International Journal of Engineering, Science and Mathematics
Vol. 8 Issue 11, November 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

UPPER TOTAL UNIDOMINATION NUMBER OF A PATH

V.ANANTHA LAKSHMI*
B.MAHESWARI**

ABSTRACT

The concept of total unidominating function was introduced and total unidominating functions of a path are studied in [8]. Minimal total unidominating functions and upper total unidomination number were introduced in [9]. In this paper the minimal unidominating functions of a path are studied and the upper total unidomination number of a path is found.

KEYWORDS:

Total unidominating function,
Minimal total unidominating function,
Upper total unidomination number.

Author correspondence:

V.Anantha Lakshmi,
Department of Mathematics,
P.R.Govt. College (A), Kakinada-533004, A. P, India

B.Maheswari
Professor(Retd.), Department of Applied Mathematics,
S.P.MahilaVisvavidyalayam(Women's University), Tirupati-517502, A. P, India

1. INTRODUCTION

Graph Theory is developing rapidly with its applications to other branches of Mathematics, Social Sciences, Physical Sciences and Technology. In which the theory of domination introduced by Berge [2] and Ore [6] is a rapidly growing area of research. Several graph theorists, Allan and Laskar [1], Cockayne and Hedetniemi [3], SampathKumar [7] and others have contributed significantly to the theory of domination.

Recently dominating functions in domination theory have received much attention. Hedetniemiet.al. [5] introduced the concept of dominating functions. The concept of total dominating functions was introduced by Cockayne et.al. [4]. The concept of total unidominating function was introduced by the authors in [8]. Minimal total unidominating functions and upper total unidomination number were introduced in [9].

In this paper the minima total unidominating functions of a path are studied and the upper total unidomination number of a path is found and the results obtained are illustrated.

2. UPPER TOTAL UNIDOMINATION NUMBER OF A PATH

In this section the upper total unidomination number of a path is discussed.First the concepts of minimal total unidominating functions and upper total unidomination number are defined as follows.

Definition 2.1: Let $G(V, E)$ be a connected graph. A function $f: V \to \{0,1\}$ is said to be a **total unidominating function,** if

$$\sum_{u \in N(v)} f(u) \geq 1 \ \forall\, v \in V \ and\ f(v) = 1,$$

$$\sum_{u \in N(v)} f(u) = 1 \ \forall\, v \in V \ \ and \ \ f(v) = 0,$$

where $N(v)$ is the open neighbourhood of the vertex v.

Definition 2.2: Let $G(V, E)$ be a connected graph. A total unidominating function $f: V \to \{0,1\}$ is called a **minimal total unidominating function** if for all $g < f$, g is not a total unidominating function.

Definition 2.3: The **upper total unidomination number** of a connected graph $G\,(V, E)$ is defined as max $\{f(V)/f$ is a minimal total unidominating function$\}$. It is denoted by $\Gamma_{tu}(G)$.

Theorem 2.1: The upper total unidomination number of a path P_n is

$$\Gamma_{tu}(P_n) = \begin{cases} 2 & if\ n = 2, \\ \left\lfloor \frac{5n}{7} \right\rfloor & if\ n > 2. \end{cases}$$

Proof: Let P_n be a path with vertex set $V = \{v_1, v_2, \dots, v_n\}$.

To find upper total unidomination number of P_n, the following seven cases arise.

Case1: Let $n \equiv 0(mod\ 7)$.

Define a function $f: V \to \{0, 1\}$ by

$$f(v_i) = \begin{cases} 1 & for\ i \equiv 2,3,4,5,6(mod\ 7), \\ 0 & for\ \ i \equiv 0,1(mod\ 7). \end{cases}$$

Now we prove that f is a minimal total unidominating function.

Subcase 1: Let $i \equiv 0(mod\ 7)$ and $i \neq n$. Then $f(v_i) = 0$.

Now $\sum_{u \in N(v_i)} f(u) = f(v_{i-1}) + f(v_{i+1}) = 1 + 0 = 1.$

For $i = n$ we have $\sum_{u \in N(v_n)} f(u) = f(v_{n-1}) = 1.$

Subcase 2: Let $i \equiv 1(mod\ 7)\ and\ i \neq 1$. Then $f(v_i) = 0$.

