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FIXED CHARGE MULTI-OBJECTIVE TRANSPORTATION PROBLEM AND ITS SOLUTION BY GREY SITUATION DECISION MAKING THEORY

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ABSTRACT

The fixed charge problem is an important application of the class of mixed integer programming problems, which form the basis of many business and industry problems wherein a charge is associated with performing an activity at a nonzero level which does not depend on the level of the activity. The fixed charge can also be expressed as the set up time of getting a machine into service or the fixed elapsed time before a certain operation can commence. The fixed charge problem has been widely applied in many decision-making and optimization problems like the warehouse or plant location decisions, wherein there is a charge associated with opening the facility; and in transportation problems where there are fixed charges for transporting goods between supply points and demand points. Now a day different approach is available for find solution of fixed charge multi-objective transportation problem. In this paper we suggest grey situation decision making theory based method using membership function for find efficient solution of fixed charge multi-objective transportation problem with help of numerical example, the proposed method is illustrated.

KEYWORDS:

Fixed charge multi-objective transportation problem;
Grey situation decision making theory;
Membership function;
Efficient Solution.

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1. INTRODUCTION

It is noted that in the transportation problem (TP),the cost of transportation problem is directly associated with the amount of commodity to be transported in which a fixed cost, sometimes called a set up cost, is also incurred when a distribution variable assume a positive value is called as fixed charge transportation problem (FCTP). FCTP is an extension of classical transportation problem was originally formulated by Hirsch and Danzig [1]. It may be simply stated as distribution problem in which m suppliers or warehouses or factories and n customers or destinations are to be considered. Each of the m suppliers can ship to any customer with shipping cost per unit of c_{ij} from supplier i to customer j plus a fixed cost of f_{ij} assessed for opening this route. Each supplier,

$i=1,2,.....,m$ has S_i units of supply and each customer, $j=1,2,.....,\text{n}$ demands D_j units. The objective is to determine which routes to be opened and the shipment size so that the total cost of meeting demand, given the supply constraints, is minimized. In real situation many distribution problems can only be modeled as FCTPs viz, rails, roads and trucks use freight rates having a fixed cost and a variable cost. The fixed cost may represent the cost of renting a vehicle, landing fees at airport, set up costs for machines in manufacturing environment etc.

1.1 Mathematical Formulation of Fixed Charge Multi-objective Transportation Problem (FCMOTP):

Suppose there are m origins and n destinations, the quantities of a uniform product available at the origins and required at the destinations are given. The total quantity available at the sources is precisely the same as the total quantity required at the destinations and it is possible to transport to any destination from any origin. In this problem there are k objectives which have to be minimized. The formulation of the problem is as follows:

$$\text{minimize} \sum_{i=1}^{m}\sum_{j=i}^{n}(\xi_{ij}{}^{l}x_{ij}+\eta_{ij}{}^{l}y_{ij});l=1,2,....,k, \qquad (1)$$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} \le a_i, i=1,2,.....,m,$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, j=1,2,.....,\text{n},$$

$$x_{ij} \ge 0, \forall i, j$$

$$where\ y_{ij}=0 \text{ if } x_{ij}=0 \text{ and } y_{ij}=1 \text{ if } x_{ij}>0.$$

$\xi_{ij}{}^{l}$= the units of cost of transportation of one unit of the product from origin i to destination j corresponding to k objectives i.e. $l=1,2,....,k$, $\eta_{ij}{}^{l}$= fixed cost of transportation of one unit of the product from origin i to destination j corresponding to k objectives i.e. $l=1,2,....,k$, a_i = the units of the product available at origin i, b_j = the units of the product required at destination j, x_{ij} = the number of units of the product transported from origin i to destination j.

