International Journal of Engineering, Science and Mathematics
Vol. 8 Issue 10, October 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

INTERVAL GRAPH WITH CONSECUTIVE CLIQUES OF SIZE 3 - SIGNED ROMAN DOMINATION

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ABSTRACT

The theory of Graphs is an important branch of Mathematics that was developed exponentially.Domination in graphs is rapidly growing area of research in graph theory today. It has been studied extensively and finds applications to various branches of Science & Technology.

Interval graphs have drawn the attention of many researchers for over 40 years. They form a special class of graphs with many interesting properties and revealed their practical relevance for modeling problems arising in the real world.

In this paper a study of signed Roman domination in an interval graph with consecutive cliques of size 3 is carried out.

Keywords: Signed Roman dominating function, Signed Roman domination number, Interval family, Interval graph.

Mathematical Subject Classification: 05C69.

1. INTRODUCTION

The major development of graph theory has occurred in recent years and inspired to a larger degree and it has become the source of interest to many researchers due to its applications to various branches of Science & Technology.

Domination in graphs introduced by Ore [11] and Berge [3] has become an emerging area of research in graph theory today. Many graph theorists, Allan, R.B. and Laskar, R.[2], Cockayne and Hedetniemi [4, 5] and others have contributed significantly to the theory of dominating sets, domination numbers and other related topics. Haynes et al. [7, 8] presented a survey of articles in the wide field of domination in graphs.

Recently, dominating functions in domination theory have received much attention. A purely graph – theoretic motivation is given by the fact that the dominating function problem can be seen, in a clear sense, as a proper generalization of the classical domination problem. Similarly edge dominating functions are also studied extensively.

Let $G(V,E)$ be a graph. A subset D of V is said to be a dominating set of G if every vertex in $V-D$ is adjacent to a vertex in D. The minimum cardinality of a dominating set is called the domination number and is denoted by $\gamma(G)$.

We consider finite graphs without loops and multiple edges.

2. SIGNED ROMAN DOMINATING FUNCTION

The concept of Signed dominating function was introduced by Dunbar et al. [6]. There is a variety of possible applications for this variation of domination. By assigning the values -1 or $+1$ to the vertices of a graph we can model such things as networks of positive and negative electrical charges, networks of positive and negative spins of electrons and networks of people or organizations in which global decisions can be made.

The Roman dominating function of a graph G was defined by Cockayne et.al [5]. The definition of a Roman dominating function was motivated by an article in Scientific American by Ian Stewart [9] entitled “Defend The Roman Empire!” and suggested by even earlier byReVelle [12]. Domination number and Roman domination number in an interval graph with consecutive cliques of size 3 are studied byC. Jaya Subba Reddy, M.Reddappa and B.Maheswari [10].

The concept of signed Roman dominating function was introduced by Ahangar et al. [1]. They presented various lower and upper bounds on the signed Roman domination number of a graph and characterized the graphs which have these bounds.The minimal signed Roman dominating functions of corona product graph of a path with a star is studied by Siva Parvati [13].

Let $G(V,E)$ be a graph. A signed Roman dominating function on the graph G is a function $f: V \rightarrow \{-1,1,2\}$, which satisfies the following two conditions:

(i) For each $u \in V$, $\sum_{v \in N[u]} f(v) \geq 1$;

(ii) Each vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$.

Thevalue $f(V) = \sum_{u \in v} f(u)$ is called as the weight of the function f, and it is denoted by $w(f)$. The signed Roman domination number of G,denoted by $\gamma_{sR}(G)$ is the minimum weight of a signed Roman dominating function of$G$.

Each signed Roman dominating function f on G is uniquely determined by the ordered partitions $(V_{-1}, V_1, V_2)$of $V(G)$, where $V_i = \{v \in V / f(v) = i\}$ for $i = -1, 1, 2$. Then$w(f) = -|V_{-1}| + |V_1| + 2|V_2|$.

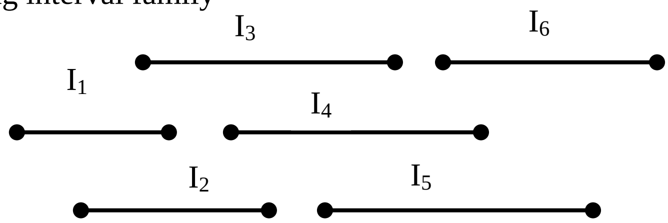
There exists a 1-1 correspondence between the functions $f : V \to \{-1, 1, 2\}$ and the ordered partitions$(V_{-1}, V_1, V_2)$of$V$. Thus we write $f = (V_{-1}, V_1, V_2)$.

