THE SPLIT DOMINATION IN ARITHMETIC GRAPHS

[1]R. SUMATHI , [2]D.L. YAMINI LATHA

[1]Research Scholar,

[2]Assistant Professor

Department of Mathematics,

SPIHER, Avadi, Chennai-54

ABSTRACT

This paper highlights about the theory of domination in graphs, more specifically on the characteristics of the split domination number of an Arithmetic Graph and has obtained important results. The split domination of these arithmetic graphs have been studied as it enables us to construct graphs with a given split domination number in a simple way. We have obtained an upper bound for the split domination number of the Vm graph as r+1,where m is a positive integer and is the canonical representation, where are distinct primes and m = $p_1^{a_1}, p_2^{a_2} \ldots\ldots p_r^{a_r}$ is the canonical representation, where, $p1, p_2, p_3, \ldots p_r$ are distinct primes and a'_i s >1.

KEYWORDS: *Domination, Graph theory, Spilt domination, Standard graphs, Arithmetic graphs*

INTRODUCTION

In Mathematics, graph theory deals the study of graphs, which are mathematical structures used to model pair wise relationships and its objects. A graph in this context is made up of vertices, nodes, or points which are connected by edges, arcs, or lines. Graph theory gives useful set of techniques for resolving real world problems particularly for different kinds for analyzing "things that are connected to other things", which applies at most everywhere. Automatic channel allocation for small wireless local area network. Analyzing communication efficiency in sensor networks with voronoi graph reducing the Complexity of Algorithms in Compression Networks. Graph labeling in communication relevant to adhoc networks and effective communication in social networks. Identification of routing algorithm with short label names with automatic routing with domination. In mathematics, graphs are useful in geometry and certain parts of topology such as, knot theory. Algebraic graph theory has close links with this graph theory. A graph structure can be prolonged by assigning a weight to each edge of the graph. Weighted graphs are used to represent structures in which pair wise connections have some numerical values.

LITERATURE REVIEW

The rigorous study of dominating sets in graph theory was begin in 1960, even though the subject has historical milestone in dating back to 1862, when the Jaenisch studied the problems of determining the minimum number of queens which are necessary to cover or dominate a n x n chess board. According to Berge (1958) defined the concept of the domination number of a graph, calling this as "coefficient of external stability". In 1962, Ore used the name 'dominating set' and 'domination number' for the same concept.

Cockayne and Hedetniemi (1977) made an interesting and extensive survey of the results know at that time about dominating sets in graphs. They have used the notation γ (G) for the domination number of a graph, which has become very popular since then.

SIGNIFICANCE OF THE STUDY

The split domination in graphs was introduced by Kulli and Janakiram. They defined the split dominating set and the split domination number and obtained several interesting results regarding the split domination number of some standard graphs. They have also obtained relations of split domination number with the other parameters such as domination number, connected domination number, vertex covering number etc., and also Sampathkumar obtained some interesting results on tensor products of graphs. Vasumath; Vangipuram and Vijayasaradhi Vangipuram obtained domination parameters in certain Graphs and also they have found an elegant method for the development of certain graph with the given domination parameter. Motivated by the study of domination and split and non-split domination we have reported some parameters of split and non-split domination number in certain graphs. This present study also throws light on the properties of the split domination number of some graphs and obtained several interesting results with regards to the split domination number of some standard graphs. Investigator has made strong intension in split domination number with other parameters such as domination number, connected domination number, verted covering number has found.

IMPORTANT DEFINITIONS:

Dominating Set: A dominating set D of a graph G is called a **split dominating set**, if the induced sub-graph <V-D> is disconnected.

Split dominating number. The **split dominating number** $\gamma_s(G)$ of G is the minimum cardinality of the split dominating set.

Arithmetic Graph: The Arithmetic graph V_m is a graph with its vertex set as the set of all divisors of m (excluding 1), where m is a natural number and $m = p_1^{a_1}, p_2^{a_2} ....p_r^{a_r}$ a canonical representation of m, where p_i's are distinct primes and and two distinct vertices a,b which are not of the same parity are adjacent in this graph if $(a,b) = p_i$, for $1 \le i \le r$.

The vertices a and b are said to be of the same parity if both a and b are the powers of the same prime, for instance $a= p^2, b = p^5$.

SPLIT DOMINATION IN ARITHMETIC GRAPH

We have develop a method of construction of graph with a given number as the split domination number of the graph. For this reason, we make use of an arithmetic graph V_m with its vertex set as the set of all divisors of m and defining the adjacent property of the arithmetic graph suitably. The split domination of these arithmetic graphs have been studied as it enables us to construct graphs with a given split domination number in a very simple way. We have found that the split domination number of the V_m graph is r + 1, where m is a positive integers and $m = p_1^{a_1}, p_2^{a_2} ....p_r^{a_r}$ is the canonical representation, where $p_1, p_2, \ldots, p_r$ are distinct primes and a_i's>1.

