International Journal of Engineering, Science and Mathematics
Vol. 8 Issue 6, June 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

INTUITIONISTIC FUZZY $\pi g\gamma^*$ CONTINUOUS MAPPINGS AND INTUITIONISTIC FUZZY $\pi g\gamma^*$ IRRESOLUTE MAPPINGS

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ABSTRACT

In this paper we have introduced intuitionistic fuzzy $\pi g\gamma^*$ continuous mappings and intuitionistic fuzzy $\pi g\gamma^*$ irresolute mappings. Some of their properties are studied.

KEYWORDS AND PHRASES: Intuitionistic fuzzy topology, intuitionistic fuzzy $\pi g\gamma^*$ closed set, intuitionistic fuzzy $\pi g\gamma^*$ continuous mappings and intuitionistic fuzzy $\pi g\gamma^*$ irresolute mappings.

1. INTRODUCTION

The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] using the notion of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy $\pi g\gamma^*$ continuous mappings and intuitionistic fuzzy $\pi g\gamma^*$ irresolute mappings and studied some of their basic properties. We provide some relations of intuitionistic fuzzy $\pi g\gamma^*$ continuous mappings and intuitionistic fuzzy $\pi g\gamma^*$ irresolute mappings between existing intuitionistic fuzzy continuous and irresolute mappings.

2. PRELIMINARIES

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$$

where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form
$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$.
Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) A = B if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X$. Also for the sake of simplicity, we shall use the notation $A = \{\langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle \}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1 \rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.
(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,
$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = [int(A)]^c$ and $int(A^c) = [cl(A)]^c$.

Definition 2.5: [6] An IFS $A = \{ \langle x, \mu_A, \nu_A \rangle \}$ in an IFTS (X, τ) is said to be an
(i) *intuitionistic fuzzy semi open set* (IFSOS in short) if $A \subseteq cl(int(A))$,
(ii) *intuitionistic fuzzy α-open set* (IFαOS in short) if $A \subseteq int(cl(int(A)))$,
(iii) intuitionistic fuzzy regular open set (IFROS in short) if $A = int(cl(A))$,

Definition 2.6: [7] The union of IFROSs is called intuitionistic fuzzy π-open set (IFπOS in short) of an IFTS (X, τ). The complement of IFπOS is called intuitionistic fuzzy π -closed set (IFπCS in short).

Definition 2.7: [6] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an
(i) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $int(cl(A)) \subseteq A$,
(ii) *intuitionistic fuzzy α-closed set* (IFαCS in short) if $cl(int(cl(A)) \subseteq A$,
(iii) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int((A)$.

Definition 2.8:[5] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy γ-open set (IFγOS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$,
(ii) intuitionistic fuzzy γ-closed set (IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$.

Definition 2.9: [11] Let A be an IFS in an IFTS (X, τ). Then
sint(A) = ∪ { G / G is an IFSOS in X and G ⊆ A },
scl(A) = ∩ { K / K is an IFSCS in X and A ⊆ K }.

Note that for any IFS A in (X, τ), we have $scl(A^c)=(sint(A))^c$ and $sint(A^c) = (scl(A))^c$.
Definition 2.10: [12] An IFS A in an IFTS (X, τ) is an
(i) intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U
and U is an IFOS in X.
(ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if cl(A) ⊆ U whenever
A ⊆ U and U is an IFROS in X.

Definition 2.11: [11] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

Definition 2.12: [10] An IFS A in (X, τ) is said to be an intuitionistic fuzzy $\pi g\gamma$* closed set (IF$\pi g\gamma$*CS in short) if cl(int(A)) ∩ int(cl(A)) ⊆ U whenever A ⊆ U and U is an IFπOS in (X, τ). The family of all IF$\pi g\gamma$*CSs of an IFTS (X, τ) is denoted by IF$\pi g\gamma$*C(X).

Result 2.13: [10] Every IFCS, IFGCS, IFRCS, IFαCS is an IF $\pi g\gamma$*CS but the converses may not be true in general.

Definition 2.14: [10] An IFS A is said to be an intuitionistic fuzzy $\pi g\gamma$*open set (IF$\pi g\gamma$*OS in short) in (X, τ) if the complement A^c is an IF$\pi g\gamma$*CS in X.
The family of all IF$\pi g\gamma$*OSs of an IFTS (X, τ) is denoted by IF$\pi g\gamma$*O(X).