3. INTERVAL GRAPH

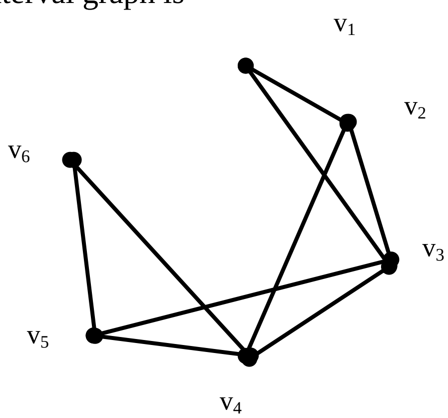
Let $I = \{I_1, I_2, I_3, \dots \dots \dots . . I_n\}$ be an interval family, where each I_i is an interval on the real line and $I_i = [a_i, b_i]$ for$i = 1, 2, 3, \dots \dots . . n$. Here a_i is called the left end point and b_i is called the right end point of I_i. Without loss of generality, we assume that all end points of the intervals in I are distinct numbers between 1 and 2n. Two intervals i = $[a_i, b_i]$ and j = $[a_j, b_j]$ are said to intersect each other if either$a_j < b_i$or $a_i < b_j$. The intervals are labelled in the increasing order of their right end points.

Let $G(V, E)$ be a graph. G is called an interval graph if there is a 1-1 correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. If i is an interval in I the corresponding vertex in G is denoted by v_i.

Consider the following interval family



The corresponding interval graph is



Interval graph

In what follows we consider interval graphs of this type. That is the interval graph which has consecutive cliques of size 3. We denote this type of interval graph by $\mathcal{G}$. Thesigned Roman domination is studied in the following for the interval graph $\mathcal{G}$.

4. RESULTS

Theorem 4.1: Let $\mathcal{G}$ be the Interval graph with n vertices, where $n \geq 6$. Then the signed Roman domination number of $\mathcal{G}$ is

$\gamma_{sR}(\mathcal{G}) = 2k + 2$ for $n = 5k + 1$, $5k + 3$, $5k + 5$

$= 2k + 3$ for $n = 5k + 2$, , $5k + 4$,

where $k = 1,2,3 \ldots \ldots$ ..respectively.

Proof: Let $\mathcal{G}$ be the interval graph with n vertices, where $n \geq 6$.

Let the vertex set of $\mathcal{G}$ be $\{v_1, v_2, v_3, v_4 \ldots \ldots \ldots \ldots \ldots .. v_n\}$.

Case 1: Suppose $n = 5k + 1$, where $k = 1,2,3 \ldots \ldots ..$.

Let $f : V \to \{-1, 1, 2\}$ and let (V_{-1}, V_1, V_2) be the ordered partition of V induced by f where $V_i = \{v \in V / f(v) = i\}$ for $i = -1, 1,2$. Then there exist a 1-1 correspondence between the functions $f : V \to \{-1, 1, 2\}$ and the ordered pairs (V_{-1}, V_1, V_2) of V.Thus we write $f = (V_{-1}, V_1, V_2)$.

Let $V_1 = \{v_1, v_4, v_6, \ldots \ldots \ldots \ldots \ldots .. v_{n-10}, v_{n-7}, v_{n-5,}, v_{n-2}\}$;

$V_2 = \{v_3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots .. v_{n-13}, v_{n-8,}, v_{n-3}, v_n\}$;

$V_{-1} = \{v_2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots .., v_{n-9}, v_{n-6,}, v_{n-4}, v_{n-1}\}$.

It was shown in [10] that V_2 is a minimumdominating set of $\mathcal{G}$. Further the set V_2 dominates V_{-1}. That is , every vertex u such that $f(u) = -1$ is adjacent to some vertex v with $f(v) = 2$.

Therefore $f = (V_{-1}, V_1, V_2)$ becomes a signed Roman dominating function of $\mathcal{G}$.

Now $|V_1| = 2k, |V_2| = 2k + 1, |V_{-1}| = 2k$.

