson parameter  $\gamma$

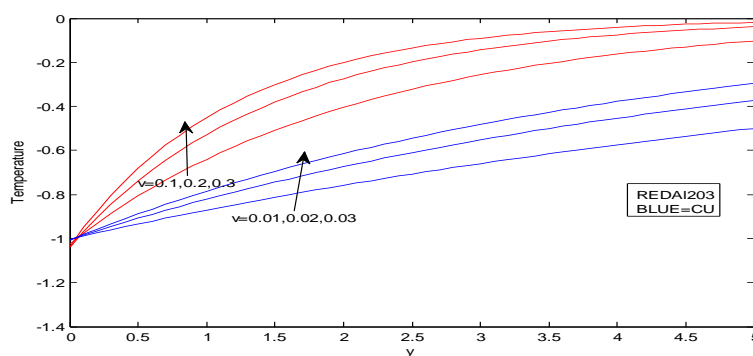


Figure 2 Temperature profile for different values of Casson parameter  $\gamma$

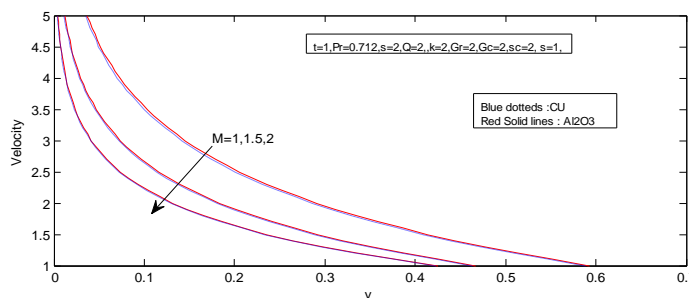




Figure 3 Velocity profile for different values of  $M$

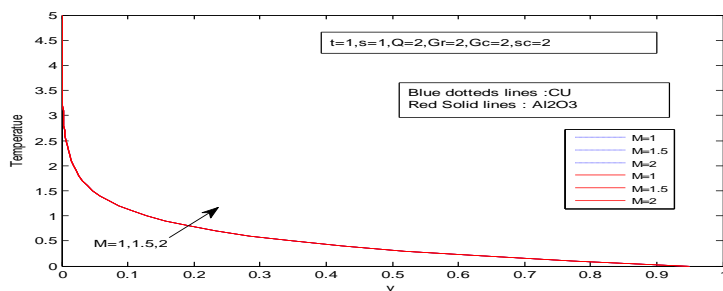


Figure 4 Temperature profile for different values of magnetic parameter  $M$

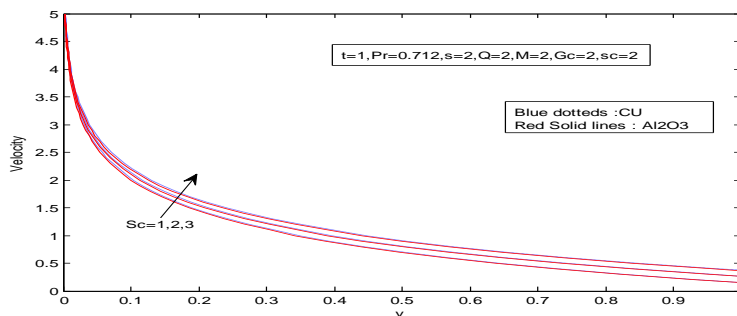


Figure 5 Velocity profile for different values of  $Gr$

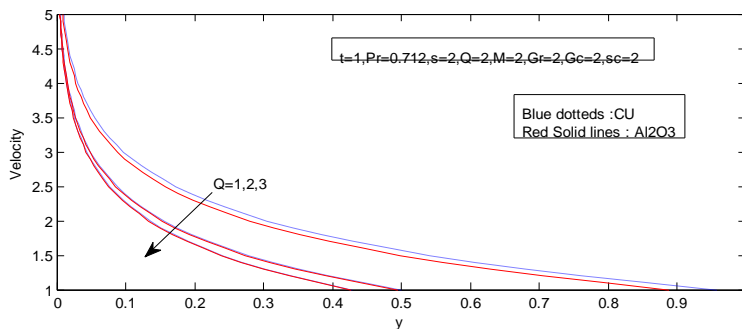


Figure 6 Velocity profile for different values of Heat source parameter  $Q$

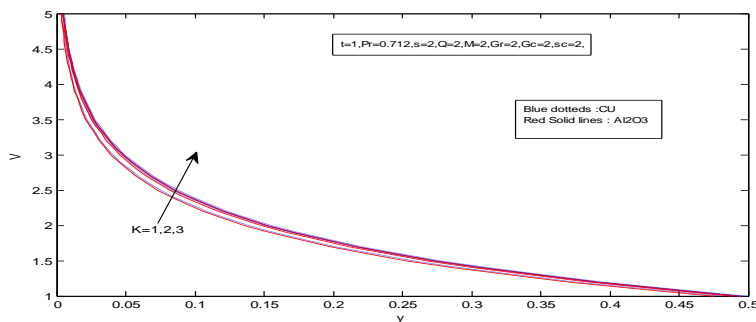


Figure 7 Velocity profile for different values of porous parameter  $K$

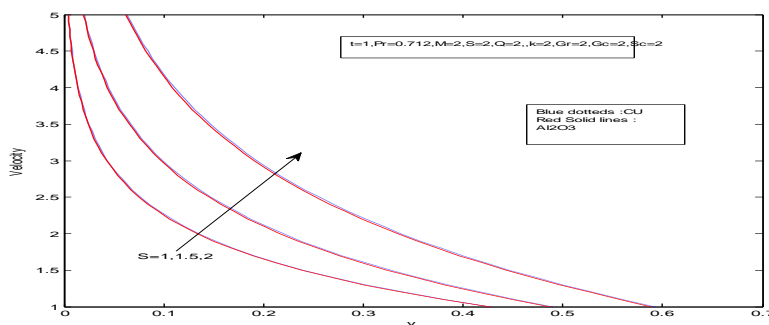


Figure 8 Velocity profile for different values of suction parameter  $S$

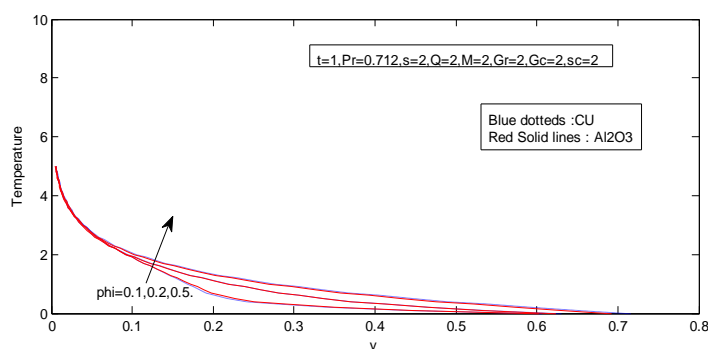


Figure 9 Temperature profile for different values of  $\phi$

#### 4. CONCLUSION

In this paper, the study of unsteady MHD flow of a Casson nano fluid past a moving vertical permeable semi infinite flat plate with constant heat source through porous medium with suction/injection was investigated. The resulting governing equations were solved by the regular multiple perturbation technique. From the study, the following remarks can be summarized.

1. The effect of increasing values of the Casson parameter is to suppress the velocity field. But the temperature is enhanced with increasing Casson parameter in case of both copper water fluid and Titanium oxide.
2. As magnetic parameter increases the velocity profile decreases where the temperature profile increases in case of both copper water fluid and Titanium oxide.
3. Both the momentum and thermal boundary layer thickness decreases in an increasing suction/injection parameter.
4. Magnitude of skin-friction reduces, but the rate of heat transfer increases with raising the suction/injection parameter.

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