Definition 2.15: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B)$ ∈ IFO(X) for every B∈ σ.

Definition 2.16: [6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be

(i) intuitionistic fuzzy semi continuous (IFS continuous in short) if $f^{-1}(B)$ ∈ IFSO(X) for every B ∈ σ.
(ii) intuitionistic fuzzy α- continuous (IFα continuous in short) if $f^{-1}(B)$ ∈ IFαO(X) for every B ∈ σ.
(iii) intuitionistic fuzzy pre continuous (IFP continuous in short) if $f^{-1}(B)$ ∈ IFPO(X) for every B ∈ σ.

Result 2.17: [6] Every IF continuous mapping is an IFα-continuous mapping and every IFα-continuous mapping is an IFS continuous mapping.

Definition 2.18: [5] A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy γ continuous* (IFγ continuous in short) if $f^{-1}(B)$ is an IFγOS in (X, τ) for every B ∈ σ.

Definition 2.19: [9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \in$ IFGCS(X) for every IFCS B in Y.

Definition 2.20: [11] A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy generalized semi continuous* (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ).

Definition 2.21: [9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy irresolute (IF irresolute in short) if $f^{-1}(B) \in$ IFCS(X) for every IFCS B in Y.

Definition 2.22: [9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized irresolute (IFG irresolute in short) if $f^{-1}(B) \in$ IFGCS(X) for every IFGCS B in Y.

Definition 2.23: An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi\gamma$*c$T_{1/2}$ (in short IF$\pi\gamma$*c$T_{1/2}$) space if every IF$\pi g\gamma$*CS in X is an IFCS in X.

Definition 2.24: An IFTS (X, τ) is an intuitionistic fuzzy $\pi\gamma$*g$T_{1/2}$ (IF$\pi\gamma$*g$T_{1/2}$) space if every IF$\pi g\gamma$*CS is an IFGCS in X.

Definition 2.25: An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi\gamma$*$T_{1/2}$ (in short IF$\pi\gamma$*$T_{1/2}$) space if every IF$\pi g\gamma$*CS in X is an IFγCS in X.

3. INTUITIONISTIC FUZZY $\pi g\gamma$* CONTINUOUS MAPPINGS

In this section I introduce intuitionistic fuzzy $\pi g\gamma$*continuous mappings and studied some of its properties.

Definition 3.1: A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy* πgγ**continuous* (IF$\pi g\gamma$* continuous in short) mapping if $f^{-1}(B)$ is an IF$\pi g\gamma$*CS in (X, τ) for every IFCS B of (Y, σ).

Example 3.2: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.3, 0.3), (0.6, 0.6) ⟩, T_2 = ⟨ y, (0.6, 0.7), (0.4, 0.2) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF$\pi g\gamma$* continuous mapping.

Theorem 3.3: *Every IF continuous mapping is an IF$\pi g\gamma$* continuous mapping but not conversely.*

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping. Let A be an IFCS in Y. Since f is IF continuous mapping, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IF$\pi g\gamma$*CS, $f^{-1}(A)$ is an IF$\pi g\gamma$*CS in X. Hence f is an IF$\pi g\gamma$*continuous mapping.

Example 3.4: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.2, 0.2), (0.5, 0.7) ⟩, T_2 = ⟨ y, (0.5, 0.4), (0.4, 0.2) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS A = ⟨ y, (0.4, 0.2), (0.5, 0.4) ⟩ is IFCS in Y. Then f^{-1}(A) is IF$\pi g\gamma$*CS in X but not IFCS in X. Therefore f is an IF$\pi g\gamma$*continuous mapping but not an IF continuous mapping.

Theorem 3.5: *Every IFS continuous mapping is an IF$\pi g\gamma$* continuous mapping but not conversely.*

Proof: Let f: (X, τ) → (Y,σ) be an IFS continuous mapping. Let A be an IFCS in Y. Then by hypothesis f^{-1}(A) is an IFSCS in X. Since every IFSCS is an IF$\pi g\gamma$*CS, f^{-1}(A) is an IF$\pi g\gamma$*CS in X. Hence f is an IF$\pi g\gamma$* continuous mapping.