Therefore $\sum_{v \in V} f(v) = \sum_{v \in V_{-1}} f(v) + \sum_{v \in V_1} f(v) + \sum_{v \in V_2} f(v)$

$$= -2k + 2k + 2k + 2 = 2k + 2.$$

Let $g = (V'_{-1},\ V'_1,\ V'_2)$ be a signed Roman dominating function of $\mathcal{G}$, where V'_2 dominates V'_{-1}. Then $g(V) = \sum_{v \in V'} g(v) = \sum_{v \in V'_{-1}} g(v) + \sum_{v \in V'_1} g(v) + \sum_{v \in V'_2} g(v)$

$$= -|V'_{-1}| + |V'_1| + 2|V'_2|$$

Since V_2 is a minimum dominating setof $\mathcal{G}$, we have $|V_2| \leq |V'_2|$. This implies that $g(V) = -|V'_{-1}| + |V'_1| + 2|V'_2| \geq -|V_{-1}| + |V_1| + 2|V_2| = f(V)$.

Therefore $f(V)$ is a minimum weightof $\mathcal{G}$, where $f(V_{-1}, V_1, V_2)$ is a signed Roman dominating function .

Thus $\gamma_{sR}(\mathcal{G}) = 2k + 2$.

Case 2: Suppose $n = 5k + 2$, where $k = 1,2,3 \ldots \ldots .$.

Now we proceed as in Case1.

Let $V_1 = \{v_1, v_4, v_6, \ldots \ldots \ldots \ldots \ldots . . v_{n-8}, v_{n-6}, v_{n-3,}, v_{n-1}\}$;

$V_2 = \{v_3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots . . v_{n-14}, v_{n-9,}, v_{n-4}, v_n\}$;

$V_{-1} = \{v_2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots . ., v_{n-10}, v_{n-7,}, v_{n-5}, v_{n-2}\}$.

ClearlyV_2 is a minimumdominating set of $\mathcal{G}$**.** Here we observe that the setV_2dominates V_{-1}.

Therefore $f = (V_{-1}, V_1, V_2)$ is a signed Roman dominating function of $\mathcal{G}$.

Now $|V_1| = 2k + 1, |V_2| = k + 1, |V_{-1}| = 2k$.

Therefore $\sum_{v \in V} f(v) = \sum_{v \in V_{-1}} f(v) + \sum_{v \in V_1} f(v) + \sum_{v \in V_2} f(v)$.

$$= -2k + 2k + 1 + 2k + 2 = 2k + 3.$$

If $g = (V'_{-1}, V'_1, V'_2)$ is a signed Roman dominating function of $\mathcal{G}$, then it follows as in Case 1, that $f(V)$ is a minimum weightof$\mathcal{G}$ for the Roman dominating function $f(V_{-1}, V_1, V_2)$.

Thus$\gamma_{sR}(\mathcal{G}) = 2k + 3$.

Case 3: Suppose $n = 5k + 3$, where $k = 1,2,3 \ldots \ldots .$.

Now we proceed as in Case1.

Let $V_1 = \{v_1, v_4, v_6, \ldots \ldots \ldots \ldots \ldots . . v_{n-9}, v_{n-7}, v_{n-4,}, v_{n-2}\}$;

$V_2 = \{v_3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots . . v_{n-15}, v_{n-10,}, v_{n-5}, v_n\}$;

$V_{-1} = \{v_2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots . ., v_{n-8}, v_{n-6,}, v_{n-3}, v_{n-1}\}$.

We have seen [10], that V_2 is a minimumdominating set of $\mathcal{G}$. Here we observe that the setV_2dominates V_{-1}.

Therefore $f = (V_{-1}, V_1, V_2)$ is a signed Roman dominating function of $\mathcal{G}$.

Now $|V_1| = 2k + 1, |V_2| = k + 1, |V_{-1}| = 2k + 1$.

Therefore $\sum_{v \in V} f(v) = \sum_{v \in V_{-1}} f(v) + \sum_{v \in V_1} f(v) + \sum_{v \in V_2} f(v)$.

$$= -2k - 1 + 2k + 1 + 2k + 2 = 2k + 2.$$

If $g = (V'_{-1}, V'_1, V'_2)$ is a signed Roman dominating function of $\mathcal{G}$, then it follows as in Case 1, that $f(V)$ is a minimum weightof$\mathcal{G}$ for the signed Roman dominating function $f(V_{-1}, V_1, V_2)$.

Thus $\gamma_{sR}(\mathcal{G}) = 2k + 2$.

