International Journal of Engineering, Science and Mathematics
Vol. 8 Issue 8, August 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

MATHEMATICAL MODEL FOR WATER POISONING DUE TO DRAIN OF POLLUTANT WATER IN LAKE BY INDUSTRY

AVHALE.P.S
Department of Mathematics,
Shivaji Arts and Science College Kannad
Dist. Aurangabad (M.H), India.

ABSTRACT

Now a day's some industry drains the poisonous pollutant water in lake directly without any process. Water from lake, Municipal Corporation supply to the city which is uses as the drinking water. If drain is close to water pump of supply then it hazardous. Now issue arise that what is safe distance between water pump of supply and drain point of west by industry where the concentration of drug is to low that which is not harmful to drink. You use advection diffusion equation for modeling the situation.

KEYWORDS: Solution of advection diffusion equation..

.Subject classification (AMS) : 35XX-35K xx , 76XX-76Rxx, 76XX-7605, 76XX-76Zxx-76Z05 and 92XX-92B

INTRODUCTION

The problem like the air pollution in cities spray drain of hazardous chemical by industry has become so severe that there is a need for timely information about changes in the concentration level. The water pollution due to dispersion is a complex problem. It covers the pollutant transport and diffusion in the atmosphere. The poisoning dispersion in the atmosphere depends on chemical features, meteorological, emission and terrain conditions. Physical and mathematical models are developed to describe the air poisoning dispersion. Physical models are small scale representations of the atmospheric flow carried out in water tunnels. Mathematical models are divided in to statistical and deterministic models. Statistical models are based on analysis of past monitoring air quality data. Deterministic models are based on a mathematical description of physical and chemical processes taking place in the atmosphere. These models are based on mathematical equations, express conservation laws of mass, momentum and energy. The deterministic models are divided in to Eulerian, Lagrangian and Gaussian models

Gaussian plume model uses a realistic description of dispersion, where it represents an analytical solution to the diffusion equation for idealized circumstances. The model assumes that the atmospheric turbulence is both stationary and homogeneous. In reality, none of these conditions is fully satisfied. Attention of researchers has been attracted by dispersion of water pollutant in many ways. Water quality mathematical model is developed by considering the rate of change of pollutant concentration in terms of average advection and turbulent diffusion which is derived from the mass conservation principle

$$\frac{\partial C}{\partial t} + \left(u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z}\right) = \frac{\partial}{\partial x}K_x\frac{\partial C}{\partial x} + \frac{\partial}{\partial y}K_y\frac{\partial C}{\partial y} + \frac{\partial}{\partial z}K_z\frac{\partial C}{\partial z} + Q + R - - - - - (1)$$

Where C= pollutant concentration; t = time; u, v, w = flow speed co-ordinate in x, y and z direction; K_x, K_y and K_z = coefficient of turbulent diffusion in x, y and z direction; Q = Source; R = sink

This Gauusian plume model we can implement with his following assumption with some modification. Many assumptions and approximations are implied in the Gaussian plume model. Some of the important assumptions are (Lyons and Scott, 1990):

1. Continuous drain from the source at a constant rate, at least for a time equal to or greater than the time of travel to the location (receptor) of interest. The plume diffusion formulae assume that release and sampling times are long compared with the travel time to receptor, so that the material is spread out in the form of a steady plume between the source and the farthest receptor. A shorter release will result in an elongated puff with a time-dependent concentration field.

2. Steady-state flow and constant meteorological conditions, at least over the time of transport (travel) from the source to the farthest receptor. This assumption may not be valid during rapidly changing meteorological conditions, such as during the passage of a front or a storm and also during the morning and evening transition periods.

3. Conservation of mass in the plume. The continuity equation satisfied by the Gaussian plume formula is a mathematical expression of the condition that the mass flow rate through any plume cross section is equal to the source emission rate. This implies that none of the material is removed through chemical reaction, gravitational settling, or deposition at the surface. All the material reaching the surface through turbulent diffusion is reflected back and none is absorbed there.

4. Gaussian or reflected Gaussian distribution of mean concentration in the lateral (cross-wind) and vertical directions at any downwind location in the plume. The assumption of Gaussian distribution in the vertical direction is somewhat questionable, but does not appear to affect adversely the model predicted ground-level concentrations.

5. A constant mean transport flow in the horizontal (x-y) plane. This implies horizontal homogeneity of flow and the underlying surface and becomes invalid over a complex terrain.

6. The variation of flow speed with height can also be considered in more accurately estimating the effective transport velocity, but this requires the knowledge of vertical concentration distribution in the plume at each receptor location. The variation of wind direction with height is ignored, although its effect on the lateral plume spread and concentration field can be considered superficially through an appropriate parameterization of σ_y.

