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VIRUS SPREAD IN AN INTUITIONISTIC FUZZY NETWORK BASED ON LAPLACIAN ENERGY

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ABSTRACT:
In this paper, we are discussed the virus spread in an intuitionistic fuzzy network and we defined the infection rate, curing rate and the sharp epidemic threshold of the virus spread in an intuitionistic fuzzy network and we illustrate these concepts with example.

AMS subject classification:
KEYWORDS: Intuitionistic fuzzy set, Virus spread, Epidemic threshold, Effective spreading rate.

1. INTRODUCTION

In this paper, we are discussed the virus spread in an intuitionistic fuzzy network. Each node(vertex) in this network is either infected or healthy. An infected node can infect its neighbors with an infection rate β, and it is cured with curing rate δ. The ratioFuzzy set was proposed by Lofti A. Zadeh in 1965 [17]. The generalization of fuzzy set, calledintuitionistic fuzzy set was proposed by Atanassov in 1986 [2]. Graph theory was found byLeonhard Euler in 1735 when he considered the "Konigsberg bridges" problem. Graph theoryhaving various applications like data mining, image segmentation, clustering, image capturing, communication networks, coding theory, group theory etc. The link structure of a webpageavailable at the website can be represented by an intuitionistic fuzzy directed graph in whichthe vertices are the links of the webpage available at the website and the directed edge fromvertex A to vertex B exists if and only if A contains a link to B. In 1973, the definition of fuzzygraph was introduced by Kafmann [9] from the Zadeh's fuzzy relations. In 1975, Rosenfeld[14] introduced another detailed definition including fuzzy vertex and fuzzy edges and severalfuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. In 1978,Gutman [7] introduced the energy of a graph as the sum of the absolute values of the eigen valuesof the adjacency matrix of a graph. The lower and upper bound for the energy of a graph are discussed in [5], [11], [8]. In 1999, the first definition of intuitionistic fuzzy graph was proposedby Atanassov [3]. The energy of fuzzy graph and its bounds are discussed in [1]. The energyof an intuitionistic fuzzy graph and its bounds are discussed in [13]. In [13], we defined theadjacency matrix of an intuitionistic fuzzy graph and we obtained the energy of an intuitionisticfuzzy graph. The lower and upper bound for the energy of an intuitionistic fuzzy graph are alsoobtained.

$$\tau = \left(\frac{\beta}{\delta}\right)$$

then virus continue and a nonzero fraction of the nodes are infected, whereas $\tau \leq \tau_c$, the epidemicdies out [15], [16].

This paper is organized as follows. In section 2, we give all the necessary definitions andtheorems related to the energy of an intuitionistic fuzzy graph. In section 3, we define thedefinition of infection rate and curing rate of the epidemic threshold of the virus spread in anintuitionistic fuzzy network. In section 4, we illustrate these concepts with example. In Section
5, we give the conclusion.

2. PRELIMINARIES

2.1. Energy of graph [7]

Definition 2.1. Let $G = (V,E)$ be a simple graph with n vertices and m edges. Let $A = [a_{ij}]$ be the adjacency matrix of G. Energy of a simple graph $G = (V,E)$ with adjacency matrix A isdefined as the sum of absolute values of eigenvalues of A. It is denoted by $E(G)$. That is,

$E(G) = \sum_{i=1}^{n} |\lambda_i|$ where λ_i is an eigen values of A, $i = 1, 2, \ldots, n$.

2.2. Energy of Fuzzy Graph [1]

Definition 2.2. Let V be a nonempty set. A fuzzy subset of V is a function $\sigma : V \to [0, 1]$. σ is called the membership function and $\sigma(v)$ is called the membership of v where $v \in V$. V_1 andV_2 be two nonempty sets and σ_1 and σ_2 be two fuzzy subsets of V_1 and V_2 respectively. Define a fuzzy subset μ of $V_1 \times V_2$ as $\mu(v_i, v_j) \leq \min\{\sigma_1(v_i), \sigma_2(v_j)\}$. Then, μ is called afuzzy relation, from σ_1 to σ_2. Suppose $\sigma_1(x) = 1, \forall x \in V_1$ and $\sigma_1(y) = 1, \forall y \in V_2$. Then$\mu$ is called a fuzzy relation from V_1 into V_2. $\mu(v_i, v_j)$ is interpreted as the strength of relationbetween v_i and v_j. Suppose $V_1 = V_2 = V$ and $\sigma_1 = \sigma_2 = \sigma$. Then μ is called a fuzzy relation on$V$. From the above definitions, it follows that binary relations on crisp sets areParticular cases of fuzzy relations. Let V be a nonempty set and σ be a fuzzy subset of$V$. Letμ be a fuzzy relation on σ. μ is said to be symmetric, if $\mu(v_i, v_j) = \mu(v_j, v_i)$ for $v_i, v_j \in V$. Afuzzy relation can also be expressed by a matrix called fuzzy relation matrix $M = [m_{ij}]$, where $m_{ij} = \mu(v_i, v_j)$.

