
ON NEUTROSOPHIC SOFT COMPACT TOPOLOGICAL SPACES

A.HaydarEş*

Abstract

In this paper, the concept of almost and near compactness on neutrosophic soft topological space have been introduced along with the investigation of their several characteristics. That's shown that the neutrosophic soft continuous image of neutrosophic soft almost compact is neutrosophic soft almost compact and it's properties developed here.

Keywords:

Neutrosophic soft sets;
Compactness on
neutrosophic soft
topological space;
Neutrosophic soft
continuous.

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1. INTRODUCTION

After the introduction of the concept of a fuzzy set by Zadeh in his classic paper [1]. C.L.Chang [2] has defined fuzzy topological spaces. In 1983, Atannasov [3] introduced the notion of intuitionistic fuzzy sets. Soft sets theory was proposed by Molodtsov [4] in 1999, as a new mathematical tool for handling problems which contain uncertainties. Maji et al [5] gave the first practical application of soft sets in decision-making problems. Shabir

and Naz [6] presented soft topological spaces and defined some concepts of soft sets on this spaces and separation axioms. Moreover, topological structure on fuzzy soft set was defined by Çoker [7], Tanay and Kandemir [8], Varol and Aygün [9]. Turanlı and Es [10] defined compactness in intuitionistic fuzzy soft topological spaces. The concept of neutrosophic set (NS) was first introduced by Smarandache [11,12] which is generalization of classical sets, fuzzy set, intuitionistic fuzzy set etc. The concept of connectedness and compactness on neutrosophic soft topological space defined by Bera and Mahapatra [13].

2. PRELIMINARIES

Hereafter, we recall some necessary definitions and theorems related to neutrosophic soft set, neutrosophic soft topological space for the sake of completeness.

Definition 2.1. [11] Let X be a space of points (objects), with a generic element in X denoted by x . A neutrosophic set A is characterized by a truth-membership function T_A , an indeterminacy-membership function

I_A , and a falsity-membership function F_A . $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real Standard or non Standard subsets of $]0,1^+[$. That is $T_A, I_A, F_A : X \rightarrow]0,1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$, $F_A(x)$ and so ,
 $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. [4] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U . Then for $A \subseteq E$, a pair (F,A) is called a soft set over U , where $F:A \rightarrow P(U)$ is a mapping.

Definition 2.3. [5] Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of neutrosophic sets (NSs) of U . Then for $A \subseteq E$, a pair (F,A) is called a neutrosophic soft set (NSS) over U , where $F:A \rightarrow NS(U)$ is a mapping.

Definition 2.4. [14] Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of neutrosophic sets (NSs) of U . Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping $f_N:E \rightarrow NS(U)$ where f_N is called approximate function of the neutrosophic soft set N . In other words, the neutrosophic soft set is a parametrized family of some elements of the set $NS(U)$ and therefore it can be written as a set of ordered pairs,

$N = \{ (e, \{ \langle x, T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) \rangle : x \in U \}) : e \in E \}$ where $T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) \in [0,1]$,

respectively the truth-membership, indeterminacy-membership, falsity-membership function obvious.

Example 2.5. [15] Let $U = \{h_1, h_2, h_3\}$ be a set of houses and $E = \{e_1(\text{beautiful}), e_2(\text{wooden}), e_3(\text{costly})\}$ be a set of parameters with respect to which the nature of houses are described.

Let

$$f_N(e_1) = \{ \langle h_1, (0.5, 0.6, 0.3) \rangle, \langle h_2, (0.4, 0.7, 0.6) \rangle, \langle h_3, (0.6, 0.2, 0.3) \rangle \};$$

$$f_N(e_2) = \{ \langle h_1, (0.6, 0.3, 0.5) \rangle, \langle h_2, (0.7, 0.4, 0.3) \rangle, \langle h_3, (0.8, 0.1, 0.2) \rangle \};$$

$$f_N(e_3) = \{ \langle h_1, (0.7, 0.4, 0.3) \rangle, \langle h_2, (0.6, 0.7, 0.2) \rangle, \langle h_3, (0.7, 0.2, 0.5) \rangle \};$$

Then $N = \{ [e_1, f_N(e_1)], [e_2, f_N(e_2)], [e_3, f_N(e_3)] \}$ is an NSS over (U, E) .