Example 3.6: Let X = { a, b }, Y = { u, v } and T_1 = ⟨ x, (0.5, 0.3), (0.5, 0.6) ⟩, and T_2 = ⟨ x, (0.6, 0.6), (0.3, 0.4) ⟩. Then τ = {$0_\sim$, T_1 ,$1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS A = ⟨ y, (0.3, 0.4), (0.6, 0.6) ⟩ is IFCS in Y. Then f^{-1}(A) is IF $\pi g\gamma$*CS in X but not IFSCS in X. Then f is IF $\pi g\gamma$* continuous mapping but not an IFS continuous mapping.

Theorem 3.7: *Every IFP continuous mapping is an IF$\pi g\gamma$* continuous mapping but not conversely.*

Proof: Let f: (X, τ) → (Y,σ) be an IFP continuous mapping. Let A be an IFCS in Y. Then by hypothesis f^{-1}(A) is an IFPCS in X. Since every IFPCS is an IF$\pi g\gamma$*CS, f^{-1}(A) is an IF$\pi g\gamma$*CS in X. Hence f is an IF$\pi g\gamma$* continuous mapping.

Example 3.8: Let X = { a, b }, Y = { u, v } and T_1 = ⟨ x, (0.2, 0.3), (0.5, 0.6) ⟩, and T_2 = ⟨ x, (0.6, 0.6), (0.3, 0.4) ⟩. Then τ = {$0_\sim$, T_1 ,$1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS A = ⟨ y, (0.3, 0.4), (0.6, 0.6) ⟩ is IFCS in Y. Then f^{-1}(A) is IF $\pi g\gamma$*CS in X but not IFPCS in X. Then f is IF $\pi g\gamma$* continuous mapping but not an IFP continuous mapping.

Theorem 3.9: *Every IFα continuous mapping is an IF$\pi g\gamma$* continuous mapping but not conversely.*

Proof: Let f: (X, τ) → (Y,σ) be an IFα continuous mapping. Let A be an IFCS in Y. Then by hypothesis f^{-1}(A) is an IFαCS in X. Since every IFαCS is an IF$\pi g\gamma$*CS, f^{-1}(A) is an IF$\pi g\gamma$*CS in X. Hence f is an IF$\pi g\gamma$* continuous mapping.

Example 3.10: Let X = { a, b }, Y = { u, v } and T_1 = ⟨ x, (0.4, 0.2), (0.6, 0.7) ⟩, T_2 = ⟨ x, (0.8, 0.8), (0.2, 0.2) ⟩ and T_3 = ⟨x, (0.4, 0.3), (0.5, 0.6) ⟩. Then τ = {$0_\sim$, T_1, T_2,$1_\sim$} and σ = { $0_\sim$, T_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS A = ⟨ y, (0.5, 0.6), (0.4, 0.3) ⟩ is IFCS in Y. Then f^{-1}(A) is IF $\pi g\gamma$*CS in X but not IFαCS in X. Then f is IFαG continuous mapping but not an IFα continuous mapping.

Theorem 3.11: *Every IFγ continuous mapping is an IF $\pi g\gamma$* continuous mapping but not conversely.*

Proof: Let f: (X, τ) → (Y,σ) be an IFγ continuous mapping. Let A be an IFCS in Y. Then by hypothesis f^{-1}(A) is an IFγCS in X. Since every IFγCS is an IF $\pi g\gamma$*CS, f^{-1}(A) is an IF $\pi g\gamma$*CS in X. Hence f is an IF $\pi g\gamma$* continuous mapping.

Example 3.12: Let X = { a, b }, Y = { u, v } and T_1 = ⟨x, (0.5, 0.6), (0.5, 0.4)⟩, T_2 = ⟨ x, (0.2, 0.2), (0.8, 0.8) ⟩ and T_3 = ⟨x, (0.4, 0.6), (0.6, 0.4) ⟩. Then τ = {$0_\sim$, T_1, T_2,$1_\sim$}

and $\sigma = \{ 0_\sim, T_3, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS A = ⟨ y, (0.6, 0.4), (0.4, 0.6) ⟩ is IFCS in Y. Then $f^{-1}(A)$ is IF$\pi g\gamma^*$CS in X but not IFγCS in X. Then f is IF $\pi g\gamma^*$ continuous mapping but not an IFγ continuous mapping.

