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ECCENTRIC CONNECTIVITY POLYNOMIAL OF WHEEL RELATED GRAPHS

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ABSTRACT

Let G = (V(G), E(G)) be a graph. The eccentric connectivity polynomial of G is defined as $EC(G,x) = \sum_{i=1}^{n} d_G(v_i).x^{\varepsilon_G(v_i)}$ where the eccentricity $\varepsilon_G(v_i)$ for a given vertex v_i of V(G), is the largest distance from v_i to any other vertices of G. In this paper, I present the eccentric connectivity polynomial of wheel related graphs namely closed helm, double wheel, flower graph, sunflower graph and a graph by sharing one vertex of wheel W_n with the path of length n.

KEYWORDS:

eccentric connectivity polynomial;
closed helm; double wheel;
flower;
sunflower.

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1. INTRODUCTION

Throughout this paper all graphs are assumed to be simple, finite and connected. For basic graph theoretical terminology I refer [4]. For a simple connected graph G = (V(G), E(G)), with n vertices and m edges, the distance between the vertices v_i and v_j of V(G), is

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equal to the length that is the number of edges of the shortest path connecting v_i and v_j. Also for a given vertex v_i of V(G) its eccentricity $\varepsilon_G(v_i)$ is the largest distance from v_i to any other vertices of G. The eccentric connectivity polynomial [2] of G is defined as $EC(G,x) = \sum_{i=1}^{n} d_G(v_i).x^{\varepsilon_G(v_i)}$. I refer the reader to [1],[3],[5] for explicit formulas for the eccentric connectivity polynomial of various families of graphs. The maximum and minimum eccentricity of all the vertices of G is called the diameter and radius of G, and is denoted by D(G) and r(G) respectively. The average eccentricity of a graph G is denoted by *ece(G)* and is defined as $ece(G) = \frac{1}{n}\sum_{i=1}^{n}\varepsilon_G(v_i)$. A *wheel graph* W_n is a graph obtained by joining all vertices of a cycle C_n to an external vertex. This external vertex may be called the central vertex of W_n and the cycle C_n may be called the rim of W_n. That is $W_n = C_n + K_1$. A *double wheel* graph DW_n is a graph defined by $2C_n + K_1$. That is, a double wheel graph is a graph obtained by joining all vertices of the two disjoint cycles to an external vertex. A *helm graph* H_n is a graph obtained by attaching a pendant edge to every vertex of the rim C_n of a wheel graph W_n. A *closed helm graph* CH_n is a graph obtained from the helm graph H_n, by joining a pendant vertex v_i to the pendant vertex v_{i+1}, where $1 \le i \le n$ and $v_{n+i} = v_i$. That is, the pendant vertices in H_n induce a cycle in CH_n. A *flower graph* Fl_n is a graph which is obtained by joining the pendant vertices of a helm graph H_n to its central vertex. A *sunflower graph* SF_n is a graph obtained by replacing each edge of the rim of a wheel graph W_n by a triangle such that two triangles share a common vertex if and only if the corresponding edges in W_n are adjacent in W_n.

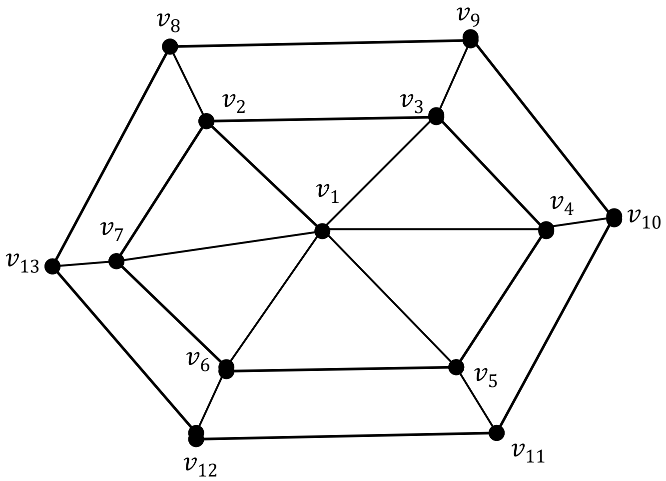
2. MAIN RESULTS

In this paper, I discuss the eccentric connectivity polynomial of wheel related graph families.

Theorem 2.1. The eccentric connectivity polynomial of closed helm graph CH_n is $EC(CH_n, x) = 7nx^3 + nx^2$ for $n \ge 4$.

Proof. The graph CH_n has 2n + 1 vertices and 4n edges with average eccentricity $ece(CH_n)$ $= \frac{6n+2}{2n+1}$. Here D(G) = 3 and r(G) = 2. In the closed helm graph CH_n, n vertices has eccentricity 3 with degree 4, n vertices has eccentricity 3 with degree 3, 1 vertex has eccentricity 2 with degree n. Hence the eccentric connectivity polynomial of CH_n is $EC(CH_n, x) = 7nx^3 + nx^2$. This is true for all $n \ge 4$.