1.2 LITERATURE REVIEW:

The fixed charge transportation problem was originally formulated by Dantzig and Hirsch [1] (1954). Then Murty [2] (1968) solved the fixed charge problem by ranking the extreme points. After that several procedures for solving Fixed charge transportation problems were developed. Also Basu*et.al.* [3],[4] (1994) developed an algorithm for the optimum time-cost trade-off in a fixed charge linear transportation problem giving same priority to cost and time. The fixed-charge transportation problem (FCTP) is an extension of the classical transportation problem in which a fixed cost is incurred, independent of the amount transported, along with a variable cost that is proportional to the amount shipped. The introduction of fixed costs in addition to variable costs results in the objective function being a step function. Therefore, fixed-charge problem are usually solved using sophisticated analytical or computer software. The transportation problem considered in the classical transportation problem is generally a two-dimensional linear transportation problem. After fixed charge bi-criteria transportation problem Thirwani et al. [5] review this algorithm and the gives the algorithm on Fixed charge bi-criteria transportation problem restricted flow is introduced. Fixed charge bi-criteria transportation problem with restricted flow which is an extension of the fixed charge bi-criteria transportation problem. In this type of problem, there is a restriction on the total flow. In the fixed charge bi-criterion transportation problem a fixed cost called the set up cost is incurred for every origin. Till date the methods obtained to solve the FCP are mainly approximate methods developed by Cooper [6], Murty [2], Cooper and Drebes [7], Walker [8] which are all based on adjacent extreme point algorithms. For small problems, Steinberg [9] provided an exact method based on the branch and bound method.Adlakha and Kowalski [10] use the Balinski approximation method introduced for the fixed charge transportation problem and apply the same for solving the fixed charge problem.

In this chapter we have developed grey situation decision making theory based technique to solve multi-objective fixed charge transportation problems and their solution.

2. Grey situation decision making theory and Fuzzy Programming Technique based approach to find solution of FCMOTP:

Consider some notations to define the variables and the sets in fixed charge multi-objective transportation problem.

Let $O=\{O_1,O_2,.......,O_m\}$ be the set of m -origins having a_i $\left(i = 1, 2,....,m\right)$ units of supply respectively. Let $D=\{D_1,D_2,.....,D_n\}$ be the set of n -destinations with $b_j(j=1,2,....,n)$ units of requirement respectively and $f_{ij}=$ fixed cost of the product transported from origin i to destination j . Let p_{ij} be the penalty associated with transporting a unit of product from i^{th} source to j^{th} destination. It may be the cost or delivery time or safety of delivery etc. x_{ij} represents the amount of quantity to be shipped from i^{th} source to j^{th} destination. The problem is to determine the transportation schedule when multiple objectives exist.

To optimize the total transportation penalty satisfying supply and demand conditions, Grey situation decision making theory is used.

If the set of m -origins $O=\{O_1,O_2,.......,O_m\}$ as the set of events, the set of n destinations $D=\{D_1,D_2,.....,D_n\}$ as the set of countermeasure, the penalty p_{ij} as the situation set denotes by $P=\{p_{ij}=(O_i,D_j)/O_i\in O,D_j\in D\}$

First of all confirm the decision making goals (objectives) and seek the corresponding effect measure matrix $U^{(k)}$ as, $U^{(k)}=\left[u_{ij}^{(k)}\right]=\begin{bmatrix} u_{11}^{(k)} & u_{12}^{(k)} & ...... & u_{1m}^{(k)} \\ u_{21}^{(k)} & u_{22}^{(k)} & ...... & u_{2m}^{(k)} \\ ...... & ...... & ...... & ...... \\ u_{n1}^{(k)} & u_{n2}^{(k)} & ...... & u_{nm}^{(k)} \end{bmatrix}$,

Where $u_{ij}^{(k)}=C_{ij}=c_{ij}+\dfrac{f_{ij}}{m_{ij}}$ where $m_{ij}=\min(a_i,b_j),\forall i,j,$ f_{ij} is fixed cost

Here, the data of decision making goals for transporting a product is the effect value $u_{ij}^{(k)}$ of situation $p_{ij}\in P$ with objective $k = 1, 2,....,s.$

Now, find the upper effect measure and lower effect measure by formula:

Upper effect measure

$$r_{ij}^{(k)}=u_{ij}^{(k)}\Big/\max_i\max_j\left\{u_{ij}^{(k)}\right\},$$

Lower effect measure
$r_{ij}^{(k)} = \min_i \min_j \left\{ u_{ij}^{(k)} \right\} \Big/ u_{ij}^{(k)}$,