Definition 2.6. [14] 1. The complement of a neutrosophic soft set N is denoted by N^c and is defined by

$$N^c = \{ (e, \{ \langle x, F_{fN(e)}(x), 1 - I_{fN(e)}(x), T_{fN(e)}(x) \rangle : x \in U \}) : e \in E \},$$

2. Let N_1 and N_2 be two NSSs over the common universe (U, E) . Then N_1 is said to be the neutrosophic soft subset of N_2 iff for each $e \in E$ and for each $x \in U$,

$$T_{fN_1(e)}(x) \leq T_{fN_2(e)}(x), I_{fN_1(e)}(x) \geq I_{fN_2(e)}(x), F_{fN_1(e)}(x) \geq F_{fN_2(e)}(x).$$

We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1 .

Definition 2.7. [14] 1. Let N_1 and N_2 be two NSSs over the common universe (U, E) . Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined as:

$$N_3 = \{ (e, \{ \langle x, T_{fN_3(e)}(x), I_{fN_3(e)}(x), F_{fN_3(e)}(x) \rangle : x \in U \}) : e \in E \} \text{ where } T_{fN_3(e)}(x) = T_{fN_1(e)}(x) \diamond T_{fN_2(e)}(x), I_{fN_3(e)}(x) = I_{fN_1(e)}(x) * I_{fN_2(e)}(x), F_{fN_3(e)}(x) = F_{fN_1(e)}(x) * F_{fN_2(e)}(x).$$

2. Their intersection is denoted by $N_1 \cap N_2 = N_4$ and is defined as:

$$N_4 = \{ (e, \{ \langle x, T_{fN_4(e)}(x), I_{fN_4(e)}(x), F_{fN_4(e)}(x) \rangle : x \in U \}) : e \in E \} \text{ where } T_{fN_4(e)}(x) = T_{fN_1(e)}(x) * T_{fN_2(e)}(x), I_{fN_4(e)}(x) = I_{fN_1(e)}(x) \diamond I_{fN_2(e)}(x), F_{fN_4(e)}(x) = F_{fN_1(e)}(x) \diamond F_{fN_2(e)}(x).$$

Definition 2.8. [13] 1. Let M and N be two NSSs over the common universe (U, E) . Then $M-N$ may be defined as, for each $e \in E$ and for each $x \in U$,

$$M-N = \{ \langle x, T_{fM(e)}(x) * F_{fN(e)}(x), I_{fM(e)}(x) \diamond (1 - I_{fN(e)}(x)), F_{fM(e)}(x) \diamond T_{fN(e)}(x) \rangle \};$$

2. A neutrosophic soft set N over (U, E) is said to be null neutrosophic soft set if T_f
 $N(e)(x)=0$, $I_{fN(e)}(x)=1$,

$F_{fN(e)}(x)=1$ for each $e \in E$ and for each $x \in U$. It is denoted by ϕ_u .

A neutrosophic soft set N over (U, E) is said to be absolute neutrosophic soft set if T_f
 $N(e)(x)=1$, $I_{fN(e)}(x)=0$,

$F_{fN(e)}(x)=0$ for each $e \in E$ and for each $x \in U$. It is denoted by 1_u .

Clearly, $\phi_u^c = 1_u$, $1_u^c = \phi_u$.

Definition 2.9. [13] Let $NSS(U, E)$ be the family of all neutrosophic soft sets over U via parameters in E and $\tau_u \subseteq NSS(U, E)$. Then τ_u is called neutrosophic soft topology on (U, E) if the following conditions are satisfied.

- (i) $\phi_u, 1_u \in \tau_u$,
- (ii) The intersection of any finite number of members of τ_u also belongs to τ_u .
- (iii) The union of any collection of members of τ_u belongs to τ_u .

Then the triple (U, E, τ_u) is called a neutrosophic soft topological space. Every member of τ_u is called τ_u -open neutrosophic soft set. An NSS is called τ_u -closed iff its complement is τ_u -open.

Definition 2.10. [13] Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M \in NSS(U, E)$ be arbitrary. Then the interior of M is denoted by M° or $\text{int}(M)$ and is defined as:

$$M^\circ = \cup \{N_1 : N_1 \text{ is neutrosophic soft open and } N_1 \subseteq M\}.$$

Definition 2.11. [13] Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $A \in NSS(U, E)$ be arbitrary. Then the closure of A is denoted by \bar{A} or $\text{cl}(A)$ and is defined as:

$$\bar{A} = \cap \{N_1 : N_1 \text{ is neutrosophic soft closed and } A \subseteq N_1\}.$$

Theorem 2.12. [13] Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $A \in NSS(U, E)$. Then, $(\bar{A})^c = (A^c)^\circ$ and $(A^\circ)^c = (\bar{A}^c)$.