Now $\sum_{u \in N(v_i)} f(u) = f(v_{i-1}) + f(v_{i+1}) = 0 + 1 = 1.$

$For\ \ i = 1\ \ we\ have$ $\sum_{u \in N(v_1)} f(u) = f(v_2) = 1.$

Subcase 3: Let $i \equiv 2(mod\ 7)$. Then $f(v_i) = 1$.

Now $\sum_{u \in N(v_i)} f(u) = f(v_{i-1}) + f(v_{i+1}) = 0 + 1 = 1.$

Subcase 4: Let $i \equiv 3,4,5(mod\ 7)$. Then $f(v_i) = 1$.

Now $\sum_{u \in N(v_i)} f(u) = f(v_{i-1}) + f(v_{i+1}) = 1 + 1 = 2 > 1.$

Subcase 5: Let $i \equiv 6(mod\ 7)$. Then $f(v_i) = 1$.

Now $\sum_{u \in N(v_i)} f(u) = f(v_{i-1}) + f(v_{i+1}) = 1 + 0 = 1.$

Hence from all the above subcases it follows that f is a total unidominating function.

Now we check for the minimality of f.

Define a function $g: V \to \{0,1\}$ by

$g(v_i) = f(v_i)$ for all $v_i \in V, i \neq k,\ k \equiv 2(mod\ 7)\ and\ \ g(v_k) = 0.$

Then by the definition of f and g it is obvious that $g < f$.

Since $k \equiv 2(mod\ 7),\ k-1 \equiv 1(mod\ 7).$ Then $g(v_{k-1}) = f(v_{k-1}) = 0.$

But $\sum_{u \in N(v_{k-1})} g(u) = g(v_{k-2}) + g(v_k) = 0 + 0 = 0 \neq 1.$

Therefore g is not a total unidominating function.

Similarly when $k \equiv 3,4,5,6(mod\ 7)$, then also we can show that g is not a total unidominating function.

Hence for all possibilities of defining a function $g < f$, we can see that g is not a total unidominating function.

Therefore f is a minimal total unidominating function.

$$\text{Now} \sum_{u \in V} f(u) = \sum_{i=1}^{n} f(v_i) = \underbrace{0+1+1+1+1+1+0}+\ldots..$$

$$+\underbrace{0+1+1+1+1+1+0} = \frac{5n}{7}.$$

Therefore $\Gamma_{tu}(P_n) \geq \frac{5n}{7} --- (1)$

Let f be a minimal total unidominating function of P_n. Then amongst seven consecutive vertices in P_n atmost five consecutive vertices can have functional value 1 and atleast two vertices must have functional value 0.

Therefore sum of the functional values of seven consecutive vertices is less than or equal to 5.

That is $\sum_{i=1}^{7} f(v_i) \leq 5, \sum_{i=8}^{14} f(v_i) \leq 5, \ldots, \sum_{i=n-6}^{n} f(v_i) \leq 5.$

$$\text{Therefore} \sum_{u \in V} f(u) = \sum_{i=1}^{7} f(v_i) + \sum_{i=8}^{14} f(v_i) + \cdots + \sum_{i=n-6}^{n} f(v_i) \leq \underbrace{5+5+\cdots+5}_{\frac{n}{7}\ times} \leq \frac{5n}{7}.$$

Since f is arbitrary, it follows that $\Gamma_{tu}(P_n) \leq \frac{5n}{7} --- (2)$

Thus from the inequalities (1) and (2), we have $\Gamma_{tu}(P_n) = \frac{5n}{7}$.

Case 2: Let $n \equiv 1(mod\ 7)$.