A fuzzy graph $G = (V, \sigma, \mu)$ is a nonempty set V together with a pair of function $(\sigma, \mu)$where σ is a fuzzy subset of V and μ is a fuzzy relation on σ. Let $M = [m_{ij}]$ be a fuzzy relationmatrix defined on μ. This represents the strength of the relation between the vertices. Hence wehave the following definitions.

The adjacency matrix A of a fuzzy graph $G = (V, \sigma, \mu)$ is a$n \times n$ matrix where $n = |V|$ defined as $A = [a_{ij}]$, where $a_{ij} = \mu(v_i, v_j)$. Note that A becomes the usual adjacency matrixwhen all the nonzero membership values are 1. i.e. when the fuzzy graph becomes a crisp graph.Let $G = (V, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. Energy of G is defined asthe sum of the absolute values of the eigen values of A.

2.3. Energy of an intuitionistic Fuzzy Graph [13]

In [13], we defined the adjacency matrix of an intuitionistic fuzzy graph and we obtained the energy of an intuitionistic fuzzy graph. The link structure of a website could be represented by an intuitionistic fuzzy directed graph. The links are considered as vertices and the path between the links are considered as edges. The weightage of the each edge are considered as the number of visitors (membership value), the number of non-visitors (non - membership value) and drop off (intuitionistic fuzzy index) among that link structure.

Definition 2.3. An intuitionistic fuzzy graph is defined as $G = (V, E, \mu, \gamma)$ where V is the set of vertices and E is the set of edges. μ is a fuzzy membership function defined on $V \times V$ and γ is a fuzzy non - membership function defined on $V \times V$. We denote $\mu(v_i, v_j)$ by μ_{ij} and $\gamma(v_i, v_j)$ by γ_{ij} such that

(i) $0 \le \mu_{ij} + \gamma_{ij} \le 1$

(ii) $0 \le \mu_{ij}, \gamma_{ij}, \pi_{ij} \le 1$, where $\pi_{ij} = 1 - \mu_{ij} - \gamma_{ij}$. Hence $(V \times V, \mu, \gamma)$ is an intuitionistic fuzzy set.