Proposition 2.13. [13] Let N_1 and N_2 be two neutrosophic soft sets over (U, E) . Then,

- (i) $(N_1 \cup N_2)^c = N_1^c \cap N_2^c$,
- (ii) $(N_1 \cap N_2)^c = N_1^c \cup N_2^c$.

Definition 2.14. [13] Let (U, E, τ_u) be a neutrosophic soft topological space and $M \in \tau_u$. A family $\Omega = \{Q_i : i \in \Gamma\}$ of neutrosophic soft sets is said to be a cover of M if $M \subseteq \cup Q_i$.

If every member of that family which covers M is neutrosophic soft open then it is called open cover of M . A subfamily of Ω which also covers M is called a subcover of M .

Definition 2.15. [13] Let (U, E, τ_u) be a neutrosophic soft topological space and $M \in \tau_u$. Suppose Ω be an open cover of M . If Ω has a finite subcover which also covers M then M is called neutrosophic soft compact.

Definition 2.16. [13] Let $\varphi : U \rightarrow V$ and $\psi : E \rightarrow E$ be two functions where E is the parameter set each of the crisp sets U and V . Then the pair (φ, ψ) is called an NSS function from (U, E) to (V, E) . We write, $(\varphi, \psi) : (U, E) \rightarrow (V, E)$.

Definition 2.17. [13] Let (M, E) and (N, E) be two NSSs defined over U and V , respectively and (φ, ψ) be an NSS function from (U, E) to (V, E) . Then,

- (1) The image of (M, E) under (φ, ψ) , denoted by $(\varphi, \psi)(M, E)$, is an NSS over V and is defined as:

$(\varphi, \psi)(M, E) = (\varphi(M), \psi(E)) = \{ \langle \psi(a), f_{\varphi(M)}(\psi(a)) \rangle : a \in E \}$ where for each $b \in \psi(E)$ and $y \in V$.

$\max_{\varphi(x)=y} \max_{\psi(a)=b} [T_{f(M)(a)}(x)],$ if $x \in \varphi^{-1}(y)$,

$T_{\varphi(M)(b)}(y) = \{ 0, \text{ otherwise.}$

$\min_{\varphi(x)=y} \min_{\psi(a)=b} [I_{f(M)(a)}(x)],$ if $x \in \varphi^{-1}(y)$,

$I_{\varphi(M)(b)}(y) = \{ 1, \text{ otherwise.}$

$\min_{\varphi(x)=y} \min_{\psi(a)=b} [F_{f(M)(a)}(x)],$ if $x \in \varphi^{-1}(y)$,

$F_{\varphi(M)(b)}(y) = \{ 1, \text{ otherwise.}$

- (2) The pre-image of (N, E) under (φ, ψ) , denoted by $(\varphi, \psi)^{-1}(N, E)$, is an NSS over U and is defined by:

$(\varphi, \psi)^{-1}(N, E) = (\varphi^{-1}(N), \psi^{-1}(E))$ where for each $a \in \psi^{-1}(E)$ and $x \in U$.

$T_{\varphi^{-1}(N)(a)}(x) = T_{fN(\psi(a))}(\varphi(x)),$

$I_{\varphi^{-1}(N)(a)}(x) = I_{fN(\psi(a))}(\varphi(x)),$

$F_{\varphi^{-1}(N)(a)}(x) = F_{fN(\psi(a))}(\varphi(x)).$

If ψ and φ are injective (surjective), then (φ, ψ) is injective (surjective).

Definition 2.18. [13] Let (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces. $(\varphi, \psi) : (U, E, \tau_u) \rightarrow (V, E, \tau_v)$ is said to be a neutrosophic soft continuous mapping if for each $(N, E) \in \tau_v$, the inverse image $(\varphi, \psi)^{-1}(N, E) \in \tau_u$ i.e., the inverse image of each open NSS in (V, E, τ_v) is also open in (U, E, τ_u) .

Theorem 2.19. [13] Let (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces. Also let, $(\varphi, \psi) : (U, E, \tau_u) \rightarrow (V, E, \tau_v)$ be a neutrosophic soft continuous mapping. If (M, E) is neutrosophic soft compact in (U, E, τ_u) , then $(\varphi, \psi)(M, E)$ is so in (V, E, τ_v) .