Define a function $f: V \to \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & for\ i \equiv 2,3,4,5,6(mod\ 7), i \neq n-3, i \neq n-2\ , \\ 0 & for\ i \equiv 0,1(mod\ 7), i \neq n-1, i \neq n. \end{cases}$$

and $f(v_{n-3}) = 0, f(v_{n-2}) = 0, f(v_{n-1}) = 1, f(v_n) = 1.$

Then this function is defined similarly as the function f defined in Case 1 and so for the vertices $v_1, v_2, \dots, v_{n-4}$ the function f is a total unidominating function. We can check easily the condition of total unidominating function for the remaining vertices $v_{n-3}, v_{n-2}, v_{n-1}, v_n$ and hence f becomes a total unidominating function.
Now we check for the minimality of f.

Define a function $g: V \to \{0,1\}$ by

$$g(u) = f(u) \quad \forall\, u \in V,\ \ u \neq v_n$$

and $g(v_n) = 0.$

Then by the definition of f and g, it is obvious that $g < f$.

Now $g(v_{n-1}) = f(v_{n-1}) = 1.$ But

$$\sum_{u \in N(v_{n-1})} g(u) = g(v_{n-2}) + g(v_n) = 0 + 0 = 0 \neq 1.$$

Therefore g is not a total unidominating function.

For all possibilities of defining a function $g < f$, we can see that g is not a total unidominating function.

Therefore f is a minimal total unidominating function.

$$\text{Now} \sum_{u \in V} f(u) = \underbrace{0+1+1+1+1+1+0}+\cdots+\underbrace{0+1+1+1+0}+\underbrace{0+1+1}$$

$$= 5\left(\frac{n-8}{7}\right) + 3 + 2 = \frac{5n-5}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Therefore $\Gamma_{tu}(P_n) \geq \left\lfloor \frac{5n}{7} \right\rfloor - - - (1)$

Let f be a minimal total unidominating function.

Suppose $n = 8$. Then the possible assignment of functional values to these eight vertices is 1,1,0,0,1,1,1,0 or 0,1,1,1,0,0,1,1, so that $f(V) = 5$ *and*

$\Gamma_{tu}(P_8) = 5 = \left\lfloor \frac{5n}{7} \right\rfloor = \left\lfloor \frac{40}{7} \right\rfloor$.

Let $n \geq 15$.

As in Case 1 of this Theorem we have $\sum_{i=2}^{n} f(v_i) \leq \frac{5(n-1)}{7}$.

Now we assign the functional value to v_1 as follows.

Suppose $f(v_1) = 0$.

Then $f(V) = f(v_1) + \sum_{i=2}^{n} f(v_i) \leq 0 + \frac{5(n-1)}{7} = \frac{5n-5}{7} = \left\lfloor \frac{5n}{7} \right\rfloor$.

Suppose $f(v_1) = 1$.

In such case among the $\frac{n-1}{7}$ sets of seven consecutive vertices, there will be one set of seven consecutive vertices whose functional values sum is 4.Otherwise the assignment makes f no more a minimal total unidominating function. Without loss of generality assume that the last set of seven consecutive vertices has functional values sum 4.

$$\text{Then } f(V) = f(v_1) + \sum_{i=2}^{n-7} f(v_i) + \sum_{i=n-6}^{n} f(v_i) \leq 1 + \frac{5(n-8)}{7} + 4 = \frac{5n-5}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Since f is arbitrary it follows that $\Gamma_{tu}(P_n) \leq \left\lfloor \frac{5n}{7} \right\rfloor$ --- (2)

Thus from the inequalities (1) and (2), we have $\Gamma_{tu}(P_n) = \left\lfloor \frac{5n}{7} \right\rfloor$.

Case 3: Let $n \equiv 2(mod\ 7)$.

Sub case 1: Let $n = 2$.

Then there is only one total unidominating function f defined by

$$f(v_1) = 1, f(v_2) = 1.$$

Thus total unidomination number of P_2 is 2.

Sub case 2: Let $n \geq 9$.

Define a function $f: V \rightarrow \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & for\ \ i \equiv 2,3,4,5,6(mod\ 7), i \neq n-3, \\ 0 & for\ \ i \equiv 0,1(mod\ 7), \quad\ \ i \neq n-1, \end{cases}$$

and $f(v_{n-3}) = 0,\ f(v_{n-1}) = 1.$

On similar lines to Case 1 we can verify that f is a total unidominating function.