Theorem 3.13: *Every IFG continuous mapping is an IF $\pi g\gamma^*$ continuous mapping but not conversely.*

Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFG continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFGCS in X. Since every IFGCS is an IF $\pi g\gamma^*$CS, $f^{-1}(A)$ is an IF $\pi g\gamma^*$CS in X. Hence f is an IF $\pi g\gamma^*$ continuous mapping.

Example 3.14: Let X = { a, b }, Y = { u, v } and T_1 = ⟨ x, (0.2, 0.8), (0.3, 0.1) ⟩, and T_2 = ⟨ x, (0.1, 0), (0.4, 0.9) ⟩. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS A = ⟨ y, (0.4, 0.9), (0.1, 0) ⟩ is IFCS in Y. Then $f^{-1}(A)$ is IF$\pi g\gamma^*$CS in X but not IFGCS in X. Then f is IF $\pi g\gamma^*$ continuous mapping but not an IFG continuous mapping.

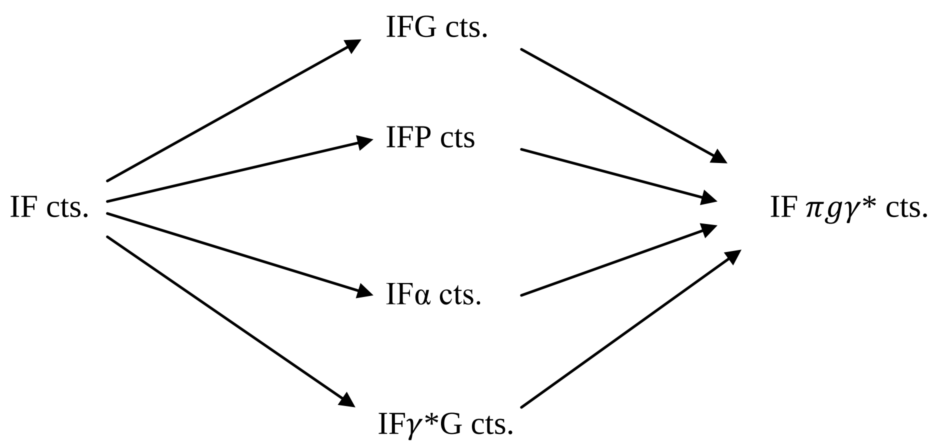
Theorem 3.15: *Every IFγ^*G continuous mapping is an IF $\pi g\gamma^*$ continuous mapping.*

Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFγ^*G continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFγ^*GCS in X. Since every IF$\gamma *$ GCS is an IF $\pi g\gamma^*$CS, $f^{-1}(A)$ is an IF $\pi g\gamma^*$CS in X. Hence f is an IF $\pi g\gamma^*$ continuous mapping.

Theorem 3.16: *A mapping f: X → Y is IF $\pi g\gamma^*$ continuous if and only if the inverse image of each IFOS in Y is an IF IF $\pi g\gamma^*$OS in X.*

Proof: Let A be an IFOS in Y. This implies A^c is IFCS in Y. Since f is IF $\pi g\gamma^*$ continuous, $f^{-1}(A^c)$ is IF$\pi g\gamma^*$CS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IF $\pi g\gamma^*$ OS in X.

The relations between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram 'cts.' means continuous.



The reverse implications are not true in general.

Theorem 3.17: *Let f: $(X, \tau) \to (Y, \sigma)$ be an IF$\pi g\gamma^*$ continuous mapping, then f is an IF continuous mapping if X is an IF$\pi\gamma^*cT_{1/2}$ space.*

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IF$\pi g\gamma$*CS in X, by hypothesis. Since X is an IF$\pi\gamma$*c$T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 3.18: *Let f: (X, τ) → (Y, σ) be an IF$\pi g\gamma$*continuous function, then f is an IFG continuous mapping if X is an IFγ*g$T_{1/2}$ space.*
Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is IF$\pi g\gamma$*CS in X, by hypothesis. Since X is an IFγ*g$T_{1/2}$ space, $f^{-1}(A)$ is an IFGCS in X. Hence f is an IFG continuous mapping.

Theorem 3.19: *Let f : (X, τ) → (Y, σ) be an IF$\pi g\gamma$* continuous mapping and g : (Y, σ) → (Z, δ) is IF continuous, then g o f : (X, τ) → (Z, δ) is an IF$\pi g\gamma$* continuous mapping.*

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since f is an IF$\pi g\gamma$*continuous mapping, $f^{-1}(g^{-1}(A))$ is an IF$\pi g\gamma$*CS in X. Hence g o f is an IF$\pi g\gamma$* continuous mapping.