And achieve the consistent matrix of effect measure $R^{(k)}$ by using upper effect measure or lower effect measure as

$$R^{(k)} = \left[r_{ij}^{(k)} \right] = \begin{bmatrix} r_{11}^{(k)} & r_{12}^{(k)} & \ldots\ldots & r_{1m}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \ldots\ldots & r_{2m}^{(k)} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ r_{n1}^{(k)} & r_{n2}^{(k)} & \ldots\ldots & r_{nm}^{(k)} \end{bmatrix}.$$

Subtract each data of comprehensive matrix $R^{(k)}$ of effect measure from 1 to convert combine maximization objective in minimization form. This will give us solution of multi-objective transportation problem with fixed charge.It can be obtained by using any standard technique.
In fuzzy programming technique, first find the lower bound as L_k and the upper bound as U_k for the K^{th} objective function $Z_k, k = 1, 2, \ldots\ldots, K$ where U_k is the highest acceptable level of achievement for objective k , L_k the aspired level of achievement for objective k and $d_k = U_k - L_k$ the degradation allowance for objective k .

When the aspiration levels for each of the objective have been specified, a fuzzy model is formed and it is converted into a crisp model. Here, we first utilized Grey situation decision making theory to find the lower effect measure $r_{ij}^{(k)}$ and upper effect measure $r_{ij}^{(k)}$ and accomplish the consistent matrix of effect measure $R^{(k)} = [r_{ij}^{(k)}]$ for each objective k. These matrices of each objective are utilized as a cost matrix of each objective in fuzzy programming technique and solution are obtained. So here Grey situation decision making theory is utilized for normalization of data.

The solution of FCMOTP can be obtained by the following steps:

Step-1:.Find the lower effect measure $r_{ij}^{(k)}$ and upper effect measure $r_{ij}^{(k)}$ and accomplish the consistent matrix of effect measure $R^{(k)} = [r_{ij}^{(k)}]$ for each objective k by using

$$u_{ij}^{(k)} = C_{ij} = c_{ij} + \frac{f_{ij}}{m_{ij}} \text{ ,where } m_{ij} = \min(a_i, b_j), \text{for } i = 1, 2, \ldots.., m \text{ and } j = 1, 2, \ldots.., n \quad .$$

f_{ij} is fixed cost

Step-2: Solve the single-objective transportation problem K times with consistent matrix of effect measure $R^{(k)} = [r_{ij}^{(k)}]$ by taking one objective at a time

Step-3: Determined the corresponding values for every objective at each solution derived. According to each solution and value for every objective,
The pay-off matrix can be found as follows:

Table 1: Pay-off matrix of FCMOTP

	$z_1(x)$	$z_2(x)$	......	$z_K(x)$
$X^{(1)}$	z_{11}	z_{12}	......	z_{1K}
$X^{(2)}$	z_{21}	z_{22}	......	z_{2K}
...	...	...	......	...
$X^{(k)}$	z_{k1}	z_{k2}	......	z_{kK}

Where, $X^{(1)}, X^{(2)}, \ldots\ldots, X^{(k)}$ are the isolated optimal solutions of the K different transportation problems for K different objective function

Step- 4: Define a membership function $\mu(Z_k)$ for the k^{th} objective function.

Step -5: Convert the MOTP of the problem, obtained in step 3, into the following crisp Model;

Maximize λ,

Subject to the constraints

$$\lambda \le \mu(Z_k),$$

$$\sum_{j=1}^{n} X_{ij} = a_i \ , \ i = 1, 2, \ldots\ldots, m.$$

$$\sum_{i=1}^{m} X_{ij} = b_j \ , j = 1, 2, \ldots\ldots, n.$$

$$X_{ij} \ge 0, \forall i, j. \quad \lambda \ge 0.$$

If we use a linear membership function, the crisp model can be simplified as:

Maximize λ

Subject to the constraints

$$Z_k + \lambda(U_k - L_k) \le U_k, k = 1, 2, \ldots.., K.$$

$$\sum_{j=1}^{n} X_{ij} = a_i \ , \ i = 1, 2, \ldots\ldots, m.$$

$$\sum_{i=1}^{m} X_{ij} = b_j \ , j = 1, 2, \ldots\ldots, n.$$

$$X_{ij} \ge 0, \forall i, j. \quad \lambda \ge 0.$$

If we will use the hyperbolic membership function then an equivalent crisp model for

the fuzzy model can be formulated as:

Maximize λ,

Subject to the constraints

$$\lambda \le \frac{1}{2} \frac{e^{\left\{\left(\frac{U_k + L_k}{2}\right) - Z_k(X)\right\}a_k} - e^{-\left\{\left(\frac{U_k + L_k}{2}\right) - Z_k(X)\right\}a_k}}{e^{\left\{\left(\frac{U_k + L_k}{2}\right) - Z_k(X)\right\}a_k} + e^{-\left\{\left(\frac{U_k + L_k}{2}\right) - Z_k(X)\right\}a_k}} + \frac{1}{2} \ if \ k = 1, 2, \ldots.., K.$$

$$where \ a_k = \frac{6}{U_k - L_k}.$$

$$\sum_{j=1}^{n} X_{ij} = a_i \ , \ i = 1, 2, \ldots\ldots, m.$$

$$\sum_{i=1}^{m} X_{ij} = b_j \ , j = 1, 2, \ldots\ldots, n.$$

$$X_{ij} \geq 0, \forall i, j. \quad \lambda \geq 0.$$

Step-6:. Solve the crisp model by an appropriate mathematical programming algorithm

Step-7:.The solution obtained in step -6 will be the compromise solution of the FCMOTP.

2.1 Algorithm for finding solution of fixed charge MOTP with membership function using MGSD theory:

Input
Output
Solution of FCMOTP
Compute the efficient solution of FMOTP using the optimization model of objective weight.
Solve MOTP
begin
Step-1 Read: problem
while problem = FCMOTP do,
for k=1 to s do,
enter effect measure matrix $U^{(k)}$

$$u_{ij}^{(k)} = C_{ij} = c_{ij} + \frac{f_{ij}}{m_{ij}}, \text{where } m_{ij} = \min(a_i, b_j),$$

for $i = 1, 2, \ldots.., m$ and $j = 1, 2, \ldots.., n$

f_{ij} is fixed cost

end

Step-2 Find the lower effect measure $r_{ij}^{(k)}$ and upper effect measure $r_{ij}^{(k)}$ and accomplish the consistent matrix of effect measure $R^{(k)} = [r_{ij}^{(k)}]$.

for k=1 to s do

$$r_{ij}^{(k)} = \min_i \min_j \left\{u_{ij}^{(k)}\right\} \Big/ u_{ij}^{(k)},$$

$$r_{ij}^{(k)} = u_{ij}^{(k)} \Big/ \max_i \max_j \left\{u_{ij}^{(k)}\right\},$$

$$R^{(k)} = \left[r_{ij}^{(k)}\right] = \begin{bmatrix} r_{11}^{(k)} & r_{12}^{(k)} & \ldots\ldots & r_{1m}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \ldots\ldots & r_{2m}^{(k)} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ r_{n1}^{(k)} & r_{n2}^{(k)} & \ldots\ldots & r_{nm}^{(k)} \end{bmatrix},$$

end

Step-3: Convert combine maximization objective in minimization form for all objectives.
Step-4: Find optimal solution to each objective by using simplex method.
Step-5: Find pay off matrix by using each objective solution.
Step-6: Define linear as well as hyperbolic membership function using payoff matrix.
Step-7: Developed single objective transportation problem using fuzzy linear membership function and hyperbolic function.
Step-8: Solve model developed in step-4 and find compromise solution.

2.2 NUMERICAL ILLUSTRATIONS:

To illustrate the efficiency of the proposed method,consider numericalillustration-1presented in [11]:

Numerical illustration: 1 (from [11])

Consider the multi objective fixed charge transportation problem:

$$\left.\begin{array}{l} min\ z = (z_1, z_2), \\ \text{Subject to} \\ x_{11} + x_{12} + x_{13} = 80, \\ x_{21} + x_{22} + x_{23} = 30, \\ x_{11} + x_{21} = 40, \\ x_{12} + x_{22} = 50, \\ x_{13} + x_{23} = 20, \\ x_{ij} \geq 0 \text{ for } i = 1,2 \text{ and } j = 1,2,3. \\ y_{ij} = 0 \text{ if } x_{ij} = 0, \\ y_{ij} = 1 \text{ if } x_{ij} > 0. \\ \text{where } Z_k = \sum_{i=1}^{2}\sum_{j=i}^{3}(c_{ijk}x_{ij} + f_{ijk}y_{ij}), k \in K = \{1,2\} \end{array}\right\} \quad (2)$$