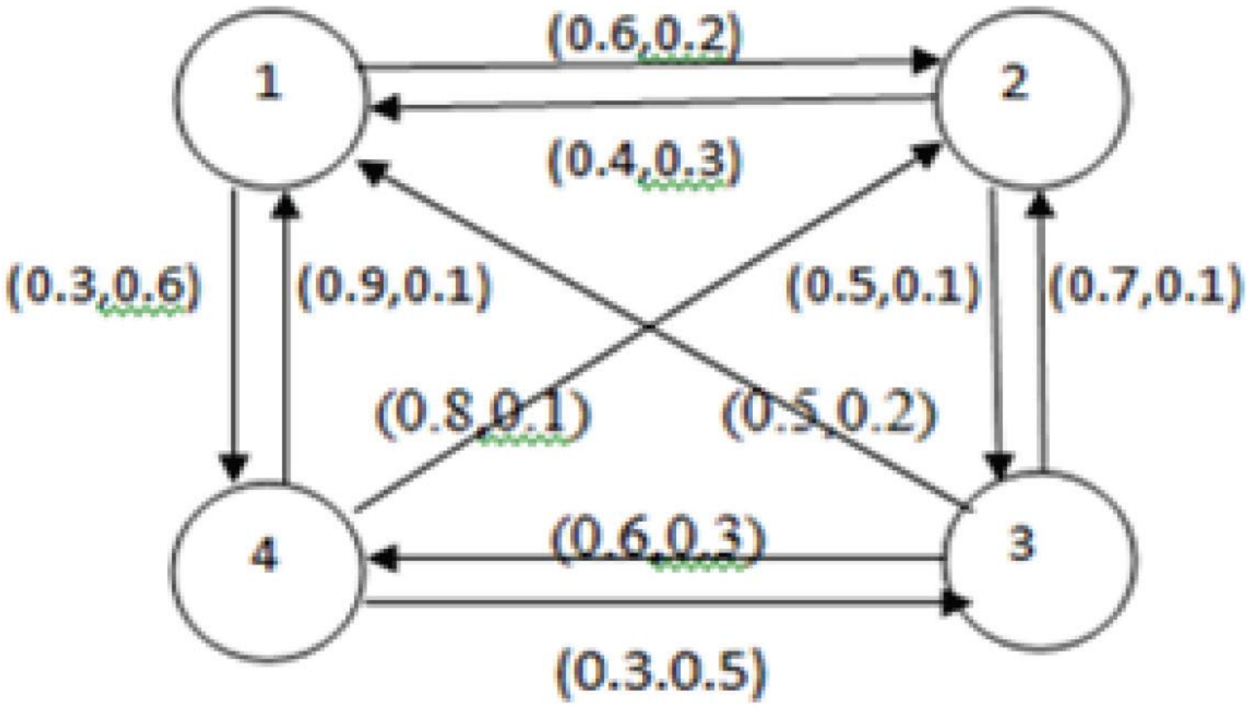

Figure 1: G_1**, An intuitionistic fuzzy graph.**

Definition 2.5. An intuitionistic fuzzy adjacency matrix of an intuitionistic fuzzy graph is definedas the adjacency matrix of the corresponding intuitionistic fuzzy graph. That is for an intuitionisticfuzzy graph $G = (V,E, \mu, \gamma)$, an intuitionistic fuzzy adjacency matrix is defined by $A(IG) = [a_{ij}]$, where $a_{ij} = (\mu_{ij}, \gamma_{ij})$. Note that μ_{ij} represents the strength of the relationship between v_i and v_j and γ_{ij} represents the strength of the non-relationship between v_i and v_j.

Example 2.6. Laplacian Energy of an intuitionistic fuzzy graph in Fig. 1,

The adjacency matrix is $$A\,(IG) = \begin{bmatrix} 0 & (0.6,0.2) & 0 & (0.3,0.6) \\ (0.4,0.3) & 0 & (0.5,0.1) & 0 \\ (0.5,0.2) & (0.7,0.1) & 0 & (0.6,0.3) \\ (0.9,0.1) & (0.8,0.1) & (0.3,0.5) & 0 \end{bmatrix}$$

Definition 2.7. The adjacency matrix of an intuitionistic fuzzy graph can be written as twomatrices one containing the entries as membership values and the other containing the entriesas non-membership values. i.e. $A(IG) = (A(\mu),A(\gamma))$, where $A(\mu) = (\mu_{ij})$ is the matrixrepresenting the membership values and $A(\gamma) = (\gamma_{ij})$ is the matrix representing the non-membershipvalues.

$$A\,(\mu) = \begin{bmatrix} 0 & 0.6 & 0 & 0.3 \\ 0.4 & 0 & 0.5 & 0 \\ 0.5 & 0.7 & 0 & 0.6 \\ 0.9 & 0.8 & 0.3 & 0 \end{bmatrix}$$

$$D\,(\mu) = \begin{bmatrix} 1.8 & 0 & 0 & 0 \\ 0 & 2.1 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

$$L\,(\mu) = D(G) - A(G) = \begin{bmatrix} 1.8 & -0.6 & 0 & -0.3 \\ -0.4 & 2.1 & -0.5 & 0 \\ -0.5 & -0.7 & 0.8 & -0.6 \\ -0.9 & -0.8 & -0.3 & 0.9 \end{bmatrix}$$

and

$$A(\gamma)=\begin{bmatrix}0 & 0.2 & 0 & 0.6\\ 0.3 & 0 & 0.1 & 0\\ 0.2 & 0.1 & 0 & 0.3\\ 0.1 & 0.1 & 0.5 & 0\end{bmatrix} D(\gamma)=\begin{bmatrix}0.6 & 0 & 0 & 0\\ 0 & 0.4 & 0 & 0\\ 0 & 0 & 0.6 & 0\\ 0 & 0 & 0 & 0.9\end{bmatrix}$$

$$L(\gamma)=\begin{bmatrix}0.6 & -0.2 & 0 & -0.6\\ -0.3 & 0.4 & -0.1 & 0\\ -0.2 & -0.1 & 0.6 & -0.3\\ -0.1 & -0.1 & -0.5 & 0.9\end{bmatrix}$$

Definition 2.8. The eigen values of an intuitionistic fuzzy adjacency matrix $A(IG)$ is defined
as (X, Y) where X is the set eigen values of $A(\mu)$ and Y is the set of eigen values of $A(\gamma)$.