Now we check for the minimality of f.

Define a function $g: V \to \{0,1\}$ by

$g(u) = f(u)\ \forall\ u \in V, u \neq v_{n-1}$ and $g(v_{n-1}) = 0.$

Then by the definitions of f and g it is obvious that $g < f$ and for $g(v_{n-2}) = 0$,we have

$$\sum_{u \in N(v_{n-2})} g(u) = g(v_{n-3}) + g(v_{n-1}) = 0 + 0 = 0 \neq 1.$$

Therefore g is not a total unidominating function.

Thus for all possibilities of defining a function $g < f$, we can see that g is not a total unidominating function.

Therefore f is a minimal total unidominating function.

$$\text{Now} \sum_{u \in V} f(u) = \underbrace{0+1+1+1+1+1+0}+\cdots+\underbrace{0+1+1+1+1+0}+\underbrace{0+1+1}$$

$$= 5\left(\frac{n-9}{7}\right) + 6 \ = \frac{5n-3}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Therefore $\Gamma_{tu}(P_n) \geq \left\lfloor \frac{5n}{7} \right\rfloor - - - (1)$

Let f be a minimal total unidominating function.

Suppose $n = 9$. Then the possible assignment of functional values to these nine vertices is 1,1,0,0,1,1,1,1,0 or 0,1,1,1,1,0,0,1,1, so that $f(V) = 6\ \ and$

$$\Gamma_{tu}(P_9) = 6 = \left\lfloor \frac{5n}{7} \right\rfloor = \left\lfloor \frac{45}{7} \right\rfloor.$$

Let $n \geq 16.$

As in Case 1 of this Theorem we have $\sum_{i=3}^{n} f(v_i) \leq \frac{5(n-2)}{7}.$

Since f is a minimal total unidominating function, the assignment of functional values to v_1, v_2isas follows.

Suppose $f(v_1) = 0, f(v_2) = 1$.

Then $f(V) = f(v_1) + f(v_2) + \sum_{i=3}^{n} f(v_i) \leq 0 + 1 + \frac{5(n-2)}{7} = \frac{5n-3}{7} = \left\lfloor \frac{5n}{7} \right\rfloor$.

Suppose $f(v_1) = 1, f(v_2) = 1$.

Then as in Case 2 we have

$$\sum_{i=3}^{n} f(v_i) = \sum_{i=3}^{n-7} f(v_i) + \sum_{i=n-6}^{n} f(v_i) \leq \frac{5(n-9)}{7} + 4$$

Therefore $f(V) = f(v_1) + f(v_2) + \sum_{i=3}^{n-7} f(v_i) + \sum_{i=n-6}^{n} f(v_i)$

$$\leq 1 + 1 + \frac{5(n-9)}{7} + 4 = \frac{5n-3}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Since f is arbitrary, it follows that $\Gamma_{tu}(P_n) \leq \left\lfloor \frac{5n}{7} \right\rfloor - - - (2)$

Thus from the inequalities (1) and (2), we have$\Gamma_{tu}(P_n) = \left\lfloor \frac{5n}{7} \right\rfloor$.

Case 4: Let $n \equiv 3 (mod\ 7)$.

Define a function $f: V \to \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & for\ i \equiv 2,3,4,5,6 (mod\ 7), \\ 0 & for\ i \equiv 0,1 (mod\ 7). \end{cases}$$

On similar linesto Case 1 we can verifythat f is a minimal total unidominating function.

$$\text{Now} \sum_{u \in V} f(u) = \underbrace{0+1+1+1+1+1+0} + \cdots + \underbrace{0+1+1+1+1+1+0} +$$

$$\underbrace{0+1+1} = 5\left(\frac{n-3}{7}\right) + 2 = \frac{5n-1}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Therefore $\Gamma_{tu}(P_n) \geq \left\lfloor \frac{5n}{7} \right\rfloor - - - (1)$

Let f be a minimal total unidominating function.