Case 4: Suppose $n = 5k + 4$, where $k = 1,2,3 \ldots \ldots .$

Now we proceed as in Case1.

$$\text{Let } V_1 = \{v_1, v_4, v_6, \ldots \ldots \ldots \ldots \ldots v_{n-8}, v_{n-5}, v_{n-3}, v_n\};$$

$$V_2 = \{v_3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots v_{n-16}, v_{n-11}, v_{n-6}, v_{n-1}\};$$

$$V_{-1} = \{v_2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots, v_{n-9}, v_{n-7}, v_{n-4}, v_{n-2}\}.$$

Obviously V_2 is a minimumdominating set of $\mathcal{G}$. Here we observe that the set V_2dominates V_{-1}.

Therefore $f = (V_0, V_1, V_2)$ is a signed Roman dominating function of $\mathcal{G}$.

Now $|V_1| = 2k + 2, |V_2| = k + 1, |V_{-1}| = 2k + 1$.

Therefore $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$$= -2k - 1 + 2k + 2 + 2k + 2 = 2k + 3.$$

If $g = (V'_{-1}, V'_1, V'_2)$ is a signed Roman dominating function of $\mathcal{G}$, then it follows as in Case 1, that $f(V)$ is a minimum weightof $\mathcal{G}$ for the signed Roman dominating function $f(V_{-1}, V_1, V_2)$.

Hence $\gamma_{sR}(\mathcal{G}) = 2k + 3$.

Case 5: Suppose $n = 5k + 5$, where $k = 1,2,3 \ldots \ldots .$

Now we proceed as in Case1.

$$\text{Let } V_1 = \{v_1, v_4, v_6, \ldots \ldots \ldots \ldots \ldots v_{n-9}, v_{n-6}, v_{n-4}, v_{n-1}\};$$

$$V_2 = \{v_3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots v_{n-17}, v_{n-12}, v_{n-7}, v_{n-2}\};$$

$$V_{-1} = \{v_2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots, v_{n-8}, v_{n-5}, v_{n-3}, v_n\}.$$

Clearly V_2 is a minimumdominating set of $\mathcal{G}$and the set V_2 dominates V_{-1}.

Therefore $f = (V_{-1}, V_1, V_2)$ is a signed Roman dominating function of $\mathcal{G}$.

Now $|V_1| = 2k + 2, |V_2| = k + 1, |V_{-1}| = 2k + 2$.

Therefore $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$$= -2k - 2 + 2k + 2 + 2k + 2 = 2k + 2.$$

If $g = (V'_{-1}, V'_1, V'_2)$ is a signed Roman dominating function of $\mathcal{G}$, then it follows as in Case 1, that $f(V)$ is a minimum weightof $\mathcal{G}$ for the signed Roman dominating function $f(V_{-1}, V_1, V_2)$.

Hence $\gamma_{sR}(G) = 2k + 2$.

Theorem 4.2: Let $\mathcal{G}$ the an interval graph with n vertices, where $2 < n < 6$. Then $\gamma_{sR}(G) = 1$ for $n = 4$

$= 2$for$n = 3, 5$.

Proof: Let $\mathcal{G}$ be the interval graph with n vertices, where $2 < n < 6$.

Case 1: Suppose $n = 3$. Let v_1, v_2, v_3 be the vertices of $\mathcal{G}$.

Let $V_1 = \{v_1\}$; $V_2 = \{v_2\}$; $V_{-1} = \{v_3\}$

Obviously V_2 is a minimumdominating set of $\mathcal{G}$, and V_2 dominates V_{-1}.

Therefore $f = (V_{-1}, V_1, V_2)$ is a signed Roman dominating functionof $\mathcal{G}$.

And $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$= -1 + 1 + 2\text{x}1 = 2$.

Thus $\gamma_{sR}(\mathcal{G}) = 2$.

Case 2: Suppose $n = 4$. Let v_1, v_2, v_3, v_4 be the vertices of $\mathcal{G}$.

Let $V_1 = \{v_2\}$;$V_2 = \{v_3\}$; $V_{-1} = \{v_1, v_4\}$.

Clearly V_2 is a minimumdominating set of $\mathcal{G}$ and V_2dominates V_{-1}.

Therefore $f = (V_{-1}, V_1, V_2)$ is a signed Roman dominating function of $\mathcal{G}$.

And $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$= -2 + 1 + 2\text{x}1 = 1$.