Theorem 3.20: *Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IF$\pi\gamma$*c$T_{1/2}$space.*
*(i) f is an IF$\pi g\gamma$*continuous mapping*
*(ii) If B is an IFOS in Y then $f^{-1}(B)$ is an IF$\pi g\gamma$*OS in X*
(iii) $f^{-1}(intB) \subseteq int(cl(int(f^{-1}(B)))$ for every IFS B in Y.

Proof: (i) ⇒ (ii): is obviously true.
(ii) ⇒ (iii): Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IF$\pi g\gamma$*OS in X. Since X is an IF$\pi\gamma$*c$T_{1/2}$ space, $f^{-1}(int(B))$ is an IFOS in X. Therefore $f^{-1}(int\ B) = int(f^{-1}(int(B)) \subseteq int(cl(int(f^{-1}(B))))$.
(iii) ⇒ (i): Let B be an IFCS in Y. Then its complement B^c is an IFOS in Y. By hypothesis $f^{-1}(int\ (B^c)) \subseteq int(cl(int(f^{-1}(B^c))))$. This implies $f^{-1}(B^c) \subseteq int(cl(int(f^{-1}(B^c))))$. Hence $f^{-1}(B^c)$ is an IFαOS in X. Since every IFαOS is an IF$\pi g\gamma$*OS, $f^{-1}(B^c)$ is an IF$\pi g\gamma$*OS in X. Therefore $f^{-1}(B)$ is an IF$\pi g\gamma$*CS in X. Hence f is an IF$\pi g\gamma$* continuous mapping.

Theorem 3.21: *Let f: (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X is an IF$\pi\gamma$*c$T_{1/2}$ space.*
(i) *f is an IF$\pi g\gamma$* continuous mapping*
(ii) *$f^{-1}(B)$ is an IF$\pi g\gamma$*CS in X for every IFCS B in Y*
(iii) *$cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS B in Y.*

Proof: (i) ⇒ (ii): is obviously true.
(ii) ⇒ (iii): Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IF$\pi g\gamma$*CS in X. Since X is an IF$\pi\gamma$*c$T_{1/2}$ space, $f^{-1}(cl(A))$ is an IFCS in X. Therefore $cl(f^{-1}(cl(A)) = f^{-1}(cl(A))$. Now $cl(int(cl(f^{-1}(A)))) \subseteq cl(int(cl(f^{-1}(cl(A))))) \subseteq f^{-1}(cl(A))$.
(iii) ⇒ (i): Let A be an IFCS in Y. By hypothesis $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFαCS in X and hence it is an IF$\pi g\gamma$*CS in X. Therefore f is an IF$\pi g\gamma$* continuous mapping.

Theorem 3.22: *Let f: (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X is an IF$\pi\gamma$*$T_{1/2}$ space:*

(i) *f is an IF$\pi g\gamma$*continuous mapping,*
(ii) *If B is an IFOS in Y then $f^{-1}(B)$ is an IF$\pi g\gamma$*OS in X,*
(iii) *$f^{-1}(int(B)) \subseteq cl(int(f^{-1}(B))) \cup int(cl(f^{-1}(B)))$ for every IFS B in Y.*

Proof: (i) ⇒ (ii) is obviously true
(ii) ⇒ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IF$\pi g\gamma$*OS in X. Since X is an IF$\pi\gamma$*$T_{1/2}$ space, $f^{-1}(int(B))$ is an IFγOS in X. Therefore $f^{-1}(int(B)) \subseteq cl(int(f^{-1}(int(B)))) \cup int(cl(f^{-1}(int(B)))) \subseteq cl(int(f^{-1}(B))) \cup int(cl(f^{-1}(B)))$.
(iii) ⇒ (i) Let B be an IFCS in Y. Then its complement is an IFOS in Y, then $int(B^c) = B^c$. Now by hypothesis $f^{-1}(B^c)= f^{-1}(int(B^c)) \subseteq cl(int(f^{-1}(B^c))) \cup int(cl(f^{-1}(B^c)))$. Hence $f^{-1}(B^c)$ is an IFγOS in X. Since every IFγOS is an IF$\pi g\gamma$*OS, $f^{-1}(B^c)$ is an IF$\pi g\gamma$*OS in X. Thus $f^{-1}(B)$ is an IF$\pi g\gamma$*CS in X. Hence f is an IF$\pi g\gamma$*continuous mapping.