Ill ustration 2.2.



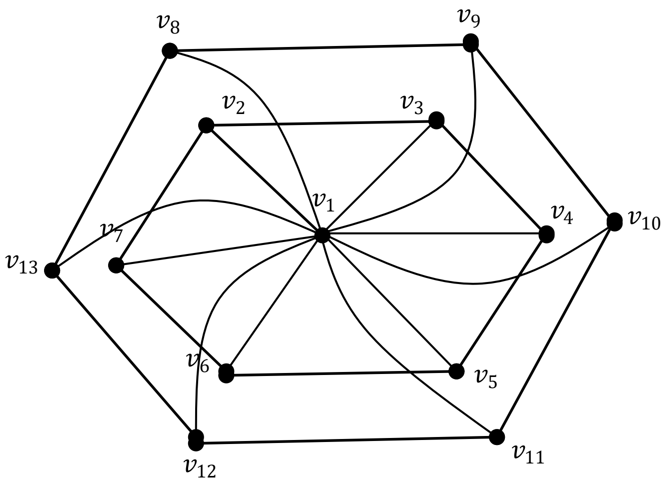
closed helm graph CH_6

Figure 2.1

Theorem 2.3. The eccentric connectivity polynomial of double wheel graph DW_n with $D(G) = 2$ and $r(G) = 1$ is $EC(DW_n, x) = 6nx^2 + 2nx$ for $n \geq 3$.

Proof. The graph DW_n has $2n + 1$ vertices and $4n$ edges with $ece(DW_n) = \frac{4n+1}{2n+1}$. In DW_n, $2n$ vertices has eccentricity 2 with degree 3, 1 vertex has eccentricity 1 with degree $2n$. Hence the eccentric connectivity polynomial of DW_n is $EC(DW_n, x) = 6nx^2 + 2nx$. This is true for all $n \geq 3$.

Illustration 2.4.



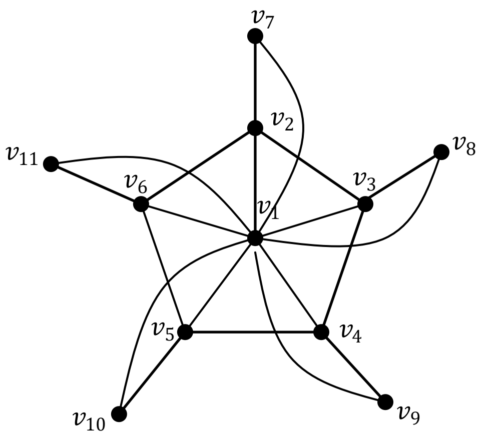
double wheel graph DW_6

Figure 2.2

Theorem 2.5. The eccentric connectivity polynomial of flower graph Fl_n is $EC(Fl_n, x) = 8nx^2 + 2nx$ for $n \geq 3$.

Proof. The graph Fl_n has $2n + 1$ vertices and $4n$ edges with $ece(Fl_n) = \frac{4n+1}{2n+1}$. Here $D(G) = 2$ and $r(G) = 1$. In Fl_n, $2n$ vertices has eccentricity 2 with degree 4, 1 vertex has eccentricity 1 with degree 2n. Hence the eccentric connectivity polynomial of Fl_n is $EC(Fl_n, x) = 8nx^2 + 2nx$. This is true for all $n \geq 3$.

Illustration 2.6.



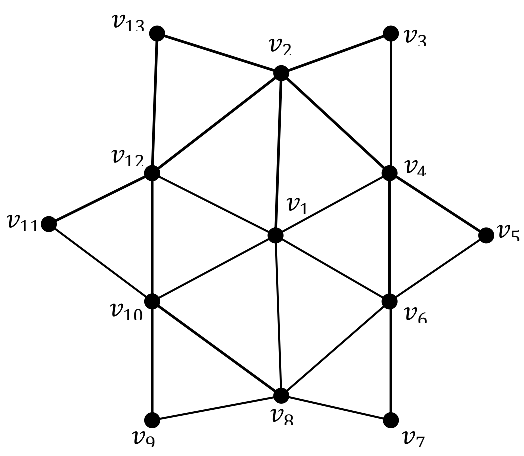
flower graph FL_5

Figure 2.3

Theorem 2.7. The eccentric connectivity polynomial of sunflower graph SF_n with $D(G) = 4$ and $r(G) = 2$ is $EC(SF_n, x) = nx^2 + 5nx^3 + 2nx^4$ for $n \geq 6$.

Proof. The graph SF_n has $2n + 1$ vertices and $4n$ edges with $ece(SF_n) = \frac{7n+2}{2n+1}$. In SF_n, n vertices has eccentricity 3 with degree 5, n vertices has eccentricity 4 with degree 2 and 1 vertex has eccentricity 2 with degree n. Hence the eccentric connectivity polynomial of SF_n is $EC(SF_n, x) = nx^2 + 5nx^3 + 2nx^4$. This is true for all $n \geq 6$.