The direct costs c_{ijk} and the fixed charges f_{ijk} for the two objectives Z_1 and Z_2 are given below:

$$c_{ij1} = \begin{bmatrix} 7 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}, c_{ij2} = \begin{bmatrix} 2 & 3 & 7 \\ 3 & 6 & 6 \end{bmatrix}, f_{ij1} = \begin{bmatrix} 20 & 24 & 21 \\ 19 & 20 & 25 \end{bmatrix}, f_{ij2} = \begin{bmatrix} 20 & 23 & 18 \\ 16 & 15 & 20 \end{bmatrix}.$$

Solution:

(1) Using formula $C_{ij} = c_{ij} + \frac{f_{ij}}{m_{ij}}$, we have

$$U^{(1)} = \begin{bmatrix} 7.5 & 1.48 & 2.05 \\ 3.6333 & 4.6667 & 2.25 \end{bmatrix}, U^{(2)} = \begin{bmatrix} 2.5 & 3.46 & 7.9 \\ 3.5333 & 6.5 & 7 \end{bmatrix}.$$

(2) For transporting a product, goals are less than its the batter, so use lower effect measure. So the lower effect measure for first data $r_{11}^{(1)} = \frac{\min_1 \min_1 \{u_{11}\}}{u_{11}} = \frac{1.48}{7.5}$

Similarly, obtain lower effect measure for each data. Therefore the consistent matrices of effect measure and Subtract each data from 1 to convert combine maximization objective in minimization form therefore,we have values

$$R^{(1)} = \begin{bmatrix} 0.8027 & 0 & 0.278 \\ 0.3807 & 0.6829 & 0.0889 \end{bmatrix}, \mathrm{R}^{(2)} = \begin{bmatrix} 0 & 0.2775 & 0.6835 \\ 0.2924 & 0.4677 & 0.4952 \end{bmatrix}.$$

(3) Find solutions for each objective of multi-objective transportation problem from consistent matrix of effect measure using simplex method

For first objective: The optimal allocations are

$x_{11}=10, x_{12}=50, x_{13}=20, x_{21}=30,$

Apply these allocations to first and second objective therefore we have

$$Z_1\left(X^1\right)=25.008, Z_2\left(X^1\right)=36.317$$

For second objective:

$x_{11}=40, x_{12}=40, x_{22}=10, x_{23}=20,$

Apply these allocations to first and second objective therefore we have

$$Z_1\left(X^2\right)=40.715, Z_2\left(X^2\right)=25.681$$

$$\text{Pay-off matrix} = \begin{bmatrix} 25.008 & 36.317 \\ 40.175 & 25.681 \end{bmatrix}$$

$$U_1 = \max(25.008, 40.715) = 40.715,$$
$$L_1 = \min(25.008, 40.715) = 25.008,$$
$$U_1 - L_1 = 15.707, U_1 + L_1 = 65.723$$
$$U_2 = \max(36.317, 25.681) = 36.317,$$
$$L_2 = \min(36.317, 25.681) = 25.681,$$
$$U_2 - L_2 = 10.636, U_2 + L_2 = 61.998$$

Applying fuzzy linear membership function, we get Solution of this illustration,

The optimal allocations are:

$x_{11}=26.66018, x_{12}=50, x_{13}=3.339816, x_{21}=13.339816, x_{23}=16.66018.$

Using these allocations we have $U^{(1)}=366.75093, U^{(2)}=429.7898283.$

With degree of satisfaction $\lambda=0.7529664$.