Theorem

If m = $p_1^{a_1}, p_2^{a_2}$.......$p_r^{a_r}$ where a_i's>1, for i = 1,2,...............r, then $\gamma_s(V_m) \leq$ r +1, where r is the number of distinct prime factors of m.

Proof: Let m = $p_1^{a_1}, p_2^{a_2}$.......$p_r^{a_r}$, where a_i's> 1, for $1 \leq i \leq r$ and $p_1,p_2,..........p_r$ are distinct primes. The set of vertices D = {$p_1,p_2,..........p_r$, $p_1p_2..........p_r$}, is a split dominating set.For, if v is any vertex in V-D, then v is of the form $p_1^{b_1}, p_2^{b_2}$.......$p_r^{b_r}$, where $0 \leq b_i \leq a_i$ not all b_i's are '0' at the same time. Then the vertex $p_1^{b_1}, p_2^{b_2}$.......$p_r^{b_r}$ is adjacent with p_i and in D. Thus D is dominating set of V_m. Further this is also a split dominating set.For, the vertex $p_1^{a_1}, p_2^{a_2}$.......$p_r^{a_r}$ is not adjacent with any vertex in < V - D >. Hence D is a split dominating set. Further this is also a minimal split dominating set.For, if we remove any vertex v from D, then v is of form either p_i for $1 \leq i \leq r$ or $p_1p_2..........p_r$.

If v is of the form p_i,say for the sake of definiteness, v = p_i. If we remove any vertex p_i,then the vertex $p_1^{a_1}, p_2^{a_2}$.......$p_r^{a_r}$ is adjacent with p_i. So D-{v} is not a split dominating set. On the other hand if we remove $p_1p_2..........p_r$, from D, the vertices p_1^2, p_1^3.......$p_1^{a_1}, p_2^2, p_2^3$,...... $p_2^{a_2}$ etc., are not adjacent with any vertex in D-{v}. This means that D - {$p_1p_2..........p_r$} is not a dominating set. Hence D is a minimal split dominating set. Also in view of the definition of the arithmetic graph and the form of the set of vertices being divisors of m and in view of the definition of the adjacency of any two vertices in V_m, it follows that D is a split dominating set of minimum cardinality. Hence $\gamma_s(V_m) \leq$ r + 1, where r is the core of 'm'.

Construction of a Graph with the given Split Domination Number

With the help of the above theorem we will now construct a graph with the given split domination number. If we are in need to construct a graph with a given split domination number 't', we proceed as follows: Choose m = $p_1^{a_1}, p_2^{a_2}$.......$p_r^{a_r}$, where p_i's are distinct primes and a_i's> 1.

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Illustration:

The construction of a graph with a given split domination number as 3:

(i)Given t = 3; we have t - 1= 2, choose any two primes p_1, p_2 and let m = $p_1^2 p_2^2$.

The 'm' divides vertices of 'Vm' (expect 1):

V= $\{p_1, p_2, p_1^2, p_2^2, p_1, p_2, p_1 p_2^2, p_1^2 p_2, p_1^2 p_2^2\}$

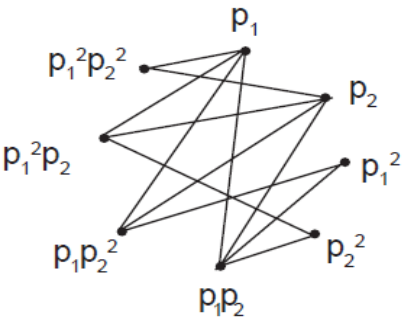

Figure (1)V_m graph with m = $p_1^2 p_2^2$

D = $\{p_1, p_2, p_1 p_2\}$ is the minimum split dominating set, since < V-D > is disconnected $\gamma_s(V_m)$ = 3.

(ii) Given t = 3, we have t-1= 2, choose any two primes P_1, P_2 and let m = $p_1^2 p_2^3$

The vertices of V_m are the divisors of m :

V= $\{p_1, p_2, p_1^2, p_2^2, p_2^3, p_1 p_2, p_1 p_2^2, p_1 p_2^3, p_1^2 p_2, p_1^2 p_2^2, p_1^2 p_2^3\}$

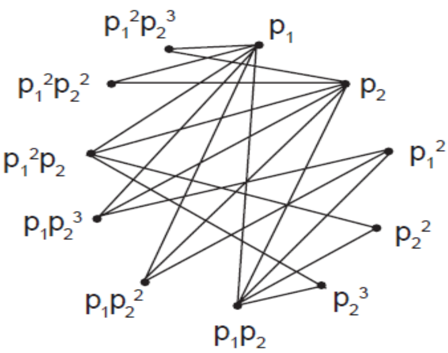

Figure (2) V_m graph with m = $p_1^2 p_2^3$

D = {p_1,p_2,p_1,p_2} is the minimum split dominating set, Since < V-D > is disconnected $\gamma_s(V_m) = 3$

CONCLUSION

In context of current research the investigator discussed about the spilt domination in arithmetic graphs with some illustrations. These constructions are quite useful in the applications of domination theory in real life situations.

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