7. Strong enough flows to make turbulent diffusion in the direction of flow negligible in comparison with mean transport. This assumption, also known as the slender-plume approximation, which is implicit in the Gaussian plume model, generally becomes invalid very close to the source where material diffuses up-flow of the source due to longitudinal velocity fluctuations. The assumption becomes invalid farther and farther away from the source as mean flow becomes weaker and vanishes entirely (e.g., under extremely stable and free convection conditions).

In steady state above equation is reduced to model

$$u\frac{\partial C}{\partial x} - w_{set}\frac{\partial C}{\partial z} = \frac{\partial}{\partial x}K_x\frac{\partial C}{\partial x} + \frac{\partial}{\partial z}K_z\frac{\partial C}{\partial z} + Q\delta(x)\delta(y)\delta(z-H) - - - - - - - - (2)$$

With boundary condition

Boundary conditions $K_z\frac{\partial c}{\partial z} + W_{set}C = W_{dep}C$ at $z = 0(deposition)$

$C \to 0$ as $x, y \to \infty$ and $z \to \infty$

Particles settle due to gravity at speed W_{set} (m/s). Sedimentation occurs at the ground at speed W_{dep} (m/s).

Gaussian plume solution to this problem is given by

$$C(x,y,z) = \frac{Q}{2\pi U \sigma_y \sigma_z} e\left(-\frac{y^2}{2\sigma_y^2}\right)\left\{\exp\left(-\frac{(z-H)}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)}{2\sigma_z^2}\right)\right\} ----------(3)$$

Where, U is the constant velocity of the flow. σ_y and σ_z are the parameters of the normal distribution in y and y directions.

At pollutant source is always at ground surface or a source located at the ground (z=0) the previous equation reduces to

$$C(x,y,0) = \frac{Q}{\pi U \sigma_y \sigma_z} e\left(-\frac{y^2}{2\sigma_y^2} - \frac{H^2}{2\sigma_z^2}\right) ------------(4)$$

2 LINE SOURCE MODELS

A line source can be considered as a superposition of point sources. The solution for finite line source can be obtained by integrating point source solution from $y_s = y_1$ to y_2 with unit

source strength Q_i with the same n γ 's as in point source in different boundary conditions

$$C(x,y,z) = \int_{y_1}^{y_2} C(x, y_s, z)$$

3 AREA SOURCE MODELS

The concentration of air pollutants due to a finite area source at (x, y, z) is calculated as a Superposition of finite line sources extending from x_1 to x_2 in x direction is obtained as

$$C(x,y,z) = \int_{x_1}^{x_2} C(x - x_s, y, z)dx_s ------------(5)$$

where $C(x - x_s, y, z)$ is equivalent to the finite line source as obtained in section 2

However, the source strength Q_i is replaced by Q_a in with same γ_n's as in point source.

For an infinite area source with uniform strengthQ_{at}, the solution is obtained as:

$$C(x,z) = \int_{x_1}^{x_2} C(x - x_s, z)dx_s ------------(6)$$

where $C(x - x_s, z)$ is equivalent to the cross advection integrated concentration.

CONCLUSION

High concentration of pollutant in water is harmful to health of living animal. We tried to write expression for concentration using advection diffusion equation and tried to find the solution of this problem.

REFERENCES

1. Prashant Chandra1, Jaipal 1, V.K.Katiyar *Mathematical model for dispersion of air pollutant considering settling of particles and dry deposition* Int. J. of Mathematical Sciences and Applications, Vol. 1, No. 3, September 2011, 1591-1595

2 Marie Emmanuel Ntigura Habingabwa, Fidèle Ndahay and Fredrik Berntsson *Air Pollution Tracking using PDEs* Rwanda Journal,: Mathematical Sciences, Engineering and Technology Volume 27 Series C, 2012, 63-69

3 Oliver Maruntalu, Gheorghe Lazaroiu,Dana Andreya Bondrea

Mathematical model for air pollutants dispersion emitted by fuel combustion U.P.B. Sci. Bull., Series D, Vol. 77, Iss. 4, 2015, ISSN 1454-2358.

4 Adel A. Abdel-Rahman *On the atmospheric dispersion and gaussian plume model* 2nd International Conference on waste management, water pollution, air pollution, indoor climate (WWAI'08) Corfu, Greece, October 26-28, 2008

5 *Marek Slesicki Application of mathematical modelling methods in the protection of groundwater environment* Journal of water and land development
J. Water Land Dev. No. 13b, 2009: 31–39