Thus $\gamma_{sR}(\mathcal{G}) = 1$.

Case 3: Suppose $n = 5$. Let v_1, v_2, v_3, v_4, v_5 be the vertices of $\mathcal{G}$.

Let $V_1 = \{v_2, v_5\}$; $V_2 = \{v_3\}$; $V_{-1} = \{v_1, v_4\}$.

Again V_2 is a minimumdominating set of $\mathcal{G}$ and the set V_2dominates V_{-1}.

Therefore $f = (V_{-1}, V_1, V_2)$ is a signed Roman dominating function of $\mathcal{G}$.

Therefore $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$= -2 + 2 + 2 \times 1 = 2$.

Thus $\gamma_{sR}(\mathcal{G}) = 2$.

Theorem 4.3: Let $\mathcal{G}$ the an interval graph with n vertices , where $n \geq 6$. Then $\gamma_{sR}(\mathcal{G}) = \gamma_R(\mathcal{G}) + 1forn = 5k + 1,\ 5k + 2,$ and $5k + 4,$ where $k = 1, 2, 3, \ldots \ldots \ldots$respectively.

Proof : Let $\mathcal{G}$ be the interval graph with n vertices , where $n \geq 6$.

Then by [10], we have

$\gamma_R(\mathcal{G}) = 2k + 2$, for $n = 5k + 2, 5k + 4$, where $k = 1, 2, 3 \ldots \ldots \ldots \ldots$

$= 2k+1$, for $n = 5k+1$, where $k = 1,2,3 \dots\dots\dots\dots$

Now by Theorem 4.1, we have

$\gamma_{sR}(\mathcal{G}) = 2k+3$, for $n = 5k+2, 5k+4$, where $k = 1,2,3 \dots\dots\dots\dots$

$= 2k+2$, for $n = 5k+1$, where $k = 1,2,3 \dots\dots\dots\dots$

For $n = 5k+2, 5k+4$, where $k = 1,2,3 \dots\dots\dots\dots$

$$\gamma_{sR}(\mathcal{G}) = 2k+3$$

$$= (2k+2)+1 = \gamma_R(\mathcal{G})+1$$

Again for $n = 5k+1$, where $k = 1,2,3 \dots\dots\dots\dots$

$$\gamma_{sR}(\mathcal{G}) = 2k+2$$

$$= (2k+1)+1 = \gamma_R(\mathcal{G})+1$$

Theorem 4. 4: Let $\mathcal{G}$ be the Interval graph with n vertices , where $n \geq 8$.Then $\gamma_{sR}(\mathcal{G}) = \gamma_R(\mathcal{G})$ for $n = 5k+3, 5k+5$, where $k = 1,2,3, \dots\dots\dots.$ respectively.

Proof : Let $\mathcal{G}$ be the interval graph with n vertices , where $n \geq 8$.

Suppose $n = 5k+3, 5k+5$, where $k = 1,2,3, \dots\dots\dots.$

Then $\gamma_{sR}(\mathcal{G}) = 2k+2$ and $\gamma_R(\mathcal{G}) = 2k+2$,(by [10])

Hence $\gamma_{sR}(\mathcal{G}) = \gamma_R(\mathcal{G})$.

Theorem 4.5: Let $\mathcal{G}$ be the interval graph with n vertices , where $n \geq 6$. Then $\gamma_{sR}(\mathcal{G}) = 2\gamma(\mathcal{G})$,for= $5k+1, 5k+3, 5k+5$,where $k = 1,2,3 \dots\dots$ respectively.

Proof: Let $\mathcal{G}$ be the interval graph with n vertices , where $n \geq 6$.

Suppose $n = 5k+1, 5k+3, 5k+5$ where $k = 1,2,3 \dots\dots$ respectively.

Then by Theorem 4.1, the signed Roman domination number is

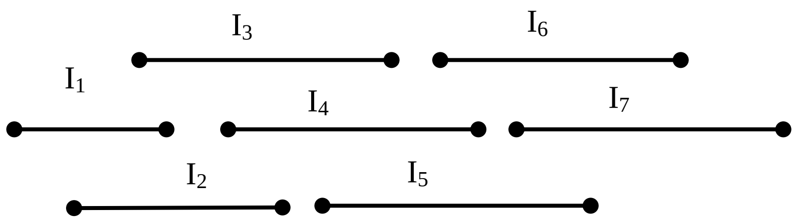
$$\gamma_{sR}(\mathcal{G}) = 2k+2$$

$= 2(k+1) = 2\gamma(\mathcal{G})$ (by[10])

