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"MODELLING OF LOCAL STABILITY ANALYSIS CONSISTING ONE DIFFERENTIAL AND TWO DIFFERENTIAL EQUATIONS FOR THE ANALYSIS OF CONTINUOUS TIME MODELS TOWARDS SPECIFIC APPLICATIONS"

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ABSTRACT

Keywords:

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In general, there is a prediction that all the dynamic systems are exhibiting multi-dimensional aspects, irrespective of the various fields of engineering science and technology. Most probably, the variations in dynamic behavior of any model may influence the behavioral pattern in terms of its own terminology for any technical features and advancements. In this context, a viable theoretical modelling is proposed for the analysis of continuous time models addressing the issues of "local stability "consisting of one dimensional as well as two-dimensional differential equations by considering the fundamentals of simple dynamic pattern, geometric growth, oscillations, attractors, geometric decay, damped oscillation and divergent oscillation. The basic concepts of equilibrium and stability for continuous time models can be presented in terms of one/two differential equations for any particular applications. a particular continuous time model ,initially *proposed by accommodating "general theory" for the system may be influencing many parameters involved in the functioning of any operating systems in terms of mechanically, chemically, electrically and electronically also, out of which fields which are composed of mechanical elements are involved more "oscillations" and "vibrations". It is necessary to exhibit the analogy of the locally linear methods developed for discrete-time-problems, in particular for any one of the application with "dynamic patterns".*

It is predicted that the local stability analysis of multiple differential equation systems is rather more mathematically demanding than that of the rest of the models meant for the analysis of continuous –time approaches.

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1. INTRODUCTION

In order to measure the complexities in dynamic behavior of any physical system, it is very essential to develop a model for predicting the changing behaviour of continuous models as a track record of behavioural pattern with respect to logistic model and many models which are influencing the factors for a realistic approach. In this context, a viable option is made for further stochastic model as a primary level investigation for capturing the complex strategies in continuous time model, in particular for vibration and oscillation to the maximum possible extent.

1.The logistic model:

It is commonly used to describe the resource limited population growth, relates the rate of change of population, to its concurrent size X, therefore,

$$\frac{dX}{dt} = G(X)$$

$$= r(X)(1 -)\frac{X}{K} ----------------------[1]$$

The equilibrium of this system (X^*), are those values of X which makes the rate of change of X equal to "0".

Hence it must satisfy,

$$G(X^*) = rX^*\left(\frac{1 - X^*}{K}\right) = 0 ------------------------[2]$$

This is true either, if $X^* = 0$, or if $\frac{X^*}{K} = 1$.

Hence the system has two equilibrium, which are termed as "extinct" (X_e^*) and "viable" (X_V^*) respectively,

Where, $X_e^* = 0$, and $X_V^* = K$ ------------------------[3]

For this rather simple model, it is necessary to determine he long-term fate of the system by direct inspection of many dynamic equations including equation [2]

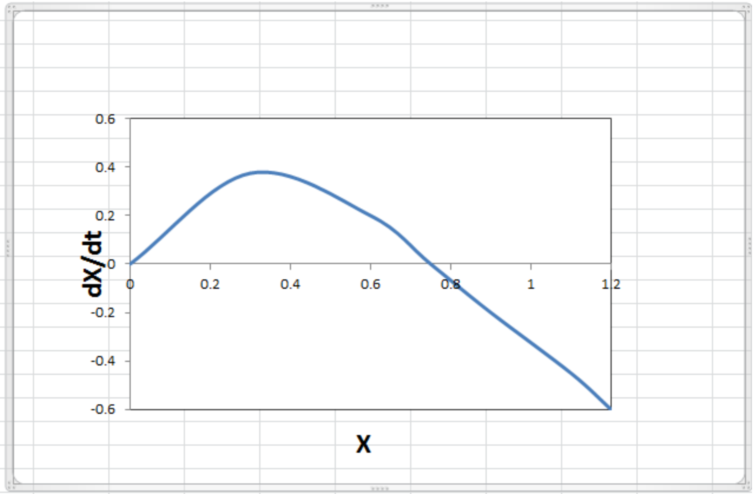

Fig 1: Population growth rate, $\frac{dX}{dt}$ against X for the logistic model with r=2.0 and K=1.0

