International Journal of Engineering, Science and Mathematics
Vol. 8 Issue 11, November 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

PLASMIC FLUID EQUATIONS VS NON-PLASMIC FLUID

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ABSTRACT

This paper is based on the system of equations of plasma fluid dynamics. These fluids flow in ionic form. So, the equations of plasma fluid dynamics remains based on electric field as well as on magnetic field. This paper has given information that the system of equations of plasma fluid dynamics will be written with the help of law of continuity, Navier Stokes law, energy conservation law, law of motion in electric as well as in magnetic field, equation of state, Maxwell's equation and thermal conduction law. When all these are taken together is known as system of equations of plasma fluid dynamics.

KEYWORDS: Equation of continuity, law of conservation of energy, Navier stocks equations, Maxwell equation of electromagnetism, equation of state, Fourier law of conduction, laws of motion (in electric as well as magnetic field).

1. INTRODUCTION

As we know that Plasma state of fluid is a type of fluid which flows in electric and in magnetic field. In plasma state, fluid particles are found in the form of ions. Thus ionic movement takes place under magnetic as well as electric field.As it is clear that every moving charge already creates magnetic field around itself. So the plasma state of fluid demands a collection of equations from different areas. These collection of equations are known as the system of equations for plasma fluid dynamics. Thus plasma state behaves as particle as well as ionic nature. So its equations of system are written on the basis of particle nature as well as on ionic nature.As, it is seen that Plasma state of fluid is very important for the development of modern life-style. We have used the transport theorem in some part of this paper to make the way easier.

Transport Theorem[1] is used in fluid dynamics to obtain equation of continuity as well as the equation of energy conservation. Let us suppose that v(t) be an arbitrary volume, which is moving with the fluid and g(x,t) be the function for the position and for time of the moving volume. Therefore we will write the statement of transport theorem as follows,

$$\frac{d}{dt}\int_{v(t)} g(x,t)dx = \int_{v(t)}\left\{\frac{\partial}{\partial t}g(x,t)+u(x,t).\nabla g(x,t)+g(x,t)divu(x,t)\right\}dx \quad ....(1)$$

2. Now we will derive the equation of continuity [2] with the help of the transport theorem, by putting $g(x,t)=\rho(x,t)$.

Let us suppose that $\rho(x,t)$ be the mass per unit volume of a fluid at point *x* and time *t*.

Therefore, the mass of any finite volume of fluid continuum will be written mathematically in any finite volume V as,

Mass of finite volume V

$$=\int_{v}\rho(x,t)dx$$

But, according to the principle of conservation of mass, the mass of moving volume of continuum of fluid does not change. Hence

$$\frac{d}{dt}\int_{v(t)}\rho(x,t)dx=0$$

Now from the transport theorem eq. (1) after putting $g(x,t)=\rho(x,t)$.

$$\int_{v(t)}\left\{\frac{\partial\rho}{\partial t}+div(\rho u)=0\right\}dx$$

or,

$$\frac{\partial\rho}{\partial t}+div(\rho u)=0 \quad ....(2)$$

It is also known as the equation of continuity. But when fluid is in incompressible then $\frac{\partial\rho}{\partial t}=0$, so equation of continuity will be $\nabla.(\rho u)=0$.

3. We try to find the equation of the conservation of linear momentum [3] in fluid flow. Let the external forces such as gravitation or electromagnetic force act on the finite volume V of fluid continuum. Let f be force per unit mass, then force which is acting on finite continuum V.

$$\int_{v}\rho f dx$$

Let the internal forces t_n acting on per unit area on volume ∂V. So, total surface force will be

$$\int_{\partial V} t_n ds$$

Now, the principle of conservation of linear momentum says that the rate of change of linear momentum of a volume continuum V will be equal to the resultant force on that volume. So the mathematical equation will be,

$$\frac{d}{dt}\int_{V(t)} \rho u dx = \int_{V(t)} \rho f dx + \int_{\partial V(t)} t_n ds$$

or $$\int_{V(t)} \rho \frac{du}{dt} dx = \int_{V(t)} \rho f dx + \int_{\partial V(t)} t_n ds \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Here t_n is known as normal stress. [3]

$t_n(x,t,n) = n(x,t)T(x,t)$. As it is considered that $T = \{T_{ij}\}$ is a matrix, is called the stress tensor. Now from the equation (3) and green theorem [4], we get

$$\int_{V(t)} \rho \frac{du}{dt} dx = \int_{V(t)} (\rho f + divT) dx$$

Or $$\rho \frac{du}{dt} = \rho f + divT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

4. Now we find the equation of energy [6] for fluid flow. From the first low of thermodynamics .[12][13]

$$\frac{d}{dt}\int_{V(t)} \rho\left(\frac{1}{2}|u|^2 + E\right)dx$$

$$= \int_{V(t)} \rho f.u dx + \int_{\partial V(t)} t_n.u ds - \int_{\partial V(t)} q.n\, ds \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Here q is the heat flux so $-q.n$ is the heat flux into volume and E is the specific internal energy.