Theorem 3.23: *Let f: (X, τ) → (Y, σ) be a mapping. Let f is an IF$\pi g\gamma$* continuous mapping, Then $(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS B in Y, if X is an IF$\pi\gamma$*c$T_{1/2}$ space.*

Proof: Let f: (X, τ) → (Y, σ) be a mapping. Let f is an IF$\pi g\gamma$* continuous mapping. That is $f^{-1}(B)$ is an IF$\pi g\gamma$*CS in X for every IFCS B in Y. Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IF$\pi g\gamma$*CS in X. Since X is an IF$\pi\gamma$*c$T_{1/2}$ space, $f^{-1}(cl(A))$ is an IFCS in X. Therefore $cl(f^{-1}(cl(A)) = f^{-1}(cl(A))$. Now $int(cl(f^{-1}(A))) \subseteq int(cl(f^{-1}(cl(A)))) \subseteq f^{-1}(cl(A))$.

4. INTUITIONISTIC FUZZY $\pi g\gamma$* IRRESOLUTE MAPPINGS

In this section we introduce intuitionistic fuzzy $\pi g\gamma$* irresolute mapping and studied some of its properties.

Definition 4.1: A mapping f: (X, τ) → (Y, σ) is called an *intuitionistic fuzzy $\pi g\gamma$* irresolute* (IF$\pi g\gamma$* irresolute) mapping if $f^{-1}(A)$ is an IF$\pi g\gamma$*CS in (X, τ) for every IF$\pi g\gamma$*CS A of (Y, σ).

Theorem 4.2: *Let f: (X, τ) → (Y, σ) be an IF $\pi g\gamma$* irresolute, then f is IF $\pi g\gamma$* continuous mapping.*
Proof: Let f be IF$\pi g\gamma$* irresolute mapping. Let B be any IFCS in Y. Since every IFCS is an IF$\pi g\gamma$*CS, B is an IF$\pi g\gamma$*CS in Y. By hypothesis $f^{-1}(B)$ is an IF $\pi g\gamma$*CS in X. Hence f is an IF$\pi g\gamma$* continuous mapping.

Theorem 4.3: *Let f: (X, τ) → (Y, σ) be an IF IF$\pi g\gamma$* irresolute, then f is an IF irresolute mapping if X is an IF$\pi\gamma$*c$T_{1/2}$ space.*
Proof: Let A be an IFCS in Y. Then A is an IF IF$\pi g\gamma$*CS in Y. Therefore $f^{-1}(A)$ is an IF IF$\pi g\gamma$*CS in X, by hypothesis. Since X is an IF$\pi\gamma$*c$T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF irresolute mapping.

Theorem 4.4: *Let f: (X, τ) → (Y, σ) be an IF$\pi g\gamma$* irresolute mapping, then f is an IFG continuous mapping if X is an IF$\pi\gamma$*g$T_{1/2}$ space.*
Proof: Let A be an IFCS in Y. Then A is an IF$\pi g\gamma$*CS in Y. Therefore $f^{-1}(A)$ is an IF$\pi g\gamma$*CS in X, by hypothesis. Since X is an IF$\pi\gamma$*g$T_{1/2}$ space, $f^{-1}(A)$ is an IFGCS in X. Hence f is an IFG continuous mapping.

Theorem 4.5: *Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IFπγ*c$T_{1/2}$ spaces.*

(i) f is an IFπgγ irresolute mapping*

*(ii) f^{-1}(B) is an IFπgγ*OS in X for each IFπgγ*OS in Y*

(iii) cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) for each IFS B of Y.

Proof: (i) ⇒ (ii): is obvious from the definition.