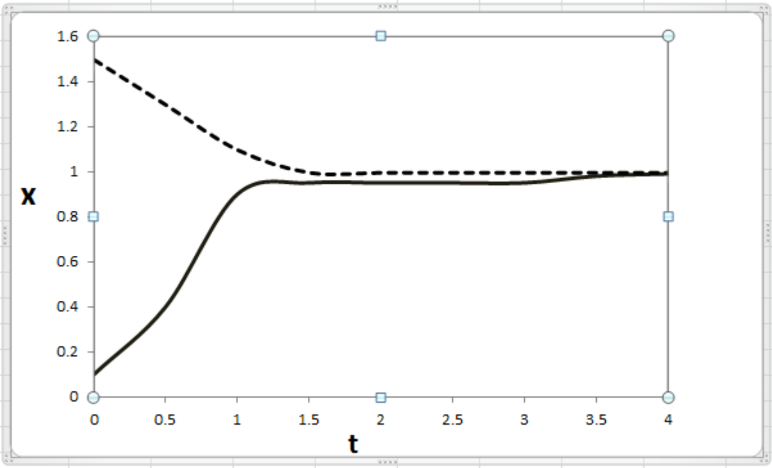

Fig 2: Typical numerical solution

As it is shown in the figure $\frac{dX}{dt}$ is positive for all values of X between “0” and “K”, and negative for all values greater than K. Hence, if we start a sum with X infinitesimally above zero, it will increase as shown in the figure by the continuous line. The “extinct” equilibrium is therefore clearly and promptly unstable.

In any run started with 0<X(0)<K, X will be initially increasing. However , an X approaches K the rate of increase diminishes- by reaching exactly zero, when X=K thus, any run started with a value of X between “0” and “K” will eventually settle to the models as well as viable equilibrium values(K). Whereas, suppose, if we start a run with X(0)>K, the $\frac{dX}{dt}$ is initially negative ,so “X” initially decreases, as shown by the dotted lines in the figure , but as “X” gets smaller the rate of reduction dimensions and incidentally or eventually X→K. Therefore, it is predicted that, the “viable” equilibrium is thus “globally stable” for constrained attributes to the maximum possible extent.

1.Local stability analysis (one differential equation):

The “analysis by inspection” carried out for the logistic model can be extended to any model defined a single differential equation, all that needed is to constant the equilibrium. However, such techniques do not readily generalize to system with more than one state variable, for which we need the strategy and analogy of the local linear methods , we develop for especially the discrete-time-model related problems, of course as a preliminary, it is necessary to show , how those techniques operates for a one-variable system.

Let us consider a general, one-variable system, defined by a dynamic equation,

$$\frac{dX}{dt} = G(x) ----------------------------------[4]$$

Where equilibrium starts X*mm, therefore satisfy,

$$G(X^*) = 0 ------------------------------------[5]$$

We can represent small derivation from equilibrium by,

$$X(t) \equiv X(t)\text{-}X^* ---------------------------[6]$$

And it is noted that $\frac{dx}{dt} = \frac{dX}{dt}$ and remembering that G(X*)=0, and using equation

$$F(X_0 + x) \approx FX_0 + X\left(\frac{dF}{dx}\right)X = X_0$$

To appropriate G(X*+x), we can state the dynamics of the deviation as,

$$\frac{dx}{dt} = \lambda x$$

Where $\lambda = \frac{dG}{dX}_{X=X^*}$ ---------------------------------------[7]

This is considered as a equation for definite, explicit exponential growth or decline as well, i.e. $\frac{dX}{dt} = rX$, i.e. geometric growth as decline is not simply a properly of the model with a single state variable. Indeed, any dynamic model whose balance equations or update lines are strictly "linear"vin the state variable normally exhibits this so as of behavior, in line with many suspicious predictions in the analogy forever.

The solution can be,

$$x(t) = x(0)e^{\lambda t} -------------------------------[8]$$

We can notice that the equilibrium is stable, if the quantity "λ" (the eigenvalue),is negative and unstable if it is positive.