Applying fuzzy hyperbolic membership function, we get Solution of this illustration **(where** $a_1=a_2=1$**):**

The optimal allocations are:

$x_{11}=26.66018, x_{12}=50, x_{13}=3.339816, x_{21}=13.339816, x_{23}=16.66018.$

Using these allocations we have $U^{(1)}=366.75093, U^{(2)}=429.7898283.$

With degree of satisfaction $\lambda=0.95416$

Table 2: Comparison of Grey situation decision making theory approach with other approaches:

membership function	Grey situation decision making theory approach	Another approach
Linear	Solution: $Z_1=366.75093, Z_2=429.7898283.$ With degree of satisfaction $\lambda=0.7529664$.	A compromise method for solving fuzzy multi objective fixed charge transportation problem[11]: $Z_1=373, Z_2=439.$
Hyperbolic	Solution: $Z_1=366.75093, Z_2=429.7898283.$ With degree of satisfaction $\lambda=0.95416$	A compromise method for solving fuzzy multi objective fixed charge transportation problem[11]: $Z_1=373, Z_2=439.$

Numerical illustration: 2 (from [11])
Consider the multi objective fixed charge transportation problem:
Constraints same as equation no. (2) InNumerical illustration: 1
The direct costs c_{ijk} and the fixed charges f_{ijk} for the two objectives Z_1 and Z_2 are given below:

$$c_{ij1}=\begin{bmatrix}8&5&3\\6&8&4\end{bmatrix}, c_{ij2}=\begin{bmatrix}4&7&9\\7&9&9\end{bmatrix}, f_{ij1}=\begin{bmatrix}22&25&23\\21&27&26\end{bmatrix}, f_{ij2}=\begin{bmatrix}22&25&20\\18&15&21\end{bmatrix}.$$

SOLUTION:

(1) Using formula $C_{ij}=c_{ij}+\dfrac{f_{ij}}{m_{ij}}$,we have

$$U^{(1)}=\begin{bmatrix}8.55&5.5&4.15\\6.7&8.9&5.3\end{bmatrix}, U^{(2)}=\begin{bmatrix}4.55&7.5&10\\7.6&9.5&10.05\end{bmatrix}$$

(2) For transporting a product, goals are less than its the batter, so use lower effect measure. So the lower effect measure for first data $r_{11}^{(1)}=\dfrac{\min\limits_{1}\min\limits_{1}\{u_{11}\}}{u_{11}}=\dfrac{4.15}{8.55}$

Similarly, obtain lower effect measure for each data. Therefore the consistent matrices of effect measure and Subtract each data from 1 to convert combine maximization objective in minimization form therefore we have values

$$R^{(1)}=\begin{bmatrix}0.5146&0.2455&0\\0.209&0.4045&0.217\end{bmatrix}, \mathrm{R}^{(2)}=\begin{bmatrix}0&0.3933&0.545\\0.4013&0.2105&0.2438\end{bmatrix}$$

(3) Find solutions for multi-objective transportation problem from consistent matrix of effect measure.

For first objective: The optimal allocations are

$x_{11}=10, x_{12}=50, x_{13}=20, x_{21}=30,$

Apply these allocations to first and second objective therefore we have

$$Z_1\left(X^1\right)=23.691, Z_2\left(X^1\right)=42.604$$

For second objective: The optimal allocations are

$x_{11}=40, x_{12}=40, x_{22}=10, x_{23}=20,$

Apply these allocations to first and second objective therefore we have

$Z_1\left(X^2\right)=38.789, Z_2\left(X^2\right)=22.713$

$$\text{Pay-off matrix}=\begin{bmatrix}23.691&42.604\\38.789&22.713\end{bmatrix}$$

$$U_1=\max(38.789, 23.691)=38.789$$
$$L_1=\min\left(38.789, 23.691\right)=23.691,$$
$$U_1-L_1=15.098, U_1+L_1=62.48$$
$$U_2=\max\left(42.604, 22.713\right)=42.604,$$
$$L_2=\min\left(42.604, 22.713\right)=22.713,$$
$$U_2-L_2=19.891, U_2+L_2=65.317$$

Applying fuzzy linear membership function, we get Solution of this illustration,

The optimal allocations are:

$x_{11} = 24.29974, x_{12} = 50, x_{13} = 5.700259, x_{21} = 15.7002, x_{23} = 14.29974.$

Using these allocations we have $U^{(1)} = 687.39881, U^{(2)} = 805.600314.$

With degree of satisfaction $\lambda = 0.5050308.$

Applying fuzzy hyperbolic membership function, we get Solution of this illustration **(where** $a_1 = a_2 = 1$**):**