Now we use theorem of stress [5] whose statement is

$$\int_{\partial V(t)} F T_{ji} n_j ds = \int_{V(t)} \left[T_{ji}F_j + \rho F\left(\frac{du_i}{dt} - f_i\right)\right]dx$$

Here, F is a function of position and time. We put $F = u_i$ in stress theorem [5] and get,

$$\int_{\partial V(t)} u_i T_{ji} n_j ds = \int_{V(t)} \left(T_{ji}u_{ij} + \rho u_i \frac{du_i}{dt} - \rho f_i u_i\right)dx$$

We rearrange the terms and use transport theorem in (5)

$$\frac{d}{dt}\int_{V(t)} \rho\frac{1}{2}|u|^2 dx = \int_{V(t)} \rho\frac{1}{2}\frac{d\,|u|^2}{dt}dx = \int_{V(t)} \rho f_i u_i dx - \int_{V(t)} T_{ji}u_{i,j}dx + \int_{\partial V(t)} u_i(t_n)_i ds \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Now, from the equations (5), (6), the transport theorem (1) as well as from the green theorem [16] we get,

$$\int_{V(t)} \left(\rho\frac{dE}{dt} + \nabla.q - T:(\nabla u)\right)dx = 0$$

Here, $T:(\nabla u)$ represents the scalar product of T and ∇u.

Or, $\rho\frac{dE}{dt} = -\nabla.q + T:(\nabla u)$

5. Thus we get the conservation laws of fluid dynamics as follow

$$\left.\begin{aligned} &\frac{d\rho}{dt} + \rho\nabla.u = 0 \\ &\rho\frac{du}{dt} - \nabla.T - \rho f = 0 \\ &\rho\frac{dE}{dt} + \nabla.q - T:(\nabla u) = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Now we use the Fourier law [7] of heat conductivity

$$q = -k\nabla\theta, \qquad k \geq 0$$

As k is the thermal conductivity of the fluid, then energy equation will be

$$\rho\frac{dE}{dt} = \nabla.(k\nabla\theta) + T:(\nabla u)$$

6. When we substitute the stress tensor into the system of equations (7) we get following field equations

$$\left.\begin{aligned} &\frac{d\rho}{dt} = -\rho\nabla.u \\ &\rho\frac{d\rho}{dt} = -\nabla p + (\lambda+\mu)\nabla\, div\, u + \mu\,\Delta u + pf \\ &\rho\frac{dE}{dt} = -p\, div\, u + \rho\phi - \nabla.q \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Here $\rho\phi$ is the dissipation function of mechanical energy per unit mass.

Now we assume that the density of fluid ρ be uniform and incompressible

div u = 0

As the specific internal energy of the fluid becomes proportional to its temperature.

$E = C_r\theta$ here $C_r > 0$ (constant).

The dissipation function of mechanical energy per unit mass will be

$$\rho\phi = \lambda(div u)^2 + 2\mu D : D$$

7. Now, the system of field equations will be reduced into,

$$\left.\begin{aligned} &\frac{\partial\rho}{\partial t} + u.\nabla\rho = 0,\ div\ u = 0 \\ &\rho\left(\frac{\partial u}{\partial t} + (u.\nabla)u\right) = -\nabla p + \mu\,\Delta u + \rho f \\ &\rho C_r\left(\frac{\partial\theta}{\partial t} + u.\nabla\theta\right) = 2\mu D : D + k\Delta\theta \end{aligned}\right\} \quad \text{..........................(9)}$$

8. Again, we denote $h = \frac{\mu}{\rho}$, $l = {}^{k}\!/_{\rho}$, then we reduce equation (9) as follow

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} + (u.\nabla)u = -\frac{1}{\rho}\nabla p + h\Delta u + f \\ &div\ u = 0 \\ &C_r\left(\frac{\partial\theta}{\partial t} + u.\nabla\theta\right) = 2hD : D + l\Delta\theta \end{aligned}\right\} \quad \text{........................ (10)}$$

when the body forces f do not depend on temperature, then these equations

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} + (u.\nabla)u = -\frac{1}{\rho}\nabla\rho + h\Delta u + f \\ &div\ u = 0 \end{aligned}\right\} \quad \text{........................... (11)}$$

are known as Navier – Stokes equations of viscous incompressible fluids with uniform density. Now we will use these above equations along with electric and magnetic field equation to get, Plasma field fluid equation.