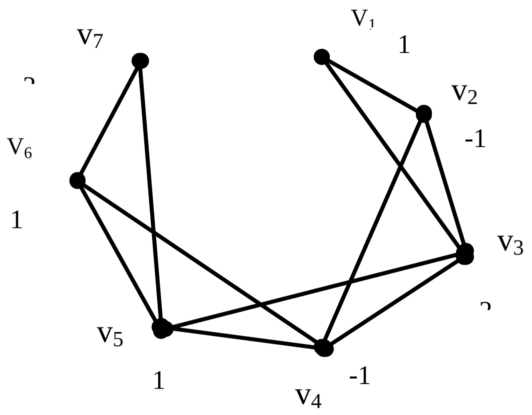
Thus $\gamma_{sR}(\mathcal{G}) = 2\gamma(\mathcal{G})$.

5. ILLUSTRATIONS

Illustration 1: n =7



Interval family



Interval graph

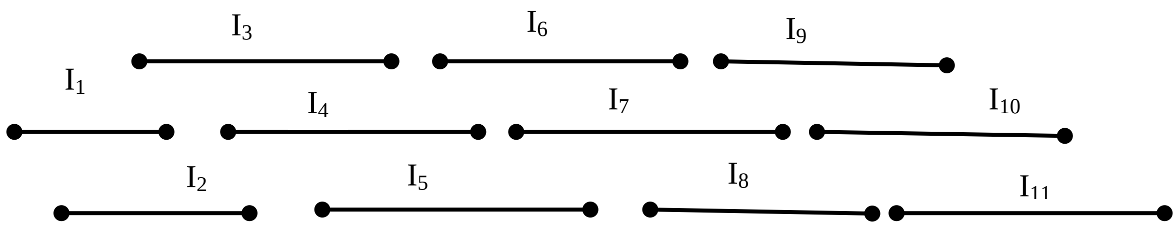
$D = \{v_3, v_7\}$and $\gamma(G) = 2.$

$V_1 = \{v_1, v_5, v_6\}; V_2 = \{v_3, v_7\}; V_{-1} = \{v_2, v_4\}$

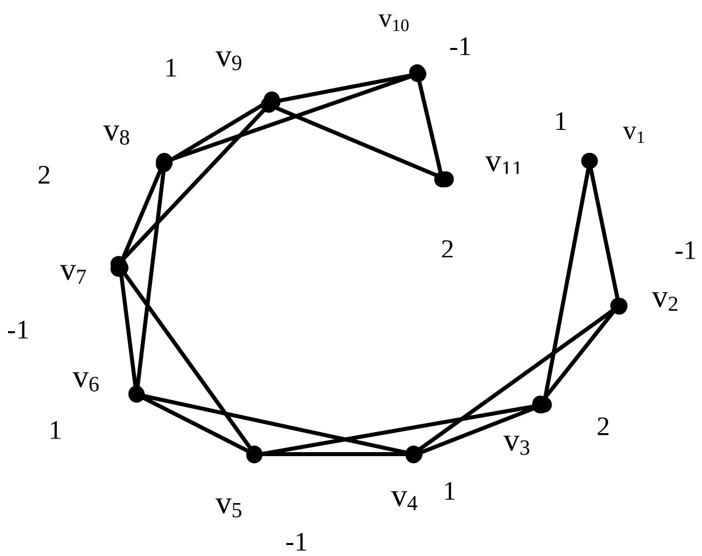
$\sum_{v \in V} f(v) = |V_{-1}|.-1 + |V_1|.1 + |V_2|.2 = -1(2) + 1(3) + 2(2) = 5 = f(V)$

Therefore $\gamma_{sR}(G) = 5.$

Illustration 2: n =11



Interval family



Interval graph

$D = \{v_3, v_8, v_{11}\}$and $\gamma(G) = 3.$

$V_1 = \{v_1, v_4, v_6, v_9\}$; $V_2 = \{v_3, v_8, v_{11}\}$; $V_{-1} = \{v_2, v_5, v_7, v_{10}\}$

$$\sum_{v \in V} f(v) = |V_{-1}|.-1 + |V_1|.1 + |V_2|.2 = -1(4) + 1(4) + 2(3) = 6 = f(V)$$

Therefore $\gamma_{sR}(G) = 6.$

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