3. NEUTROSOPHIC SOFT ALMOST COMPACTNESS AND NEUTROSOPHIC SOFT NEAR COMPACTNESS

Here, the Notion of almost compactness and near compactness on neutrosophic soft topological space is developed with some basic theorems.

Definition 3.1. (a) A neutrosophic soft topological space (U, E, τ_u) is called neutrosophic soft almost compact iff every open cover of (U, E, τ_u) has a finite subcollection whose closures cover (U, E, τ_u) , or equivalently, every open cover contains a finite subcollection whose closures form a cover of (U, E, τ_u) .

(b) A neutrosophic soft topological space (U, E, τ_u) is called neutrosophic soft nearly compact iff every open cover of (U, E, τ_u) has a finite subcollection such that the interiors of closures of neutrosophic soft sets in this subcollection covers (U, E, τ_u) .

Example 3.2. Let $U = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau_u = \{\phi_u, 1_u, N_1, N_2, N_3, N_4\}$, where N_1, N_2, N_3, N_4 being neutrosophic soft sets are defined as following:

$$\begin{aligned} f_{N_1}(e_1) &= \{ \langle h_1, (1, 0, 1) \rangle, \langle h_2, (0, 0, 1) \rangle \}; \\ f_{N_1}(e_2) &= \{ \langle h_1, (0, 1, 0) \rangle, \langle h_2, (1, 0, 0) \rangle \}; \\ f_{N_2}(e_1) &= \{ \langle h_1, (0, 1, 0) \rangle, \langle h_2, (1, 1, 0) \rangle \}; \\ f_{N_2}(e_2) &= \{ \langle h_1, (1, 0, 1) \rangle, \langle h_2, (0, 1, 1) \rangle \}; \\ f_{N_3}(e_1) &= \{ \langle h_1, (1, 1, 1) \rangle, \langle h_2, (0, 1, 1) \rangle \}; \\ f_{N_3}(e_2) &= \{ \langle h_1, (0, 1, 0) \rangle, \langle h_2, (0, 1, 1) \rangle \}; \\ f_{N_4}(e_1) &= \{ \langle h_1, (1, 1, 0) \rangle, \langle h_2, (1, 1, 0) \rangle \}; \\ f_{N_4}(e_2) &= \{ \langle h_1, (1, 0, 0) \rangle, \langle h_2, (0, 1, 1) \rangle \}; \end{aligned}$$

Here $N_1 \cap N_1 = N_1, N_1 \cap N_2 = \phi_u, N_1 \cap N_3 = N_3, N_1 \cap N_4 = N_3, N_2 \cap N_2 = N_2, N_2 \cap N_3 = \phi_u, N_2 \cap N_4 = N_2, N_3 \cap N_3 = N_3, N_3 \cap N_4 = N_3, N_2 \cap N_4 = N_4$, and $N_1 \cup N_1 = N_1, N_1 \cup N_2 = \phi_u, N_1 \cup N_3 = N_1, N_1 \cup N_4 = 1_u, N_2 \cup N_2 = N_2, N_2 \cup N_3 = N_4, N_2 \cup N_4 = N_4, N_3 \cup N_3 = N_3, N_3 \cup N_4 = N_4, N_4 \cup N_4 = N_4$;

Corresponding t-norm and s-norm are defined as $a * b = \max\{a+b-1, 0\}$ and $a \diamond b = \min\{a+b, 1\}$. Then τ_u is a neutrosophic soft topology on (U, E) and so (U, E, τ_u) is a neutrosophic soft topological space over (U, E) [13].

The family $\{N_1, N_2, N_3, N_4\}$ is an open cover of (U, E, τ_u) . Since $\text{cl}(N_1 \cup N_2) = \text{cl}(N_1 \cup N_2) = 1_u$, (U, E, τ_u) is neutrosophic soft almost compact topological space. Also, since $\text{int}(\text{cl}(N_1 \cup N_2)) = \text{int}(\text{cl}(N_1 \cup N_2)) = 1_u$, (U, E, τ_u) is neutrosophic soft nearly compact topological space.

It is clear that in neutrosophic soft topological spaces we have the following implications:
neutrosophic soft compact \rightarrow neutrosophic soft nearly compact \rightarrow neutrosophic soft almost compact.

Theorem 3.3. A neutrosophic soft topological space (U, E, τ_u) is called neutrosophic soft almost compact iff each family $\Omega = \{Q_i : i \in I\}$ of neutrosophic soft open sets in (U, E, τ_u) having the finite intersection property we have $\bigcap_{i \in I} \text{cl}(Q_i) \neq \phi_u$.