The optimal allocations are:

$x_{11} = 24.29974, x_{12} = 50, x_{13} = 5.700259, x_{21} = 15.7002, x_{23} = 14.29974.$

Using these allocations we have $U^{(1)} = 687.39881, U^{(2)} = 805.600314.$

Table 3: Comparison of Grey situation decision making theory approach with other approaches:

membership function	Grey situation decision making theory approach	Another approach
Linear	Solution: $Z_1 = 687.39881, Z_2 = 805.600314$ With degree of satisfaction $\lambda = 0.5050308.$	A compromise method for solving fuzzy multi objective fixed charge transportation problem[11]: $Z_1 = 724, Z_2 = 746.$
Hyperbolic	Solution: $Z_1 = 687.39881, Z_2 = 805.600314$ With degree of satisfaction $\lambda = 0.515088.$	A compromise method for solving fuzzy multi objective fixed charge transportation problem[11]: $Z_1 = 724, Z_2 = 746.$

Numerical illustration: 3 (from [11])

Consider the multi objective fixed charge transportation problem

Constraints same as equation no. (2) In Numerical illustration: 1

The direct costs c_{ijk} and the fixed charges f_{ijk} for the two objectives Z_1 and Z_2 are given below:

$$c_{ij1} = \begin{bmatrix} 9 & 11 & 5 \\ 7 & 15 & 9 \end{bmatrix}, c_{ij2} = \begin{bmatrix} 6 & 10 & 12 \\ 8 & 14 & 11 \end{bmatrix}, f_{ij1} = \begin{bmatrix} 27 & 26 & 25 \\ 28 & 29 & 27 \end{bmatrix}, f_{ij2} = \begin{bmatrix} 28 & 26 & 22 \\ 25 & 18 & 24 \end{bmatrix}.$$

Solution:

(1) Using formula $C_{ij} = c_{ij} + \frac{f_{ij}}{m_{ij}}$, we have

$$U^{(1)} = \begin{bmatrix} 9.675 & 11.56 & 6.25 \\ 7.9333 & 15.9667 & 10.35 \end{bmatrix}, U^{(2)} = \begin{bmatrix} 6.7 & 10.52 & 13.1 \\ 8.8333 & 14.6 & 12.2 \end{bmatrix}$$

(2) For transporting a product, goals are less than its the batter, so use lower effect measure. So the lower effect measure for first data $r_{11}^{(1)} = \frac{\min_1 \min_1 \{u_{11}\}}{u_{11}} = \frac{6.25}{9.675}$

Similarly obtain lower effect measure for each data. Therefore the consistent matrices of effect measure and Subtract each data from 1 to convert combine maximization objective in minimization form therefore we have values

$$R^{(1)}=\begin{bmatrix}0.354 & 0.4593 & 0\\ 0 & 0.5031 & 0.3961\end{bmatrix}, R^{(2)}=\begin{bmatrix}0 & 0.3631 & 0.4885\\ 0.2451 & 0.395 & 0.276\end{bmatrix}.$$

(3) Find solutions for multi-objective transportation problem from consistent matrixof effect measure

For first objective: The optimal allocations are

$$x_{11}=10, x_{12}=50, x_{13}=20, x_{21}=30,$$

Apply these allocations to first and second objective therefore we have

$$Z_1\left(X^1\right)=26.505, Z_2\left(X^1\right)=35.17$$

For second objective: The optimal allocations are

$$x_{11}=40, x_{12}=40, x_{22}=10, x_{23}=20,$$

Apply these allocations to first and second objective therefore we have

$$Z_1\left(X^2\right)=45.485, Z_2\left(X^2\right)=23.994$$

$$Pay-off\ \ matrix=\begin{bmatrix}26.505 & 35.170\\ 45.485 & 23.994\end{bmatrix},$$