Definition 2.9. The energy of an intuitionistic fuzzy graph $G = (V,E, \mu, \gamma)$ is defined as

$$\left(\sum_{\lambda_i\in x}|\lambda_i|, \sum_{\delta_i\in y}|\delta_i|\right)$$

Where

$$\sum_{\lambda_i\in x}|\lambda_i|$$

is defined as the energy of the membership matrix denoted by $E(\mu(G))$ and

$$\sum_{\delta_i\in y}|\delta_i|$$

is defined as the energy of the non membership matrix denoted by $E(\gamma(G))$.

Example 2.10. For an intuitionistic fuzzy graph in Fig. 1,

Eigen values ={0,2.5172,1.7558,1.3270}

$$LE[\mu(G)]=\sum_{i=1}^{n}\left|\lambda_i-\frac{\Sigma(\mu)}{n}\right|=\left|0-\frac{5.6}{2}\right|+\left|2.5172-\frac{5.6}{2}\right|+\left|1.7558-\frac{5.6}{2}\right|+\left|1.3270-\frac{5.6}{2}\right|$$

= 2.8+0.2828+1.0442+1.473

= 5.6

Eigen values = {0,1,1,0.5}

$$LE(\gamma(G))=\left\{\left|0-\frac{2.5}{4}\right|+\left|1-\frac{2.5}{4}\right|+\left|1-\frac{2.5}{4}\right|+\left|0.5-\frac{2.5}{4}\right|\right\}$$

= 0.625+ 0.375+0.375+0.125

= 1.5

LE (G_1) = (5.6, 1.5)

LE $(\mu(G_1))$ > LE$(\gamma(G_1))$

2.4. Epidemic threshold [16]

Theorem 2.11. (Epidemic threshold) The epidemic threshold τ_c is $\tau_c = \frac{1}{\lambda_{\max(L)}}$ where $\lambda_{\max(L)}$ is the largesteigen value of the adjacency matrix L of the network.

We will prove this in two parts: the necessity of this condition in eliminating an infection andthe sufficiency of this condition for wiping out any initial infection. The corresponding theoremstatements are shown below. Following this, we will see how quickly an infection dies out if theepidemic threshold condition is satisfied.

Theorem2.12. (Necessity of epidemic threshold) In order to ensure that over time, the infection probability of each node in the graph goes to zero (that is, the epidemic dies out), we must have $\frac{\beta}{\delta} < \tau_c = \frac{1}{\lambda_{\max}(L)}$, where β is the infection rate, δ is the curing rate and $\lambda_{\max}(L)$ is the largest eigen value of the adjacency matrix L.

Theorem 2.13. (Sufficiency of epidemic threshold)

If $\frac{\beta}{\delta} < \tau_c = \frac{1}{\lambda_{\max}(L)}$, then the epidemicwill die out over time (the infection probabilities will go to zero), irrespective of the size of theintial outbreak of infection.

3. VIRUS SPREAD IN AN INTUITIONISTIC FUZZY NETWORK

In this section, we consider an intuitionistic fuzzy network $G = (V,E, \mu, \gamma)$. Let $L(IG) = (L(\mu), L(\gamma))$ be the Laplacian matrix of an intuitionistic fuzzy network. Let β be the infectionrate and δ be the curing rate of an intuitionistic fuzzy network. The ratio $\delta = \frac{\beta}{\delta}$ is an intuitionisticfuzzy effective spreading rate and τ_c is the epidemic throshold of an intuitionistic fuzzy network.If the effective spreading rate $\delta = \left(\frac{\beta}{\delta}\right) > \tau_c$, then virus continue and a nonzero fraction of thenodes are infected, whereas $\tau \le \tau_c$, the epidemic dies out. we define the following definitions.

Definition 3.1.

If $L(E(\mu(G))) \succ L(E(\gamma(G)))$

i.e. If the number of visitors are maximum in an intuitionistic fuzzy network then the spreading rate of virus will be maximum. Otherwise we can say the system or network is unstable.

Definition 3.2.