Proof. Let (U, E, τ_u) be an almost compact neutrosophic soft topological space. Consider $\Omega = \{Q_i : i \in I\}$ be a family of neutrosophic soft open sets in (U, E, τ_u) having the finite intersection property. Suppose the $\bigcap_{i \in I} \text{cl}(Q_i) = \phi_u$. Then we have $\bigcup_{i \in I} [\text{cl}(Q_i)]^c = \bigcap_{i \in I} \text{int}(Q_i^c) = 1_u$. Since (U, E, τ_u) almost compact neutrosophic soft topological space, there exists a finite subfamily $\{Q_i^c : i = 1, 2, \dots, n\}$ such that $\bigcup_{i=1}^n \text{cl}(\text{int}(Q_i^c)) = 1_u$. Hence $\bigcup_{i=1}^n \text{cl}([\text{cl}(Q_i)]^c) = \bigcup_{i=1}^n [\text{int}(\text{cl}(Q_i))]^c = 1_u \Rightarrow \bigcap_{i=1}^n \text{int}(\text{cl}(Q_i)) = \phi_u$. But from $Q_i = \text{int}(Q_i) \subseteq \text{int}(\text{cl}(Q_i))$, we see that $\bigcap_{i=1}^n Q_i = \phi_u$ which in contradiction with the finite intersection property of the family.

Next assume that (U, E, τ_u) is not almost compact. Then, a neutrosophic soft open cover of $\{Q_i : i \in I\}$, say, of (U, E, τ_u) has no finite subcover i.e., $\bigcup_{i=1}^n \text{cl}(Q_i) \neq 1_u$. Since $[\text{cl}(Q_i)]^c = \text{int}(Q_i^c)$, consists of neutrosophic soft open sets in (U, E, τ_u) and having the finite intersection property. Then by hypothesis, $\bigcap_{i=1}^n \text{cl}([\text{cl}(Q_i)]^c) \neq \phi_u \Rightarrow \bigcup_{i=1}^n [\text{cl}([\text{cl}(Q_i)]^c)] \neq 1_u$.

$(Q_i)]^c] \neq 1_u \Rightarrow \bigcup_{i=1}^n \text{int}(\text{cl}(Q_i)) \neq 1_u$ which is in contradiction with $\bigcup_{i=1}^n Q_i = 1_u$ since $Q_i \subseteq \text{int}(\text{cl}(Q_i))$ for each $i=1,2,\dots,n$.

Definition 3.4. A neutrosophic soft set N_1 is called a neutrosophic soft regular open set iff $N_1 = \text{int}(\text{cl}(N_1))$; a neutrosophic soft set N_2 is called a neutrosophic soft regular closed set iff $N_2 = \text{cl}(\text{int}(N_2))$.

Theorem 3.5. In a neutrosophic soft topological space (U, E, τ_u) the following conditions are equivalent:

- (i) (U, E, τ_u) is neutrosophic soft almost compact.
- (ii) For each family $\Omega = \{Q_i : i \in I\}$ of neutrosophic soft regular closed sets such that $\bigcap_{i \in I} Q_i = \phi_u$, there exists a finite subfamily $\Omega_1 = \{Q_i : i = 1, 2, \dots, n\}$ such that $\bigcap_{i=1}^n Q_i = \phi_u$.
- (iii) $\bigcap_{i \in I} \text{cl}(Q_i) \neq \phi_u$ holds for each family $\Omega = \{Q_i : i \in I\}$ of neutrosophic soft regular open sets having the finite intersection property.
- (iv) Each neutrosophic soft regular open cover of (U, E, τ_u) contains a finite subfamily whose closures cover (U, E, τ_u) .

Proof. The proof of this theorem follows a similar pattern to Theorem 3.3.

Definition 3.6. Let (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces. Then $(\varphi, \psi) : (U, E, \tau_u) \rightarrow (V, E, \tau_v)$ is said to be a neutrosophic soft almost continuous mapping if for each (N, E) neutrosophic soft regular open set of (V, E, τ_v) , the inverse image $(\varphi, \psi)^{-1}(N, E) \in \tau_u$ i.e., the inverse image of each neutrosophic soft regular open set in (V, E, τ_v) is neutrosophic soft open in (U, E, τ_u) .

Theorem 3.7. Let (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces and $(\varphi, \psi) : (U, E, \tau_u) \rightarrow (V, E, \tau_v)$ a neutrosophic soft almost continuous surjection mapping. If (M, E) is neutrosophic soft almost compact in (U, E, τ_u) , then $(\varphi, \psi)(M, E)$ is so in (V, E, τ_v) .