$U_1=\max(26.505, 45.485)=45.485$

$L_1=\min\left(26.505, 45.485\right)=26.505,$

$U_1-L_1=18.980, U_1+L_1=71.990$

$U_2=\max\left(35.170, 23.994\right)=35.170,$

$L_2=\min\left(35.170, 23.994\right)=23.994,$

$U_2-L_2=11.176, U_2+L_2=59.164$

Applying fuzzy linear membership function, we get Solution of this illustration,

The optimal allocations are:

$$x_{11}=22.54779, x_{12}=50, x_{13}=7.452207, x_{21}=17.45221, x_{23}=12.54779.$$

Using these allocations we have $U^{(1)}=1111.0494, U^{(2)}=1081.937749.$

With degree of satisfaction $\lambda=0.5041043.$

Applying fuzzy hyperbolic membership function, we get Solution of this illustration **(where $a_1=a_2=1$):**

The optimal allocations are:

$$x_{11}=22.54779, x_{12}=50, x_{13}=7.452207, x_{21}=17.45221, x_{23}=12.54779.$$

Using these allocations we have $U^{(1)}=1111.0494, U^{(2)}=1081.937749.$

With degree of satisfaction $\lambda=0.51231$.

Table 4: Comparison of Grey situation decision making theory approach with other approaches:

membership function	Grey situation decision making theory approach	Another approach
Linear	Solution: $Z_1=1111.0494, Z_2=1081.937749$ With degree of satisfaction $\lambda=0.5041043.$	A compromise method for solving fuzzy multi objective fixed charge transportation problem[11]: $Z_1=1178, Z_2=1083.$
Hyperbolic	Solution: $Z_1=1111.0494, Z_2=1081.937749$ With degree of satisfaction $\lambda=0.51231$.	A compromise method for solving fuzzy multi objective fixed charge transportation problem[11]: $Z_1=1178, Z_2=1083.$

3. RESULTS AND ANALYSIS:

In above tables 2, 3 and 4,comparison of grey situation decision making theory based approach with other developed approaches. These comparison table shows that the grey situation decision making theory based approach provide very efficient alternative approach to find solution of fixed charge multi-objective transportation problem and one of the benefit of this approach is you can find this solution with less computation and time.

4. CONCLUSION:

Chapter discussed Grey situation decision making theory and fuzzy programming technique based approach to find the solution of fixed charge multi-objective Transportation problem and provide an alternative approach to find the solution FCMOTP as well as other multi objective problems.

REFERENCES:

[1] Hirsch W.M., and Dantzig, G.B., The Fixed Charge Poblem. Naval Researchh Logistics Quartly, 15 (1968) 413-424.

[2] Murty, K.G., (1968), Solving the fixed-charge problem by ranking the extreme points,Operations Research, **16**, 268-279.

[3] Basu, M., Pal, B.B. and Kundu, A., (1994), An algorithm for the optimum time-cost trade-off in fixed-charge bi-criterion transportation problem, Optimization, **40**, 53-68.

[4] Basu, M., Pal, B.B. and Kundu, A., (1994), An algorithm for finding the optimum solution of solid fixed-charge transportation problem, Optimization, **31**, 283-291.

[5] Khurana, Archana, DeepaThirwani, and S. R. Arora. "An algorithm for solving fixed charge bi-criterion indefinite quadratic transportation problem with restricted flow." International Journal of Optimization: Theory, Methods and Applications 1, no. 4 (2009): 367-380.

[6] Cooper L. The fixed charge problem—I: a new heuristic method. Computers & Mathematics with Applications 1975;1:89–95.

[7] Cooper L, Drebes C. An approximate algorithm for the fixed charge problem. Naval Research Logistics Quarterly 1967;14:101–13.

[8] Walker WE. A heuristic adjacent extreme point algorithm for the xed charge problem. Management Science 1976;22:587–96.

[9] Steinberg, D.I., The Fixed Charge Problem. Naval Researchh Logistics Quartly, 17 (1970) 217-236.

[10]VeenaAdlakha,KrysztofKowaski, A simple heuristic for solving small fixed-charge transportation problems,International journal of management science,Omega 31 (2003) 205 – 211.

[11] Ratnesh R. Saxena,MuditaUpmanyu, A compromise method for solving fuzzy multi objective fixed charge transportation problem, ISSN 2008-0050 (Print), ISSN 1927-0097 (Online), Lecture Notes in Management Science (2016) Vol. 8, 8–15.