Suppose $n = 3$. Then the possible assignment of functional values to these three vertices is 1,1,0 or 0,1,1 so that $f(V) = 2$ *and* $\Gamma_{tu}(P_3) = 2 = \left\lfloor \frac{5n}{7} \right\rfloor = \left\lfloor \frac{15}{7} \right\rfloor$.

Let $n \geq 10$.

Now$n \equiv 3(mod\ 7) \Rightarrow \mathrm{n} - 3 \equiv 0(mod\ 7)$.

So by Case 1 we have $\sum_{i=1}^{n-3} f(v_i) \leq \frac{5(n-3)}{7}$.

Then for the vertices v_{n-2}, v_{n-1}, v_n we have $\sum_{i=n-2}^{n} f(v_i) = 2$.

$$\text{There fore } f(V) = \sum_{u \in V} f(u) = \sum_{i=1}^{n-3} f(v_i) + \sum_{i=n-2}^{n} f(v_i) \leq \frac{5(n-3)}{7} + 2 \leq \frac{5n-1}{7} \leq \left\lfloor \frac{5n}{7} \right\rfloor.$$

Since f is arbitrary, it follows that $\Gamma_{tu}(P_n) \leq \left\lfloor \frac{5n}{7} \right\rfloor - - - (2)$

Therefore from the inequalities (1) and (2), we have $\Gamma_{tu}(P_n) = \left\lfloor \frac{5n}{7} \right\rfloor$.

Case5: Let $n \equiv 4(mod\ 7)$.

Define a function $f: V \rightarrow \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & for\ i \equiv 2,3,4,5,6(mod\ 7), i \neq n, \\ 0 & for\ i \equiv 0,1(mod\ 7). \end{cases}$$

and $f(v_n) = 0$.

On similar lines to Case 1 we can show that f is aminimal total unidominating function.

$$\text{Now } \sum_{u \in V} f(u) = \underbrace{0+1+1+1+1+1+0}+\cdots+\underbrace{0+1+1+1+1+1+0}+ \underbrace{0+1+1+0} = \frac{5(n-4)}{7} + 2 = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Therefore $\Gamma_{tu}(P_n) \geq \left\lfloor \frac{5n}{7} \right\rfloor - - - (1)$

Let f be a minimal total unidominating function.

Suppose $n = 4$. Then the possible assignment of functional values to these four vertices is 0,1,1,0, so that $f(V) = 2$ and $\Gamma_{tu}(P_4) = 2 = \left\lfloor \frac{5n}{7} \right\rfloor = \left\lfloor \frac{20}{7} \right\rfloor$.

Let $n \geq 11$.

As in Case 1 of this Theorem we have $\sum_{i=2}^{n-3} f(v_i) \leq \frac{5(n-4)}{7}$.

Similar to Case 3 for the vertices v_{n-2}, v_{n-1}, v_n we have $\sum_{i=n-2}^{n} f(v_i) = 2$.

Now the functional value to v_1 is assigned as follows.

Suppose $f(v_1) = 0$.

$$\text{Then } f(V) = f(v_1) + \sum_{i=2}^{n-3} f(v_i) + \sum_{i=n-2}^{n} f(v_i) \leq 0 + \frac{5(n-4)}{7} + 2 = \frac{5n-6}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Suppose $f(v_1) = 1$.

Then as in Case 2 we have

$$f(V) = f(v_1) + \sum_{i=2}^{n-10} f(v_i) + \sum_{i=n-9}^{n-3} f(v_i) + \sum_{i=n-2}^{n} f(v_i)$$

$$\leq 1 + \frac{5(n-11)}{7} + 4 + 2 = \frac{5n-6}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Since f is arbitrary, it follows that $\Gamma_{tu}(P_n) \leq \left\lfloor \frac{5n}{7} \right\rfloor$ − − − (2)

From the inequalities (1) and (2), we have $\Gamma_{tu}(P_n) = \left\lfloor \frac{5n}{7} \right\rfloor$.

Case 6: Let $n \equiv 5(mod\ 7)$.