If $$L(E(\mu(G))) \succ L(E(\gamma(G)))$$

then the infection rate of an intuitionistic fuzzy network is defined as $\beta = \max_{i,j} \mu_{ij}$ and the curingrate of an intuitionistic fuzzy network is defined as $\delta = \min_{i,j} \gamma_{ij}$. The ratio $\tau = \frac{\beta}{\delta}, (\delta \neq 0)$ is the

effective spreading rate and $\tau_c = \frac{1}{\lambda_{\max} L(\mu)}$ where $\lambda_{\max} L(\mu)$ is the largest eigen value of theLaplacian matrix $L(\mu)$ of an intuitionistic fuzzy network.

Definition 3.3.

If $L(E(\mu(G))) \prec L(E(\gamma(G)))$

i.e. If the number of visitors are minimum in an intuitionistic fuzzy network then the spreading
rate of virus will be minimum. Otherwise we can say the system or network is stable.

Definition 3.4.

If
$L(E(\mu(G))) \prec L(E(\gamma(G)))$

then the infection rate of an intuitionistic fuzzy network is defined as $\beta = \min_{i,j} \mu_{ij}$ and the curing
rate of an intuitionistic fuzzy network is defined as $\delta = \max_{i,j} \gamma_{ij}$. The ratio $\tau = \frac{\beta}{\delta}$ is the effective
spreading rate and $\tau_c = \frac{1}{\lambda_{\max} L(\gamma)}$ where $\lambda_{\max} L(\gamma)$ is the largest eigen value of the Laplacian matrix $L(\gamma)$ of an intuitionistic fuzzy network.

4. NUMERICAL EXAMPLES

Example 4.1. In example 2.4,

$L(E(\mu(G_1))) \succ L(E(\gamma(G_1)))$

$\beta = \max_{i,j} |\mu_{ij}| = 2.1$

$\delta = \min_{i,j} |\gamma_{ij}| = 0.3$

$\tau = \frac{\beta}{\delta} = \frac{2.1}{0.3} = 7$

$\tau_c = \frac{1}{\lambda_{\max} L(\mu)} = \frac{1}{2.5172} = 0.3972$

Here $\tau > \tau_c$, so the virus continue and a nonzero fraction of the nodes are infected. Hence the
spreading rate of virus will be maximum.