(ii) ⇒ (iii): Let B be any IFS in Y and B ⊆ cl(B). Then f^{-1}(B) ⊆ f^{-1}(cl(B)). Since cl(B) is an IFCS in Y, cl(B) is an IFπgγ*CS in Y. Therefore f^{-1}(cl(B)) is an IFπgγ*CS in X, by hypothesis. Since X is IFπγ*c$T_{1/2}$ space, f^{-1}(cl(B)) is an IFCS in X. Hence cl(f^{-1}(B)) ⊆ cl(f^{-1}(cl(B))) = f^{-1}(cl(B)). That is cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

(iii) ⇒ (i): Let B be an IFπgγ*CS in Y. Since Y is IFπγ*c$T_{1/2}$ space, B is an IFCS in Y and cl(B) = B. Hence f^{-1}(B) = f^{-1}(cl(B)) ⊇ cl(f^{-1}(B)). But clearly f^{-1}(B) ⊆ cl(f^{-1}(B)). Therefore cl(f^{-1}(B)) = f^{-1}(B). This implies f^{-1}(B) is an IFCS and hence it is an IFπgγ*CS in X. Thus f is an IFπgγ* irresolute mapping.

Theorem 4.6: *Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) be IF IFπgγ* irresolute mappings, then g o f : (X, τ) → (Z, δ) is an IFπgγ* irresolute mapping.*

Proof: Let A be an IFπgγ*CS in Z. Then g^{-1}(A) is an IFπgγ*CS in Y. Since f is an IFπgγ* irresolute mapping, f^{-1}(g^{-1}(A)) is IFπgγ*CS in X. Hence g o f is an IFπgγ* irresolute mapping.

Theorem 4.9: *Let f: (X, τ) → (Y, σ) be an IFπgγ* irresolute and g: (Y, σ) → (Z, δ) is IFπgγ* continuous mapping, then g o f : (X, τ) → (Z, δ) is an IFπgγ* continuous mapping.*

Proof: Let A be an IFCS in Z. Then g^{-1}(A) is an IFπgγ*CS in Y. Since f is an IFπgγ* irresolute, f^{-1}(g^{-1}(A)) is an IFπgγ*CS in X. Hence g o f is an IFπgγ* continuous mapping.

References

[1] Atanassov. K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986), 87-96.

[2] Coker. D., An introduction to fuzzy topological space, Fuzzy sets and systems, 88(1997), 81-89.

[3] Coker, D., and Demirci, M., "On Intuitionistic Fuzzy Points", Notes on intuitionistic fuzzy sets 1(1995), 79-84.

[4] Gurcay, H., Coker. D., and Haydar, A., On fuzzy continuity in intuitionistic fuzzy topological spaces, jour. of fuzzy math., 5 (1997), 365-378.

[5] Hanafy, I. M., "Intuitionistic Fuzzy γ-Continuity", Canad. Math. Bull. 52(2009) 544–554.

[6] Joung Kon Jeon, Young Bae Jun, and Jin Han Park, Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity, International Journal of Mathematics and Mathematical Sciences, 19 (2005), 3091-3101.

[7] Maragathavalli, S and Ramesh, K., Intuitionistic Fuzzy π Generalized Semi Closed sets, Advances in Theoretical and Applied Sciences, 1(2012), 33-42.

[8] Riya V. M and Jayanthi D, Generalized Closed γ* On Intuitionistic Fuzzy Sets, Advances in Fuzzy Mathematics. 12 (2017), 389-410.

[9] Sakthivel, K., Intuitionistic Fuzzy Alpha Generalized Continuous Mappings and Intuitionistic Alpha Generalized Irresolute Mappings, Applied Mathematical Sciences., 4(2010), 1831 – 1842.

[10] Sakthivel K. and Manikandan M., πgγ* Closed sets in Intuitionistic Fuzzy Topological Space, International Journal of Mathematics Trends and Technology, 65(2019), 167-176.

[11] Santhi, R and Jayanthi, D., Intuitionistic fuzzy generalized semi-pre closed sets, Tripura math. Soc. 1(2009), 61-72.

[12] Santhi, R. and Sakthivel, K., Intuitionistic fuzzy generalized semi continuous mappings, Advances in Theoretical and Applied Mathematics, 5 (2009), 73-82.

[13] Thakur, S.S. and Rekha Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si Cercetari Stiintifice, Seria: Matematica, 16 (2006), 257-272.

[14] Young Bae Jun and Seok- Zun Song, Intuitionistic fuzzy semi-pre open sets and Intuitionistic semi-pre continuos mappings, jour. of Appl. Math and computing, 19(2005), 467-474.

[15] Zadeh. L. A., Fuzzy sets, Information and control, 8(1965) 338-353.

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