Define a function $f: V \rightarrow \{0, 1\}$ by

$$f(v_i) = \begin{cases} 1 & for\ i \equiv 2,3,4,5,6(mod\ 7), i \neq n, \\ 0 & for\ i \equiv 0,1(mod\ 7). \end{cases}$$

and $f(v_n) = 0$.

Then on similar lines to Case 1 we can show that f is a minimal total unidominating function.

Now $\sum_{u\in V} f(u) = \underbrace{0+1+1+1+1+1+0}+\cdots+\underbrace{0+1+1+1+1+1+0}+$

$$\underbrace{0+1+1+1+0} = 5\left(\frac{n-5}{7}\right)+3 = \frac{5n-4}{7} = \left\lfloor\frac{5n}{7}\right\rfloor.$$

Therefore $\Gamma_{tu}(P_n) \geq \left\lfloor\frac{5n}{7}\right\rfloor$ $---(1)$

Let f be a minimal total unidominating function.

Suppose $n = 5$.

Then thefunctional values to these five vertices can be assigned as $0,1,1,1,0,$ so that

$$f(V) = 3 \text{ and } \Gamma_{tu}(P_5) = 3 = \left\lfloor\frac{5n}{7}\right\rfloor = \left\lfloor\frac{25}{7}\right\rfloor.$$

Let $n \geq 12$.

As in Case 1 of this theorem we have $\sum_{i=3}^{n-3} f(v_i) \leq \frac{5(n-5)}{7}$.

As in Case 3 for the vertices v_{n-2}, v_{n-1}, v_n we have $\sum_{i=n-2}^{n} f(v_i) = 2$.

Since f is a minimal total unidominating function, the assignment of functional values to $v_1, v_2$is as follows.

Suppose $f(v_1) = 0, f(v_2) = 1$.

Then $f(V) = f(v_1) + f(v_2) + \sum_{i=3}^{n-3} f(v_i) + \sum_{i=n-2}^{n} f(v_i)$

$$\leq 0 + 1 + \frac{5(n-5)}{7} + 2 = \frac{5\text{n}-4}{7} = \left\lfloor\frac{5\text{n}}{7}\right\rfloor.$$

Suppose $f(v_1) = 1, f(v_2) = 1$.

Then as in Case 2 we have

$$\sum_{i=3}^{n-3} f(v_i) = \sum_{i=3}^{n-10} f(v_i) + \sum_{i=n-9}^{n-3} f(v_i) \leq \frac{5(n-12)}{7} + 4.$$

Therefore $f(V) = f(v_1) + f(v_2) + \sum_{i=3}^{n-10} f(v_i) + \sum_{i=n-9}^{n-3} f(v_i) + \sum_{i=n-2}^{n} f(v_i)$

$$\leq 1 + 1 + \frac{5(n-12)}{7} + 4 + 2 = \frac{5(n-12)}{7} + 8 = \frac{5n-4}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Since f is arbitrary, it follows that $\Gamma_{tu}(P_n) \leq \left\lfloor \frac{5n}{7} \right\rfloor$ – – – (2)

Thus from the inequalities (1) and (2), we have $\Gamma_{tu}(P_n) = \left\lfloor \frac{5n}{7} \right\rfloor$.

Case 7: Let $n \equiv 6(mod\ 7)$.

Define a function$f: V \to \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & for\ i \equiv 2,3,4,5,6(mod\ 7), i \neq n, \\ 0 & for\ \ i \equiv 0,1(mod\ 7). \end{cases}$$

and $f(v_n) = 0$.

On similar lines to Case 1 we can verify that f is a minimal total unidominating function.

Now $\sum_{u \in V} f(u) = \underbrace{0+1+1+1+1+1+0} + \cdots + \underbrace{0+1+1+1+1+1+0} +$

$$\underbrace{0+1+1+1+1+1+0} = 5\left(\frac{n-6}{7}\right) + 4 = \frac{5n-2}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Therefore $\Gamma_{tu}(P_n) \geq \left\lfloor \frac{5n}{7} \right\rfloor$ – – – (1)

Let f be a minimal total unidominating function.