9. Let us consider a fluid of conductivity λ , moving with the velocity v in electromagnetic field (E,B). The mass density of the fluid is ρ , the electric charge density is ρ_e and the static pressure of the fluid is p.

The Maxwell's equation[9] are

$$\left.\begin{array}{ll}\text{(Faradays Law)} & \nabla \times E = -\dfrac{\partial B}{\partial t} \\ \text{(Gauss Law of Magnetism)} & \nabla . B = 0 \\ \text{(Ampere Law)} & \nabla . \dfrac{B}{\mu} = j \\ \text{(Gauss Law of Electricity)} & \nabla . D = \rho_e \end{array}\right\} \qquad \text{.......................(12)}$$

If the fluid is perfect [10] and infinitely conducting then $\xi = \eta = 0, \lambda \to \infty$, and the motion is isentropic s = constant. The fundamental system of equations with the help of Maxwell's law of electromagnetism reduces to,

$$\left.\begin{array}{l}\dfrac{\partial B}{\partial t} = \nabla \times (v \times B) \\ \in \nabla . E = \rho_{\in} \\ \rho_a = \rho F - \nabla p + j \times B \\ \dfrac{\partial \rho}{\partial t} = \nabla . (\rho v) = 0 \\ \dfrac{ds}{dt} = 0 \\ f(p, \rho, T) = 0 \end{array}\right\} \qquad \text{.........................(13)}$$

10. Now, we join Conservation laws of fluid dynamics with fundamental system of all equations of (13)

$$\left.\begin{array}{l}\dfrac{d\rho}{dt} + \rho \nabla . u = 0 \\ \rho \dfrac{du}{dt} - \nabla . T - \rho\, f = 0 \\ \rho \dfrac{dE}{dt} + \nabla . q - T : (\nabla u) = 0 \\ \dfrac{\partial B}{\partial t} = \nabla \times (v \times B) \\ \in \nabla . E = \rho_{\in} \\ \rho_a = \rho F - \nabla p + j \times B \\ \dfrac{ds}{dt} = 0 \end{array}\right\} \qquad \text{...........................(14)}$$

$$f(p,\rho,T)=0$$

These system of equations in (14) will be a complete equations of electromagneto fluid dynamics. These equations will be based on conservation laws, electrical laws, magnetic law as well as on thermal conduction law, with equation of state. In such type of fluid, particles are found in the form of ions. Due to the movement of ionic particles, the magnetic force/field is created, that is Lorentz force or electromagnetic force [14], $\vec{F}=-e\vec{v}\times\vec{B}$. Also on every charged particle electric force already remains present in the form of F= -eE. [15]. Thus such type of fluid is known as plasma. Plasma state of fluid, thus is written mathematically by the combination of Conservation laws (mass, momentum and energy), electric force, magnetic force laws, equation of state and thermal conduction law.

11. Conclusions: We have found that the equation of Plasma fluid dynamics is collection of following equations, which is supposed as the system of equations of plasmic fluid.

1. Equation of continuity
2. Equation of conservation of linear momentum (Navier – Stokes law)
3. Equation of conservation of energy
4. Equation of state.
5. Law of motion under magnetic as well as electric field.
6. Maxwell's electromagnetic field law.
7. Law of thermal conduction.

Thus plasma nature of fluid shows two types of behaviours one is particles behaviour and other is ionic behaviour. Therefore, on the basis of these two behaviours the system of equations of plasma nature of fluid are constructed mathematically.

Equations of Non-Plasmic Fluid	Equations of Plasmic Fluid
$\frac{d\rho}{dt}+\rho\nabla.u=0$	$\frac{d\rho}{dt}+\rho\nabla.u=0$
$\rho\frac{du}{dt}-\nabla.T-\rho f=0$	$\rho\frac{du}{dt}-\nabla.T-\rho f=0$
$\rho\frac{dE}{dt}+\nabla.q-T:(\nabla u)=0$	$\rho\frac{dE}{dt}+\nabla.q-T:(\nabla u)=0$
$f(P,\rho,T)=0$	$f(P,\rho,T)=0$

$\frac{ds}{dt}=0$	$\frac{ds}{dt}=0$
	$\frac{\partial B}{dt}=\nabla\times(v\times B)$
	$\in\nabla.E=\rho_{\in}$
	$\rho_a=\rho F-\nabla p+j\times B$

ACKNOWLEDGMENT :

This research paper has been done by second author during his Ph.D. research work.

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