Proof. Let $\{(N_i, E) : i \in I\}$ be a neutrosophic soft open cover of $(\varphi, \psi)(M, E)$ i.e., $(\varphi, \psi)(M, E) \subseteq \bigcup_{i \in I} (N_i, E)$. Since (φ, ψ) is neutrosophic soft almost continuous,

$\{(\varphi, \psi)^{-1} \text{int}(\text{cl}((N_i, E))) : i \in I\}$ is a neutrosophic soft open cover of (M, E) . Since (M, E) is almost compact, there exists a finite subcover $\{(\varphi, \psi)^{-1}(N_i, E) : i=1, 2, \dots, n\}$ such that

$(M, E) \subseteq \bigcup_{i=1}^n \text{cl}((\varphi, \psi)^{-1}(\text{int}(\text{cl}(N_i, E)))) = 1_u$. Hence

$(\varphi, \psi)(M, E) \subseteq (\varphi, \psi)[\bigcup_{i=1}^n \text{cl}((\varphi, \psi)^{-1}(\text{int}(\text{cl}(N_i, E))))] = \bigcup_{i=1}^n (\varphi, \psi)[\text{cl}((\varphi, \psi)^{-1}(\text{int}(\text{cl}(N_i, E))))] = f(1_u) = 1_v$. But from $\text{int}(\text{cl}(N_i, E)) \subseteq \text{cl}(N_i, E)$ and from the neutrosophic soft almost continuity of f ,

$(\varphi, \psi)(\text{cl}((\varphi, \psi)^{-1}(\text{int}(\text{cl}(N_i, E)))) \subseteq (\varphi, \psi)((\varphi, \psi)^{-1}(\text{cl}(N_i, E))) \subseteq \text{cl}(N_i, E)$ for each $i=1, 2, \dots, n$, i.e., $\bigcup_{i=1}^n \text{cl}(N_i, E) = 1_v$. Hence, $(\varphi, \psi)(M, E)$ is neutrosophic soft almost compact also.

Definition 3.8. Let (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces. Then $(\varphi, \psi) : (U, E, \tau_u) \rightarrow (V, E, \tau_v)$ is said to be a neutrosophic soft weakly continuous mapping if for each (N, E) neutrosophic soft open set of (V, E, τ_v) ,

$$(\varphi, \psi)^{-1}(N, E) \subseteq \text{int}((\varphi, \psi)^{-1}(\text{cl}(N, E))).$$

Theorem 3.9. Let (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces and $(\varphi, \psi) : (U, E, \tau_u) \rightarrow (V, E, \tau_v)$ a neutrosophic soft weakly continuous surjection mapping. If (M, E) is neutrosophic soft compact in (U, E, τ_u) , then $(\varphi, \psi)(M, E)$ is neutrosophic soft almost compact in (V, E, τ_v) .

Proof. The proof is similar to Theorem 3.7.

Definition 3.10. Let (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces. Then $(\varphi, \psi) : (U, E, \tau_u) \rightarrow (V, E, \tau_v)$ is said to be a neutrosophic soft strongly continuous mapping if for each (M, E) neutrosophic soft set of (V, E, τ_v) ,

$$(\varphi, \psi)[\text{cl}(M, E)] \subseteq (\varphi, \psi)(M, E).$$

Theorem 3.9. Let (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces and $(\varphi, \psi) : (U, E, \tau_u) \rightarrow (V, E, \tau_v)$ a neutrosophic soft strongly continuous surjection mapping. If (M, E) is neutrosophic soft almost compact in (U, E, τ_u) , then $(\varphi, \psi)(M, E)$ is neutrosophic soft compact in (V, E, τ_v) .

Proof. By using a similar technique of the proof of Theorem 3.7, the theorem holds.

Corollary 3.12. Let (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces and $(\varphi, \psi) : (U, E, \tau_u) \rightarrow (V, E, \tau_v)$ a neutrosophic soft strongly continuous surjection mapping. If (M, E) is neutrosophic soft nearly compact in (U, E, τ_u) , then $(\varphi, \psi)(M, E)$ is neutrosophic soft compact in (V, E, τ_v) .

4. Conclusion

In this paper, the concepts of Neutrosophic soft topological spaces are introduced and studied. Some interesting properties are established. The results in this work can be extended to the Neutrosophic connectedness properties.

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