Suppose $n = 6$. Then the possibilities of assigning functional values to these six vertices are 0,1,1,1,1,0 or 1,1,0,0,1,1, so that $f(V) = 4$ and

$\Gamma_{tu}(P_6) = 4 = \left\lfloor \frac{5n}{7} \right\rfloor = \left\lfloor \frac{30}{7} \right\rfloor$.

Let $n \geq 13$.

If f is any minimal total unidominating function, then the functional values of first three vertices and the last three vertices must satisfy the following conditions.

$\sum_{i=1}^{3} f(v_i) = 2\ \ and\ \ \sum_{i=n-2}^{n} f(v_i) = 2.$

Now $n \equiv 6(mod\ 7) \Rightarrow n - 6 \equiv 0(mod\ 7)$. Then as per the discussion in Case 1,

we have $\sum_{i=4}^{n-3} f(v_i) \leq \frac{5(n-6)}{7}$.

Therefore $f(V) = \sum_{u \in V} f(u) = \sum_{i=1}^{3} f(v_i) + \sum_{i=4}^{n-3} f(v_i) + \sum_{i=n-2}^{n} f(v_i)$

$$\leq 2 + \frac{5(n-6)}{7} + 2 = \frac{5n-2}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

Since f is arbitrary, it follows that $\Gamma_{tu}(P_n) \leq \left\lfloor \frac{5n}{7} \right\rfloor$ $- - -$ (2)

Therefore from the inequalities (1) and (2), we have $\Gamma_{tu}(P_n) = \left\lfloor \frac{5n}{7} \right\rfloor$.

Thus for all possibilities of $n, n \neq 2$ we have $\Gamma_{tu}(P_n) = \left\lfloor \frac{5n}{7} \right\rfloor$ and

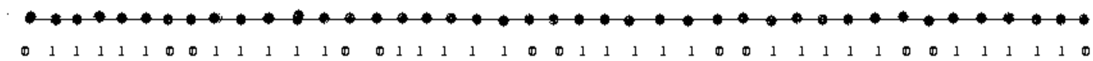
for $n = 2, \Gamma_{tu}(P_n) = 2$. ■

3. ILLUSTRATIONS

Example 3.1: Let $n = 42$.

We know that $42 \equiv 0(mod\ 7)$.

The functional values of a minimal total unidominating function f defined as in Case 1 of Theorem 2.1 for P_{42} are given at the corresponding vertices.

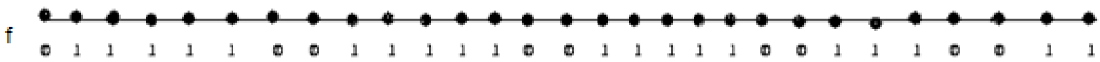


Upper total unidomination number $= \left\lfloor \frac{5 \times 42}{7} \right\rfloor = 30$. ■

Example 3.2: Let $n = 29$.

We know that $29 \equiv 1(mod\ 7)$.

The functional values of a minimal total unidominating function f defined as in Case 2 of Theorem 2.1 for P_{29} are given at the corresponding vertices.



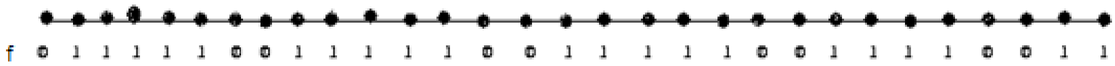
Upper total unidomination number ofP_{29} is $\left\lfloor\frac{5\ x\ 29}{7}\right\rfloor = 20.$

Example 3.3: .Let $n = 30.$

We know that 30$\equiv 2(mod\ 7).$

The functional values of a minimal total unidominating function f defined as in

Case 3 of Theorem 2.1 for P_{30} are given at the corresponding vertices.



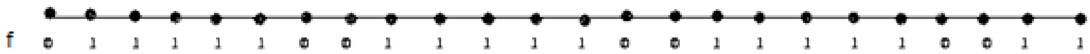
Upper total unidomination number of $P_{30}$is$\left\lfloor\frac{5\ x\ 30}{7}\right\rfloor = 21.$

Example 3.4: Let $n = 24.$

We know that $24 \equiv 3(mod\ 7).$

The functional values of a minimal total unidominating function f defined as in

Case 4 of Theorem 2.1 for P_{24} are given at the corresponding vertices.