By applying this analysis to the generic as well as logistic model, we can express,

$$\frac{dG}{dx} = r - \frac{2rX}{K} ------------------------------[9]$$

We can obtain the eigen value, for the two equilibrium ($\lambda_e and \lambda_V$) by substituting $X = X_e^*, X_V^*$,as appropriate to find,

$$\lambda_e = r, \lambda_V = -r, ------------------------------[10]$$

Since"η" is positive, the extent equilibrium is locally unstable and the vaiable equilibrium is locally unstable in agreement with one previous experiments meant for the same investigation as well to maximum possible extent predominantly.

1.Local stability analysis (two differential equation):

The local stability analysis of multiple differential equation systems is rather more mathematically demanding than the rest of the homogeneous stochastic models. At this situation, we can trace the argument for a two variable system, to understand the process at work and it becomes formation with the changes of possible outcomes.

We can deal with the general pay-prediction or resume-consume system where specified dynamics as specified a pair of differential equations,

$$\frac{dX}{dt} = G_1(X,Y) \frac{dy}{dt} = G_2(X,Y) ------------------------[11]$$

The equilibrium states of this system (X^*, Y^*) are considered as the simultaneous solution of,

$$G_1(X^*, Y^*) = 0 , G_2(X^*, Y^*) = 0 ----------------------- [12]$$

The following expression can lead to identify the importance of local stability in two-differential equation,

$$x = X - X^*$$

$$y = Y - Y^*$$

$$\frac{dx}{dt} \approx a_1 x + a_2 y , \quad \frac{dy}{dt} \approx a_3 x + a_4 y ------------------- [13]$$

$$\lambda^2 + A_1\lambda + A_2\lambda = 0 ----------------------------[14]$$

$$A_1 = -(a_1 + a_4)A_2 = a_1a_4 - a_2a_3 -----------------[15]$$

$$\lambda_1 = \frac{1}{2}\left(-A_1 + \sqrt{A_1^2 + 4A_2}\right), \lambda_2 = \frac{1}{2}\left(-A_1 - \sqrt{A_1^2 - 4A_2}\right) ---------[16]$$

$$\lambda_1 = \frac{1}{2}(-A_1 + iw), \qquad \lambda_2 = \frac{1}{2}(-A_1 - iw)$$

$$\omega = \left(\sqrt{4A_2 - A_1^2}\right) ---------------------------[17]$$

1.CONCLUSION:

Normally, an extension version of basic logistic model is proposed for complexities in a continuous-time model for accommodating the constraints of any models mathematically towards homogeneity of such equations.

$$x(t) \propto \frac{e^{\lambda_1 t} + e^{\lambda_2 t}}{2} = e^{-A_1 t}\cos(wt)$$

$$x(t) = \left(e^{\lambda_1 t} + e^{\lambda_2 t}\right)\frac{1}{2} = e^{-A_1 t}\sin(wt) -----------------[18]$$

Comparing this result with equations,

$X(t) = \bar{X} + A[\frac{e^{\lambda_1 t} + e^{\lambda_2 t}}{2}]$, then it shows that; if $A_1>0$, the derivation again decays, but this time it does so oscillating. If $A_1>0$, the small derivatives grows oscillatory. We can conclude that therules relating the behavior of small derivation to the coefficients in the characteristic equation, therefore

$A_1>0$, $4A_2<A_1^2$ → Exponential decay

$A_1<0$, $4A_2<A_1^2$ →Exponential growth

$A_1>0$, $4A_2>A_1^2$ →Damped osillation

$A_1<0$, $4A_2>A_1^2$ →Divergent osillation

Thus two variable differential equation systems are capable of exhibiting all the dynamic behaviours observed as single variable discrete time system and it can be extended for many complex analysis of mechanical, electrical and electronics application under uncertainty for a specific period of time interval by accommodating the provision for instantaneous adjustments. However the basic logic model influences the local stability analysis for accommodating the changes in the higher order terms of initializing the variable and constraints. Therefore dynamic behaviours of discrete system must be considered with suitable boundary conditions, in particular with Drichlet conditions.

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