Illustration 2.8.



sunflower graph SF_6

Figure 2.4

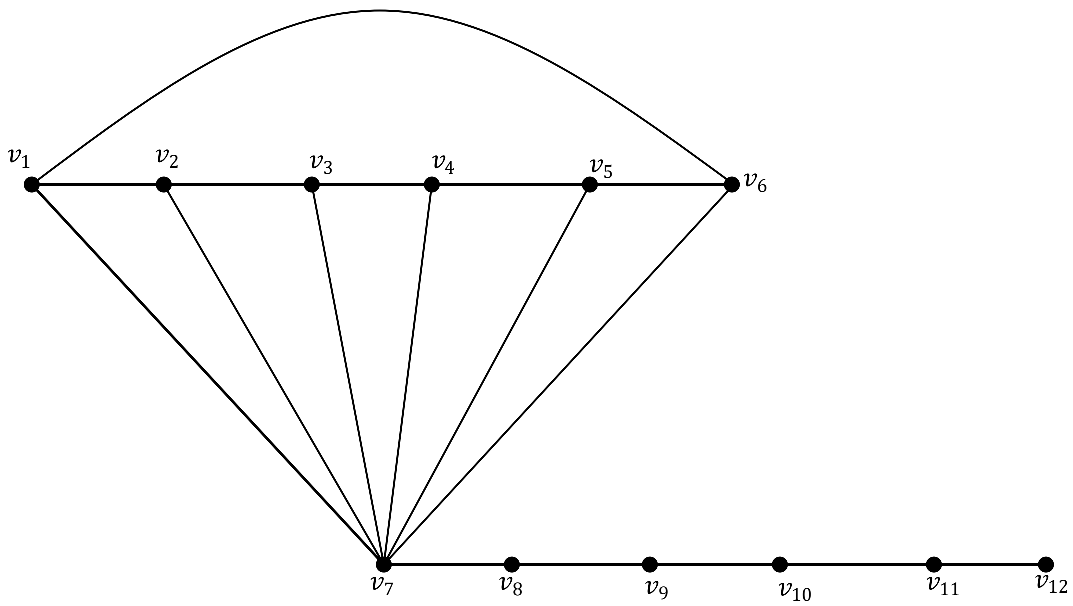
Theorem 2.9. The eccentric connectivity polynomial of a graph obtained by sharing one vertex of wheel W_n with the path of length n with diameter $D(G) = n$ and radius $r(G) = n - 1 - \left\lfloor \frac{n-2}{2} \right\rfloor$, $ece(WP_n) = 4\left[n+1+\left\lfloor \frac{n-2}{2} \right\rfloor\right]$ for odd n and $4\left[n+1+\left\lfloor \frac{n-3}{2} \right\rfloor\right]$ for even n, is given by

$$EC(WP_n, x) = \begin{cases} (3n+1)x^n + (n+3)x^{n-1} + \sum_{k=1}^{\left\lfloor \frac{n-2}{2} \right\rfloor} 4x^{n-1-k} & if\ n = 7,9,... \\ (3n+1)x^n + (n+3)x^{n-1} + \sum_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} 4x^{n-1-k} + 2x^{\left\lfloor \frac{n}{2} \right\rfloor} & if\ n = 6,8,10,... \end{cases}$$

Proof. The graph WP_n has 2n vertices and 3(n – 1) + 2 edges. For odd n ≥ 7, n vertices has eccentricity n with degree 3 and 1 vertex has eccentricity n with degree 1 and thus we obtained $(3n + 1)x^n$. Also 1 vertex has eccentricity n – 1 with degree n + 1, further one vertex has eccentricity (n – 1) with degree 2. Hence we obtained $(n + 3)x^{n-1}$. Likewise, 1 vertex has eccentricity n – 1 – k with degree 4, for k = 1, 2, 3, …, $\left\lfloor \frac{n-2}{2} \right\rfloor$. The proof is similar for n is even, n ≥ 6. Hence the eccentric connectivity polynomial of WP_n is

$$EC(WP_n, x) = \begin{cases} (3n+1)x^n + (n+3)x^{n-1} + \sum_{k=1}^{\left\lfloor \frac{n-2}{2} \right\rfloor} 4x^{n-1-k} & if\ n = 7,9,... \\ (3n+1)x^n + (n+3)x^{n-1} + \sum_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} 4x^{n-1-k} + 2x^{\left\lfloor \frac{n}{2} \right\rfloor} & if\ n = 6,8,10,... \end{cases}$$

Illustration 2.10.



W_6 with the path of length 6 attached to one vertex

Figure 2.5

3. CONCLUSION

Thus in the paper eccentric connectivity polynomial of wheel related graph families have been studied.

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