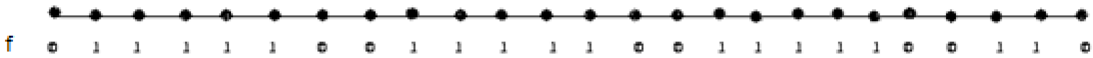
Upper total unidomination number of P_{24} is $\left\lfloor\frac{5\text{x}24}{7}\right\rfloor = \left\lfloor\frac{120}{7}\right\rfloor = 17.$

Example 3.5: Let $n = 25.$

We know that 25$\equiv 4(mod\ 7).$

The functional values of a minimal total unidominating function f defined as in

Case 5 of Theorem 2.1 for P_{25} are given at the corresponding vertices.



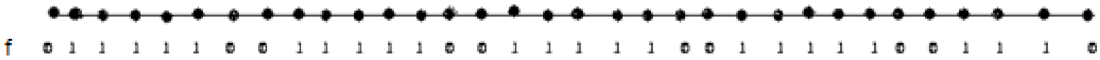
Upper total unidomination number is $\left\lfloor\frac{5\text{ x }25}{7}\right\rfloor = 17.$

Example 3.6: Let $n = 33.$

We know that $33 \equiv 5(mod\ 7).$

The functional values of a minimal total unidominating function f defined as in

Case 6 of Theorem 2.1 for P_{33} are given at the corresponding vertices.



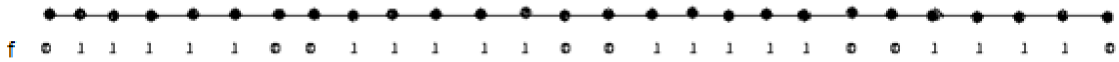
Upper total unidomination number is $\left\lfloor \frac{5\ x\ 33}{7} \right\rfloor = \left\lfloor \frac{165}{7} \right\rfloor = 23.$

Example 3.7: Let $n = 27$.

We know that $27 \equiv 6(mod\ 7)$.

The functional values of a minimal total unidominating function f defined as in

Case 7 of Theorem 2.1 for P_{27} are given at the corresponding vertices.



Upper total unidomination number is $\left\lfloor \frac{5\ x\ 27}{7} \right\rfloor = 19.$

4. REFERENCES

1. Allan, R.B.Laskar, R.C.Hedetniemi, S.T. *A note on total domination*, Discrete Math. 49(1984), 7 – 13.

2. Berge, C., *The Theory of Graphs and its Applications*, Methuen, London (1962).

3. Cockayne, E.J.Hedetniemi, S.T., *Towards a theory of domination in graphs*. Networks, 7(1977), 247 – 261.

4. Cockayne, E.J.Mynhardt, C.M.Yu, B., *Total dominating functions in trees: Minimality and Convexity*, Journal of Graph Theory, 19(1995), 83 – 92.

5. Hedetniemi, S.M.Hedetniemi, S.T.Wimer, T.V., *Linear time resource allocation algorithms for trees*. Technical report URI – 014, Department of Mathematics, Clemson University, (1987).

6. Ore, O., *Theory of Graphs*, Amer. Soc. Colloq. Publ. Vol.38. Amer. Math. Soc., Providence, RI, (1962).

7. Sampath Kumar, E., *On some new domination parameters of a graph* – A survey. Proceedings of a Symposium on Graph Theory and Combinatorics, Kochi, Kerala, India, 17 – 19 May (1991),7 – 13.

8. Anantha Lakshmi, V and Maheswari, B. – Total *Unidominating Functions of a Path,* IJCA, vol 126-No.13, (2015), 43-48.

9. V. Anantha Lakshmi, B. Maheswari - *Total Unidominating functions of a complete k-partite graph*: Open Journal of Applied & Theoretical Mathematics, Vol.2, No.4, December (2016) pp.795-805.