Example 4.2. For an intuitionistic fuzzy graph in Fig. 2,

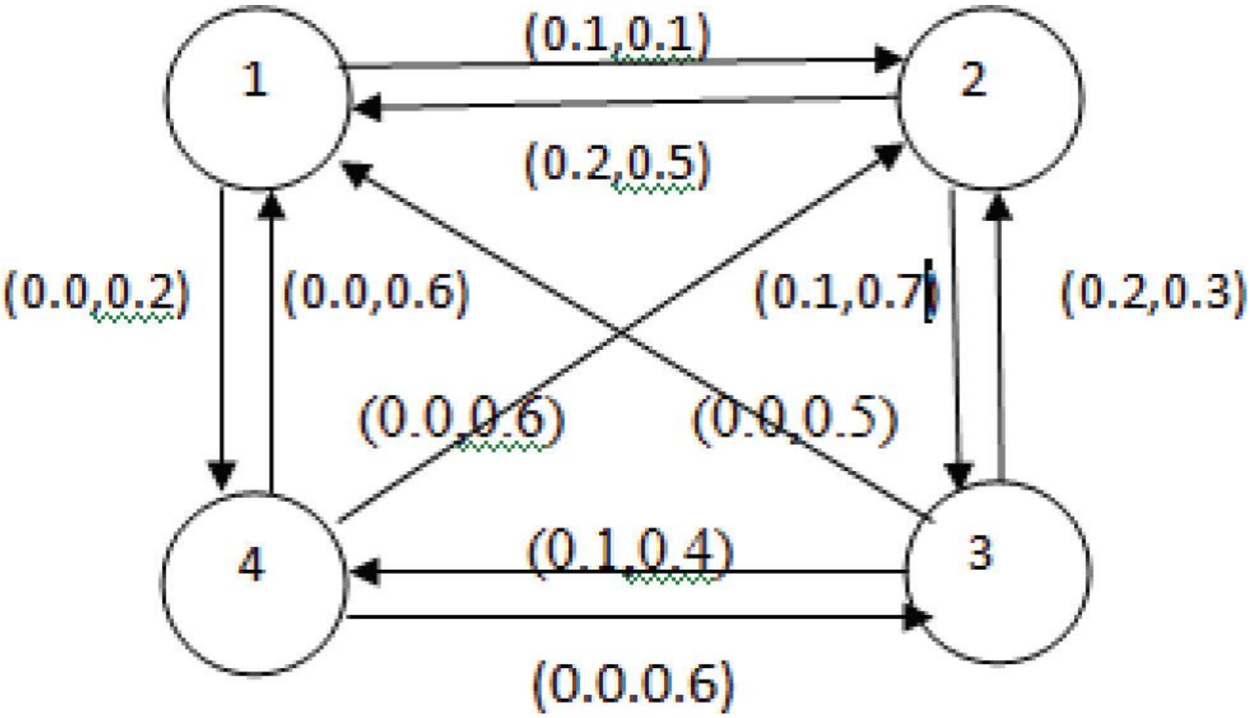

Figure 2: G_2 **,An intuitionistic fuzzy graph**.

Example4.3. In example 4.2,

The adjacency matrix is

$$A\ (IG) = \begin{bmatrix} 0 & (0.1,0.1) & 0 & (0.0,0.2) \\ (0.2,0.5) & 0 & (0.1,0.7) & 0 \\ (0.0,0.5) & (0.2,0.3) & 0 & (0.1,0.4) \\ (0.0,0.6) & (0.0,0.6) & (0.0,0.6) & 0 \end{bmatrix}$$

$$A\ (\mu) = \begin{bmatrix} 0 & 0.1 & 0 & 0 \\ 0.2 & 0 & 0.1 & 0 \\ 0 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D\ (\mu) = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$$

$$L\ (\mu) = D(G) - A(G) = \begin{bmatrix} 0.2 & -0.1 & 0 & 0 \\ -0.2 & 0.3 & -0.1 & 0 \\ 0 & -0.2 & 0.1 & -0.1 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$$

Eigen values ={0.4414,0.1586,-0.0000,0.1000}

$$LE[\mu(G)] = \sum_{i=1}^{n} \left| \lambda_i - \frac{\Sigma(\mu_i)}{n} \right| = \left|0.4414 - \frac{0.7}{4}\right| + \left|0.1586 - \frac{0.7}{4}\right| + \left|-0.0000 - \frac{0.7}{4}\right| + \left|0.1000 - \frac{0.7}{4}\right|$$

= 0.2644+0.0164+0.175+0.075

= 0.5328

$$L(E(\mu(G_2))) \prec L(E(\gamma(G_2)))$$

$$\beta = \max_{i,j} |\mu_{ij}| = 0.3$$

$$\delta = \min_{i,j} |\gamma_{ij}| = 0.1$$

$$\tau = \frac{\beta}{\delta} = \frac{0.3}{0.1} = 3$$

$$\tau_C = \frac{1}{\lambda_{\max} L(\mu)} = \frac{1}{0.4414} = 2.2655$$

$$\tau > \tau_c \ i.e 3 > 2.2655$$

$$A(\gamma)=\begin{bmatrix} 0 & 0.1 & 0 & 0.2 \\ 0.5 & 0 & 0.7 & 0 \\ 0.5 & 0.3 & 0 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0 \end{bmatrix} \quad D(\gamma)=\begin{bmatrix} 1.6 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.3 & 0 \\ 0 & 0 & 0 & 0.6 \end{bmatrix}$$

$$L(\gamma)=D(\gamma)-A(\gamma)=\begin{bmatrix} 1.6 & -0.1 & 0 & -0.2 \\ -0.5 & 1.0 & -0.7 & 0 \\ -0.5 & -0.3 & 1.3 & -0.4 \\ -0.6 & -0.6 & -0.6 & 0.6 \end{bmatrix}$$

Eigen values = {0, 1,2, 2}

$$LE(\gamma(G))=\left\{\left|0-\frac{0.7}{4}\right|+\left|1-\frac{0.7}{4}\right|+\left|2-\frac{0.7}{4}\right|+\left|2-\frac{0.7}{4}\right|\right\}$$

= 0.175+ 0.825+1.825+1.825

= 4.65

LE (G_1) = (0.5328, 4.65)

LE (μ(G_1)) > LE(γ (G_1))

Here $\tau < \tau_c$, so the epidemic dies out. Hence the spreading rate of virus will be minimum.

5. CONCLUSION

In this paper, we are discussed the virus spread in an intuitionistic fuzzy network based on Laplacian energy and we definedthe infection rate, curing rate and the sharp epidemic threshold of the virus spread in anintuitionistic fuzzy network and we illustrate these concepts with example.

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